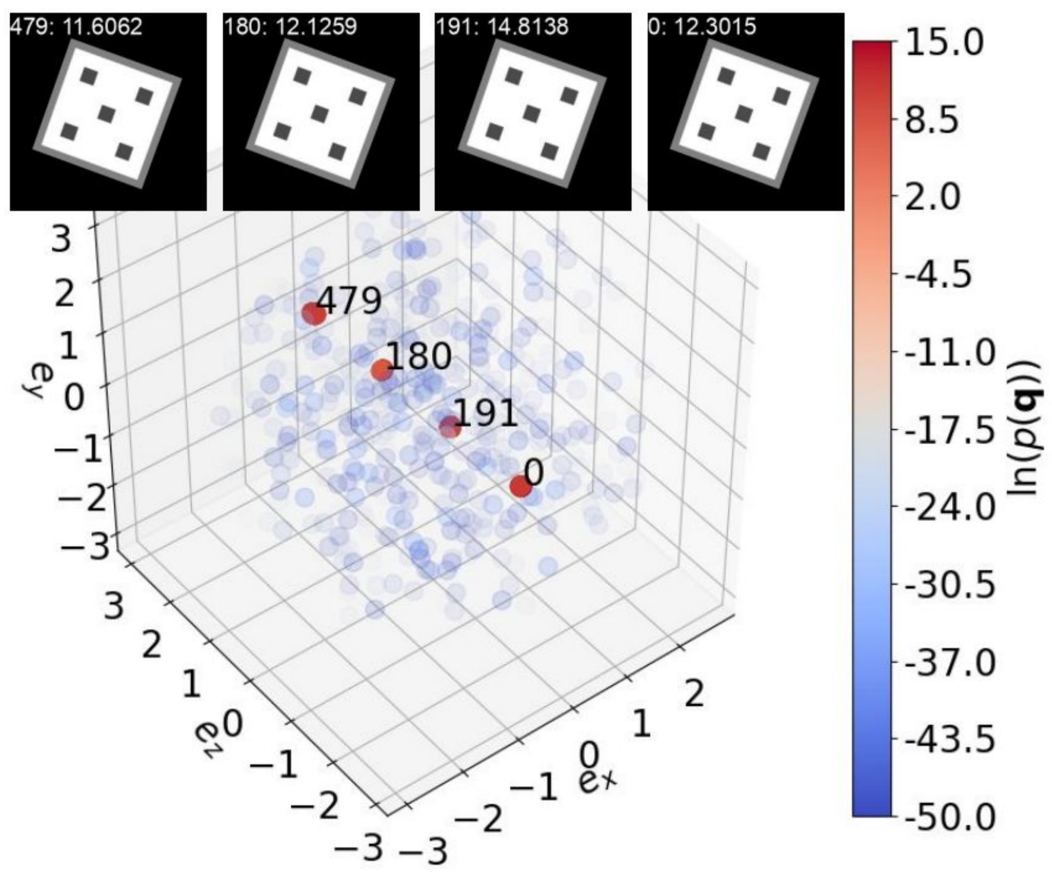
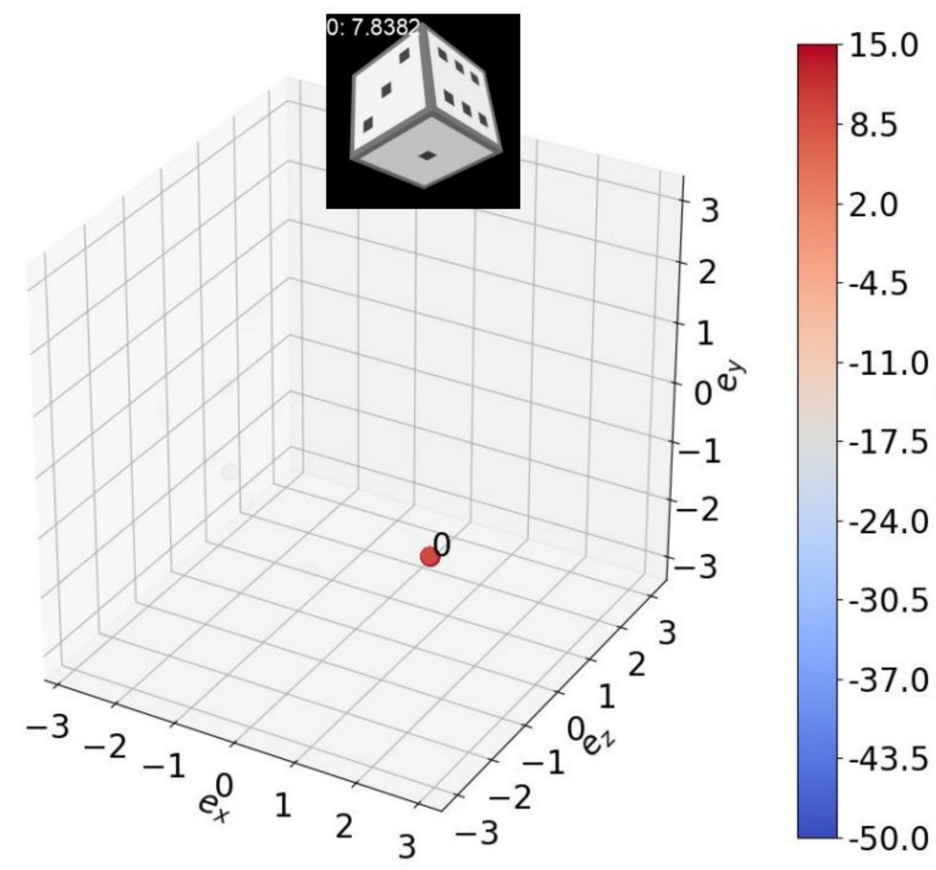
AQuaMaM: An Autoregressive, Quaternion Manifold Model for Rapidly Estimating Complex **SO**(3) Distributions

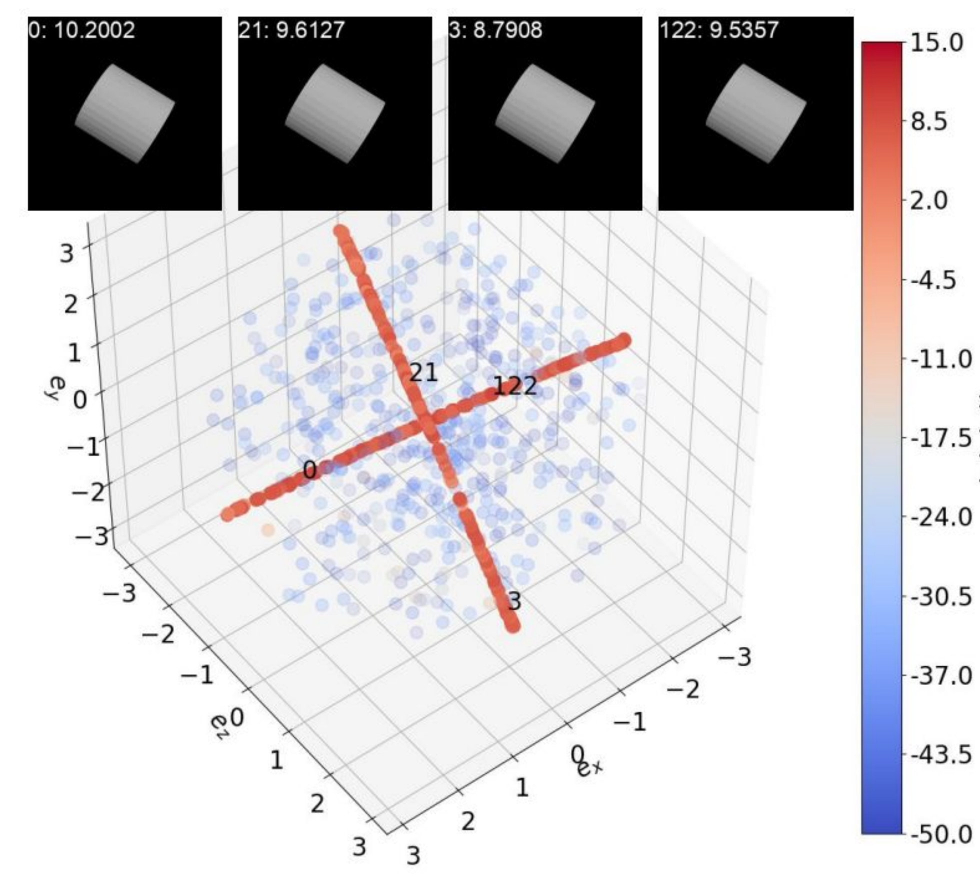
Michael A. Alcorn



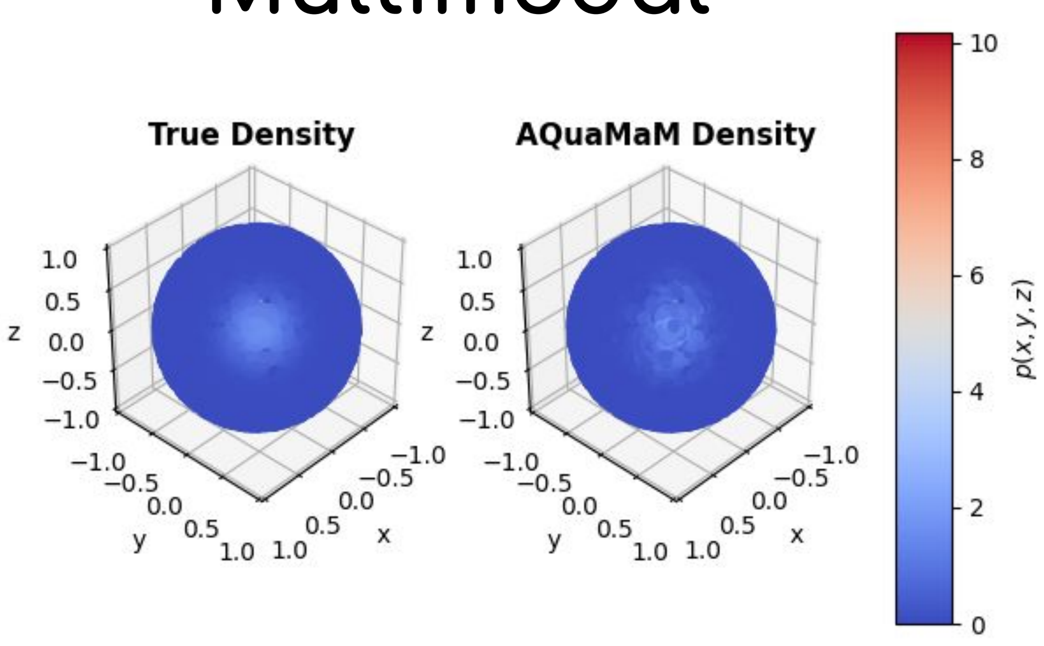
Ambiguous/ Multimodal

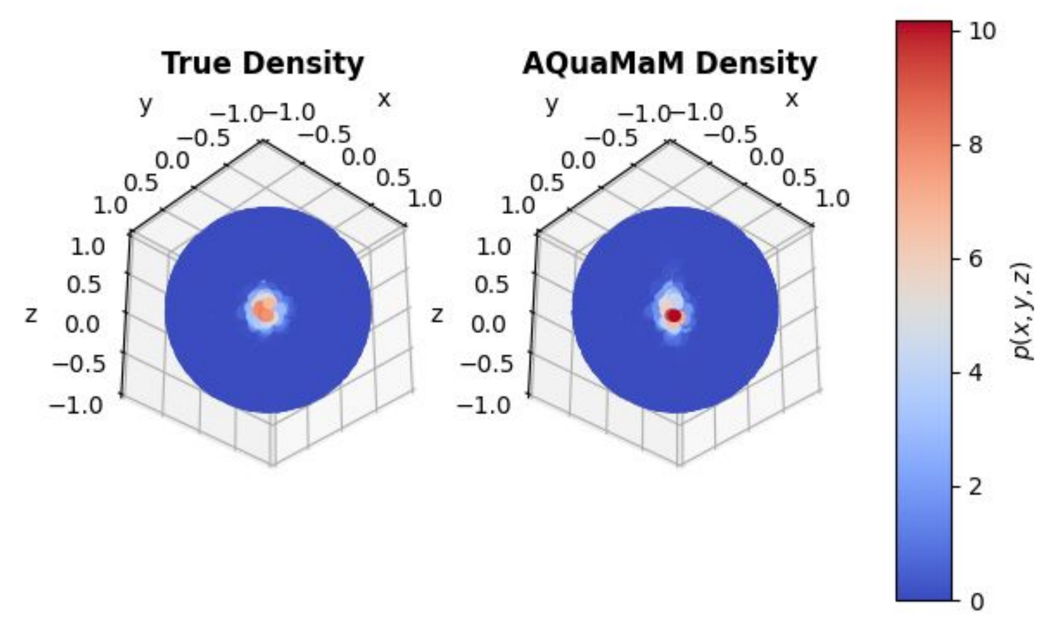


Unambiguous/ Peak



Continuous Symmetry

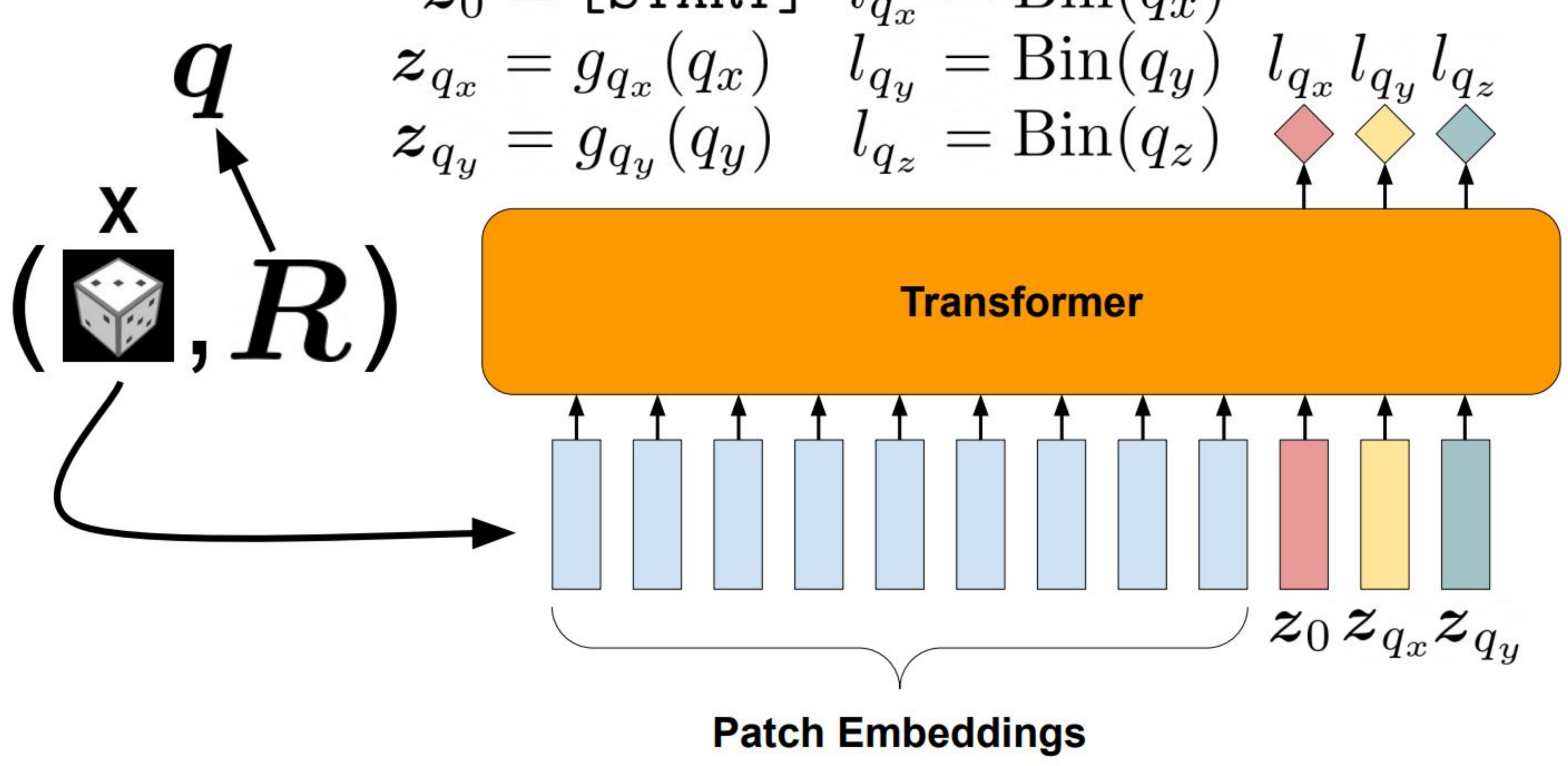


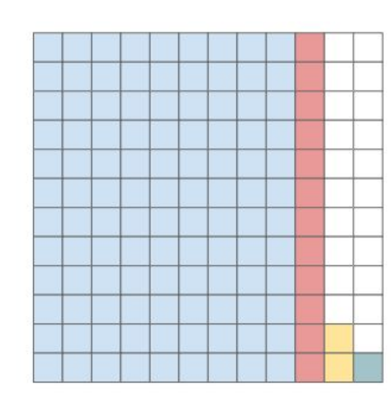


Mixture of two von Mises–Fisher distributions on a sphere

Architecture: ViT + "Quaternion Language Model"

$$\boldsymbol{z}_0 = \texttt{[START]} \quad l_{q_x} = \text{Bin}(q_x)$$
$$\boldsymbol{z}_{q_x} = g_{q_x}(q_x) \quad l_{q_y} = \text{Bin}(q_y)$$
$$\boldsymbol{z}_{q_y} = g_{q_y}(q_y) \quad l_{q_z} = \text{Bin}(q_z)$$



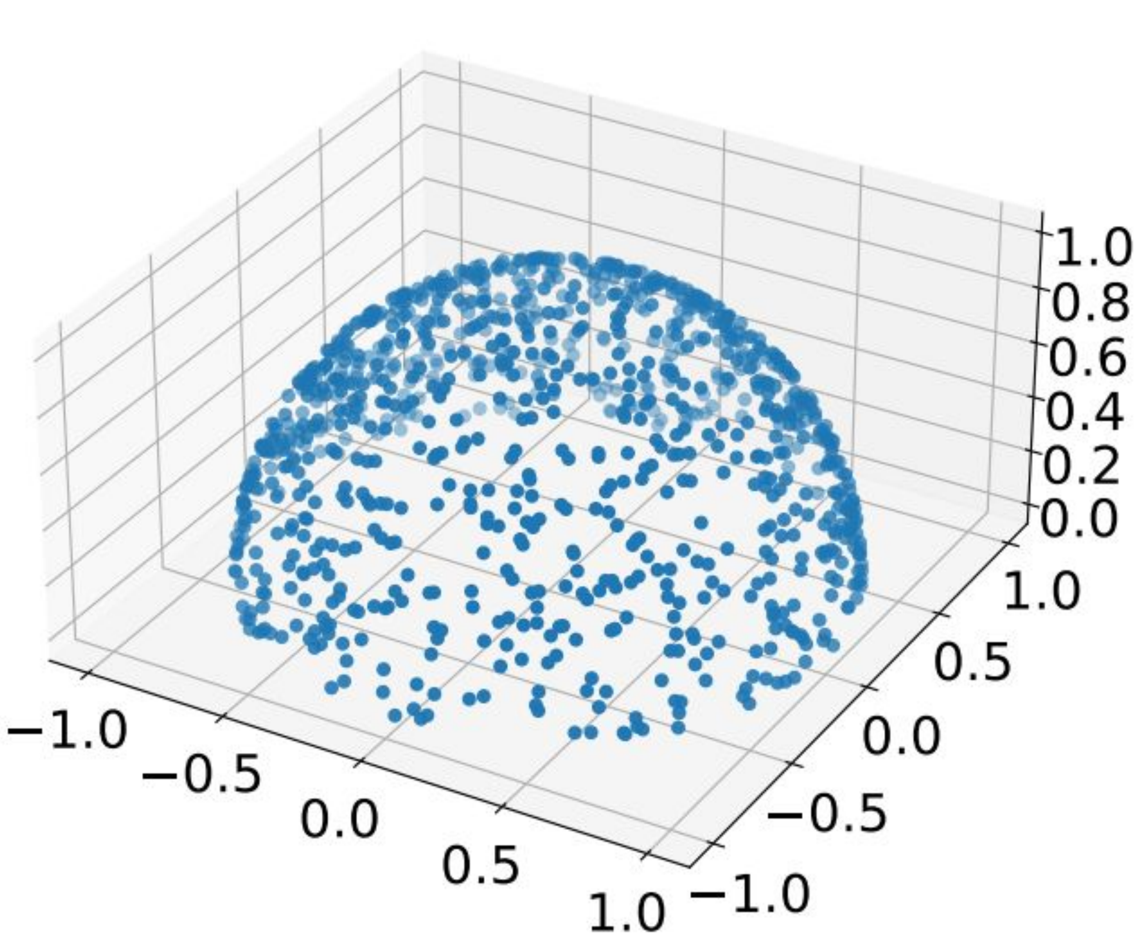


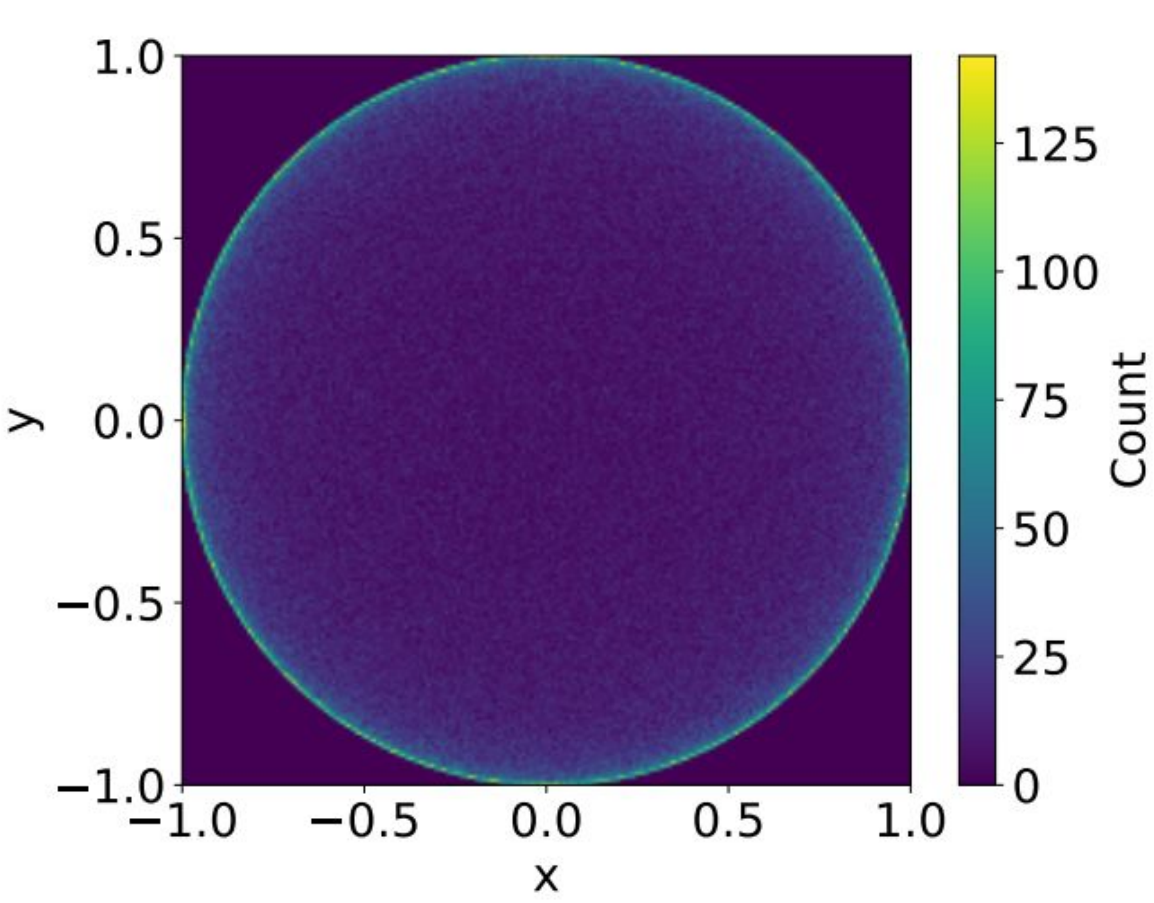
Attention Mask

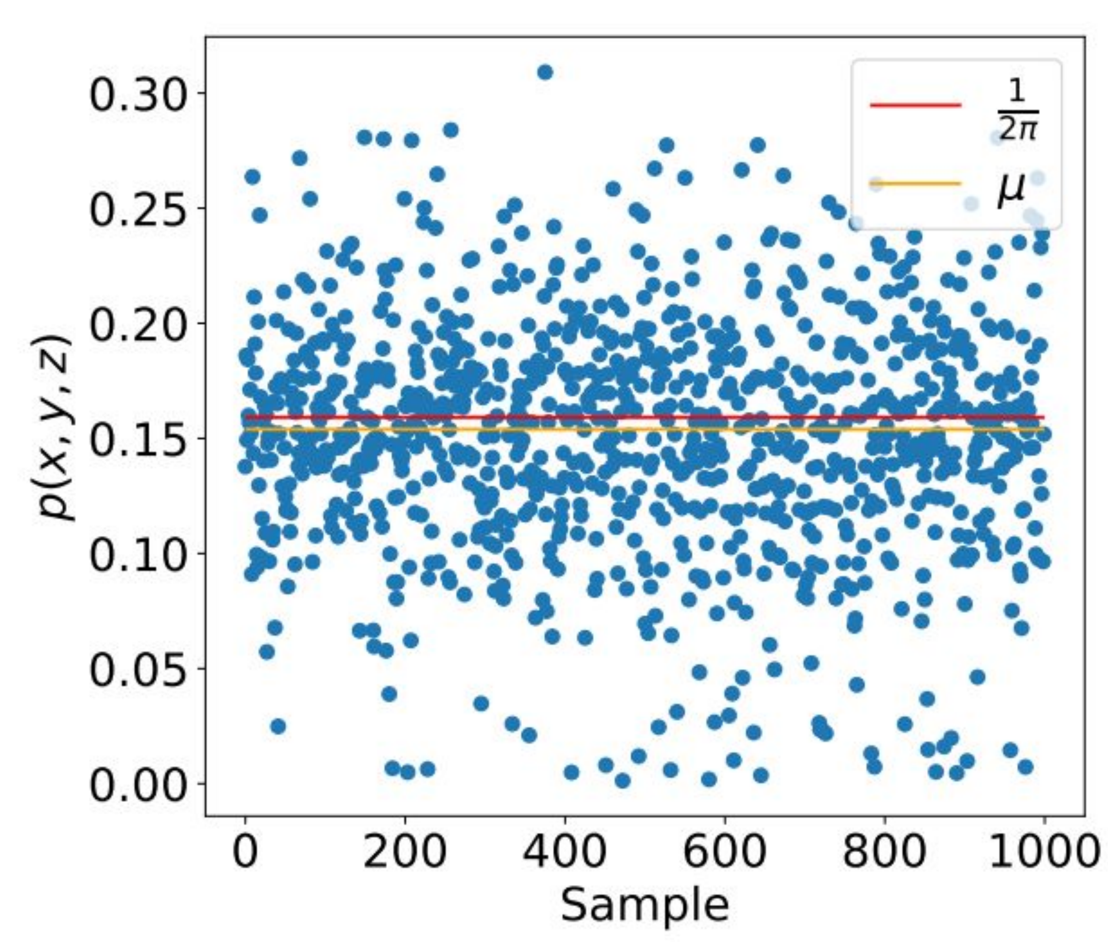
$$p(q_x, q_y, q_z) = \pi_{q_x}\frac{N}{2}\pi_{q_y}\frac{1}{\omega_{q_y}}\pi_{q_z}\frac{1}{\omega_{q_z}}$$
$$= \pi_{q_x}\pi_{q_y}\pi_{q_z}\frac{N}{2\omega_{q_y}\omega_{q_z}}$$

$$\mathcal{L} = -\sum_{d=1}^{|\mathcal{X}|} \ln \pi_{q_{d,x}} + \ln \pi_{q_{d,y}} + \ln \pi_{q_{d,z}} + \ln \frac{N q_{d,w}}{2\omega_{q_{d,y}}\omega_{q_{d,z}}}$$

Curved Hemisphere → Flat Ball (2D)







$$f(x, y) = [x, y, z]$$
$$z = \sqrt{1 - x^2 - y^2}$$

$$\boldsymbol{J} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{-x}{z} & \frac{-y}{z} \end{bmatrix}$$

$$a = \sqrt{\begin{vmatrix} 0 & 1 \\ \frac{-x}{z} & \frac{-y}{z} \end{vmatrix}^2 + \begin{vmatrix} 1 & 0 \\ \frac{-x}{z} & \frac{-y}{z} \end{vmatrix}^2 + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}^2}$$
$$= \sqrt{\frac{x^2}{z^2} + \frac{y^2}{z^2} + 1} = \sqrt{\frac{x^2 + y^2 + z^2}{z^2}} = \frac{1}{z}$$

$$p(x, y, z) = \frac{p(x, y)}{a}$$
$$= p(x, y)z$$
$$= p(x)p(y|x)z$$