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Covariate-Adjusted Deep Causal Learning for Heterogeneous Panel Data Models

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KEY TAKE AWAY INFORMATION

CoDEAL estimates unit-specific **heterogenerous** treatment effects in panel data by learning **nonlinear, unit-specific factor loadings** with a **multi-output autoencoder**, with convergence guarantees for the imputed counterfactuals.

01 The Problem

Causal panel data is a missing-data problem: a treated unit's untreated outcome is never observed. Under staggered adoption the missing entries form blocks, so they are **missing not at random**.

Existing causal matrix completion methods typically

- assume entries are missing at random (MAR),
- target the overall ATT rather than unit-specific effects,
- rely on linear factors and cannot separate nonlinear covariate effects.

02 Model: Non-linear Factor Model

$$Y_{it} = \phi_i^\star(\mathbf{F}_t^\star) + g_t^\star(\mathbf{X}_i) + \tau_i^\star \cdot \mathbb{1}\{W_{it} = 1\} + \varepsilon_{it},$$

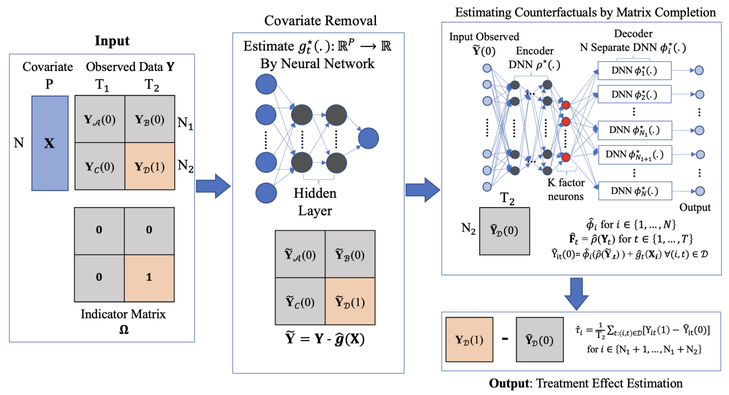
τ_i : unit-specific ATT (**Heterogeneous, static effects**), the target

$\phi_i(F_t)$: nonlinear loading of unit i on K latent factors

$g_t(X_i)$: nonlinear, time-varying covariate effect

N *(# units)*, T *(periods)* → ∞ with P *(# covariates)*, K *(latent factors)* fixed. Unifies linear factor-based causal matrix completion as special cases.

03 Method: CoDEAL



CoDEAL in the four-block design. Gray: observed; orange: missing counterfactuals to impute.

1. **Covariate adjustment.** Fit a deep ReLU network on untreated entries and subtract $\hat{g}_t(X_i)$.
2. **Nonlinear factors.** A multi-output autoencoder: a shared encoder ρ recovers the factors, and one decoder ϕ_i **per unit** learns its loading.

$$\min_{(\phi_1, \ldots, \phi_N) \circ \rho \in \mathcal{G}_{\mathrm{AE}}^{K_1}} \sum_{(i,t):\, W_{it} = 0} \left(\phi_i(\rho(\widetilde{\mathbf{Y}}_{\cdot t})) - \widetilde{Y}_{it}\right)^2$$

3. **Impute and estimate.** $\hat{Y}_{it}(0) = \hat{\phi}_i(\hat{\rho}(\tilde{Y}_{\cdot t})) + \hat{g}_t(X_i)$; $\hat{\tau}_i$ averages $Y_{it}(1) - \hat{Y}_{it}(0)$ over treated periods. Staggered adoption: repeat over four-block submatrices.

04 Theory: Estimation Consistency

Theorem 1. Under Hölder-β smoothness, sub-Gaussian noise and factor pervasiveness (n: encoder complexity),

$$\max_{N_1+1 \le i \le N} \frac{1}{T_2} \sum_{t=T_1+1}^{T} \mathbb{E}\left(\widehat{Y}_{it}(0) - \phi_i^\star(\mathbf{F}_t^\star) - g_t^\star(\mathbf{X}_i)\right)^2 = O_{\mathbb{P}}(\gamma),$$

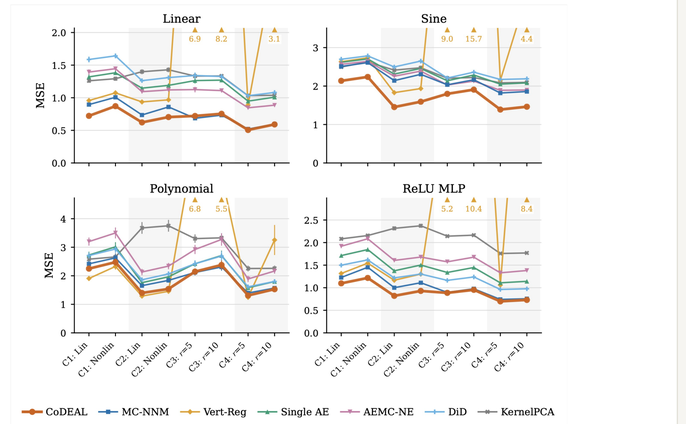
$$\gamma = \left(\frac{(N+n)T^{-\frac{2\beta}{2\beta+K}}}{n} + \frac{n}{T} + \frac{nK_1 + N}{n^2}\right) \log^4(NT) + N^{-\frac{2\beta}{2\beta+K}} \log^5 N.$$

When $\gamma = o(1)$, the unit-specific ATT estimator is consistent.

05 Simulations

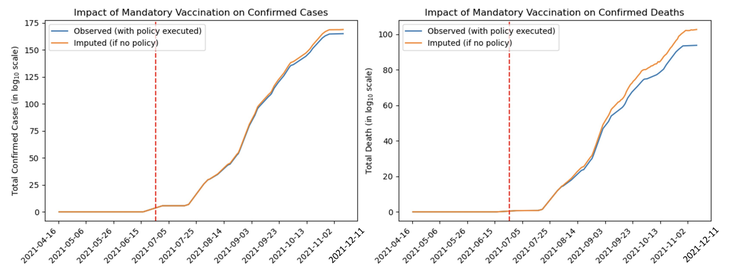
35–89%
lower MSE than benchmarks under staggered adoption with linear factors; ties MC-NNM within 5%

15–60%
lower MSE than every competing method under ReLU-based nonlinear factors



06 COVID-19 Vaccine Mandates

OxCGRT data, 64 states and territories in the US and Canada, Apr–Dec 2021 (N = 64, T = 240, P = 4).



Observed (mandate in effect) vs. CoDEAL counterfactual (no mandate), $\log_{10}$ scale, 14-day moving average.

+12.2%
confirmed cases without the mandate (29.81M vs. 26.57M observed)

+34.6%
deaths without the mandate (0.52M vs. 0.39M observed)

Next: dynamic and multiple treatments, and confidence intervals for deep causal estimators.