

# Path-specific effects for pulse-oximetry guided decisions in critical care

NeurIPS 2025

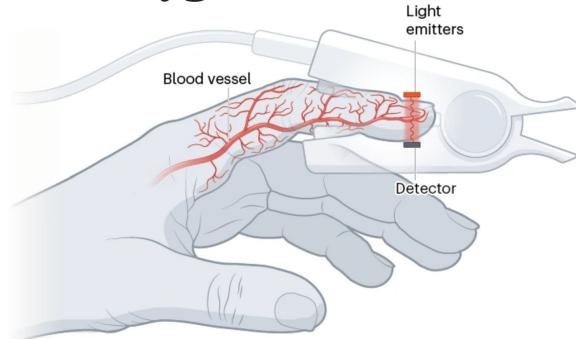
Kevin Zhang, Yonghan Jung, Divyat Mahajan,  
Karthikeyan Shanmugam, Shalmali Joshi

# Evidence of racial bias in pulse oximeter device measurements

**nature**

NEWS & VIEWS FORUM | 19 October 2022

## Skin colour affects the accuracy of medical oxygen sensors



**npr**

**COVID-19 made pulse oximeters ubiquitous. Engineers are fixing their racial bias**

Original Investigation | Pulmonary Medicine

## Analysis of Discrepancies Between Pulse Oximetry and Arterial Oxygen Saturation Measurements by Race and Ethnicity and Association With Organ Dysfunction and Mortality

An-Kwok Ian Wong, MD, PhD<sup>1,2</sup>; Marie Charpignon, MS<sup>3</sup>; Han Kim, MSE<sup>4</sup>; [et al](#)

Original Investigation

## Assessment of Racial and Ethnic Differences in Oxygen Supplementation Among Patients in the Intensive Care Unit

Eric Raphael Gottlieb, MD, MS<sup>1,2,3</sup>; Jennifer Ziegler, MD, MSc<sup>4</sup>; Katharine Morley, MD, MPH<sup>2,5</sup>; [et al](#)

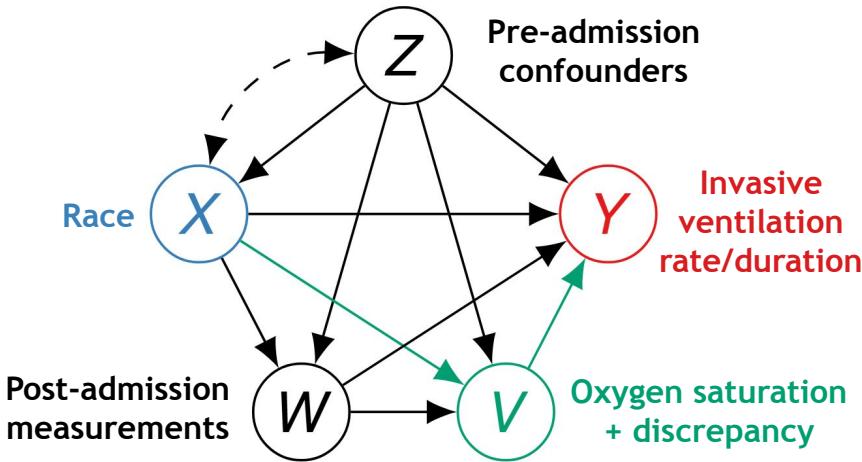
Original Investigation

## Racial and Ethnic Discrepancy in Pulse Oximetry and Delayed Identification of Treatment Eligibility Among Patients With COVID-19

Ashraf Fawzy, MD, MPH<sup>1</sup>; Tianshi David Wu, MD, MHS<sup>2,3</sup>; Kunbo Wang, MS<sup>4</sup>; [et al](#)

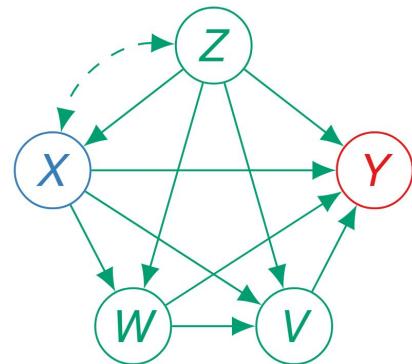
# Quantifying path-specific causal effect of pulse-oximeter discrepancies on clinical decision-making in the ICU

1. Extend existing causal mediation analysis for multiple mediators
2. Develop robust estimators with theoretical guarantees and better performance in small-sample regimes (e.g. ICU data)



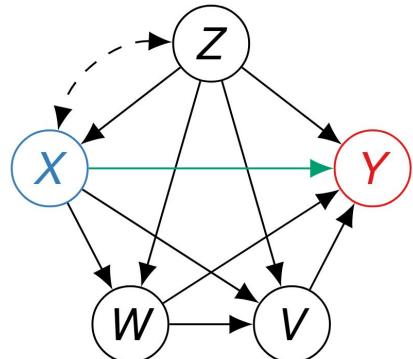
Study	Causal Analysis	Path-Spec. Analysis	Multi-Med.	ICU Data	Finite Guar.
Sjoding, 2020	Assoc.	✗	✗	✓	✗
Wong, 2021	Assoc.	✗	✗	✓	✗
Gottlieb, 2022	Part.	Part.	✗	✓	✗
Miles, 2017	✓	✓	✓	✗	✗
Tchetgen, 2012	✓	✓	✗	✗	✗
VanderWeele, 2014	✓	✓	✗	✗	✗
<b>Ours</b>	✓	✓	✓	✓	✓

# Causal fairness model with two mediators



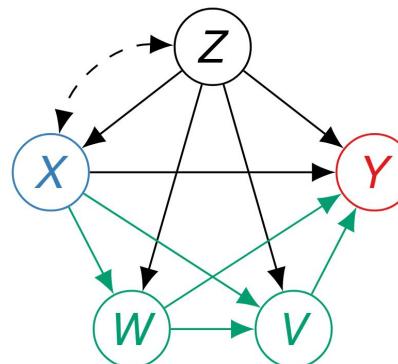
Total Effect (TE)

$$\mathbb{E}[Y_{x_1}] - \mathbb{E}[Y_{x_0}]$$



Natural Direct Effect (NDE)

$$\mathbb{E}[Y_{x_1, W_{x_0}, V_{x_0}}] - \mathbb{E}[Y_{x_0}]$$

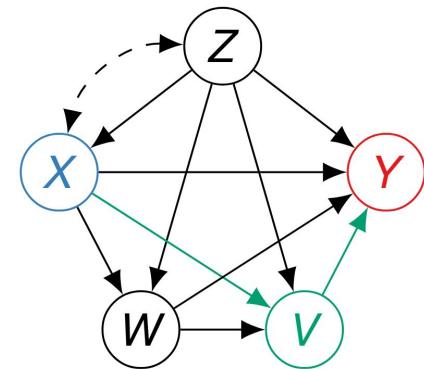


Natural Indirect Effect (NIE)

$$\mathbb{E}[Y_{x_1}] - \mathbb{E}[Y_{x_1, W_{x_0}, V_{x_0}}]$$

$$\mathbb{E}[Y_{x_1}] - \mathbb{E}[Y_{x_1, V_{x_0}, W_{x_1}}]$$

V-specific Direct Effect (VDE)



Pulse oximetry application

Pearl, UAI, 2001

## VDE doubly robust estimator

The VDE is identifiable using the back-door adjustment and given by

$$\mathbb{E}[Y_{x_1, V_{x_0, W_{x_1}}}] = \sum_{w, v, z} \mathbb{E}[Y \mid x_1, w, v, z] P(v \mid x_0, w, z) P(w \mid x_1, z) P(z). \quad (1)$$

We use the doubly robust estimator,

$$\begin{aligned} \varphi((Y, V, W, X, Z); \boldsymbol{\mu}_0, \boldsymbol{\pi}_0) &\triangleq \boldsymbol{\pi}_0^3(V, W, X, Z) \{Y - \boldsymbol{\mu}_0^3(V, W, X, Z)\} \\ &\quad + \boldsymbol{\pi}_0^2(W, X, Z) \{\boldsymbol{\mu}_0^3(V, W, x_1, Z) - \boldsymbol{\mu}_0^2(W, X, Z)\} \\ &\quad + \boldsymbol{\pi}_0^1(X, Z) \{\boldsymbol{\mu}_0^2(W, x_0, Z) - \boldsymbol{\mu}_0^1(X, Z)\} + \boldsymbol{\mu}_0^1(x_1, Z). \end{aligned}$$

$$\mathbb{E}[\varphi((Y, V, W, X, Z); \boldsymbol{\mu}_0, \boldsymbol{\pi}_0)] = \text{Eq. (1)}$$



Nuisance parameters defined following Jung, *NeurIPS*, 2024

# Theoretical guarantees of the VDE estimator

## Algorithm

### 1. (Sample-Splitting)

Take any  $L$ -fold partition of the data  
 $\mathcal{D} = \cup_{\ell=1}^L \mathcal{D}_\ell$ .

### 2. (Learning by Partitions)

Learn  $\hat{\mu}_\ell^i, \hat{\pi}_\ell^i$  using  $\mathcal{D} \setminus \mathcal{D}_\ell$  and compute  
 $\hat{\psi}_\ell \triangleq \mathbb{E}_{\mathcal{D}_\ell}[\varphi(Y, V, W, X, Z); \hat{\mu}, \hat{\pi}]$ .

### 3. (Aggregation)

Construct  $\hat{\psi} \triangleq \frac{1}{L}(\hat{\psi}_1 + \dots, \hat{\psi}_L)$ .

## Theorem 2 (informal)

Suppose the second and third moments of  $\varphi$  exist.

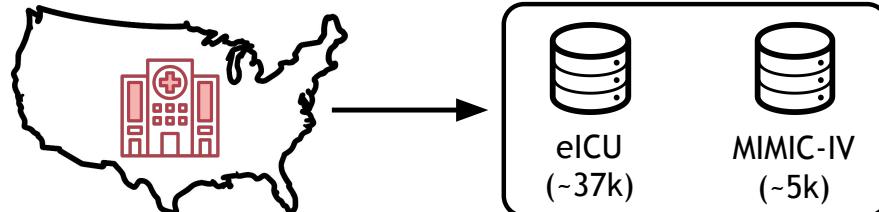
Let  $R_1 \triangleq \frac{1}{L} \sum_{\ell=1}^L (\mathbb{E}_{\mathcal{D}_\ell}[\hat{\varphi}_\ell] - \mathbb{E}_P[\varphi_0])$ . Then,

$$\hat{\psi} - \psi_0 = R_1 + \frac{1}{L} \sum_{\ell=1}^L \sum_{i=1}^3 \mathbb{E}[\{\hat{\mu}_\ell^i - \mu_0^i\} \{\pi_0^i - \hat{\pi}_\ell^i\}]$$

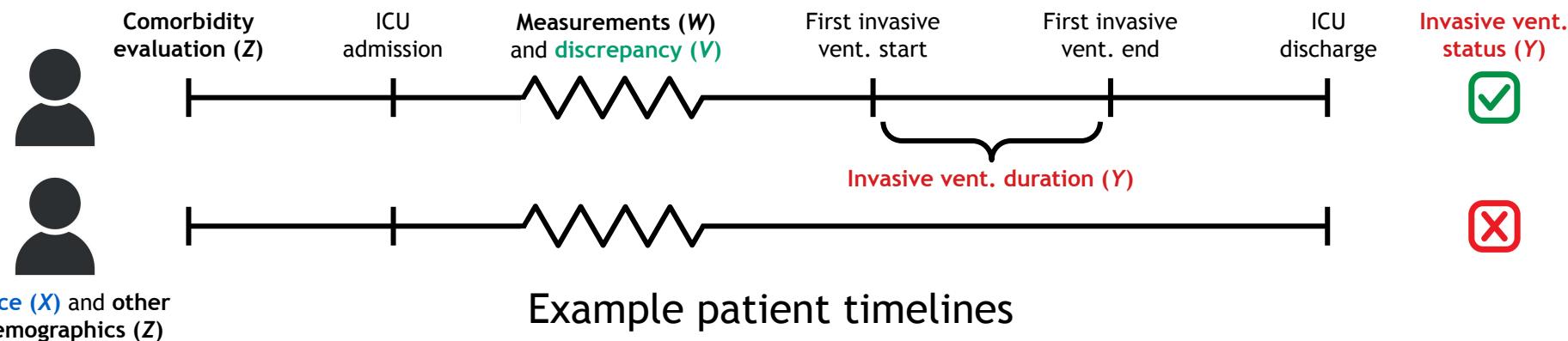
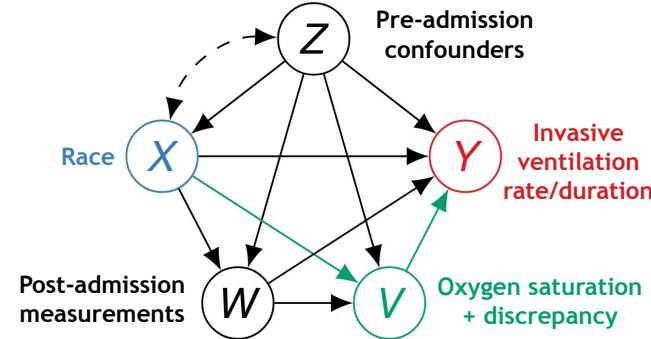
Moreover,  $R_1 \xrightarrow{p} 0$  and  $R_1$  is asymptotically normal.

Since  $\mathbb{E}[\pi_0^i] = 1$ , we use the self-normalized (SN) estimator,  $\hat{\pi}_{\text{SN}}^i \leftarrow \hat{\pi}^i / \mathbb{E}_{\mathcal{D}}[\hat{\pi}^i]$ , as this improves the estimation stability in practice.

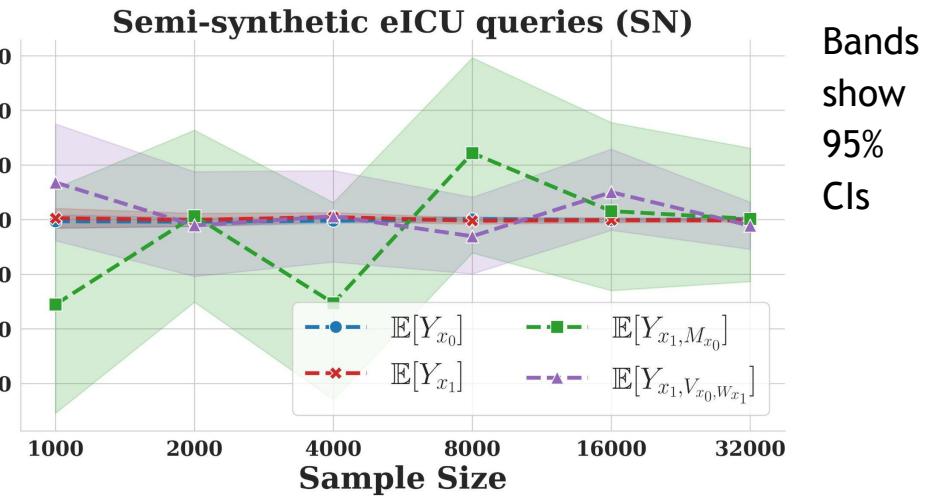
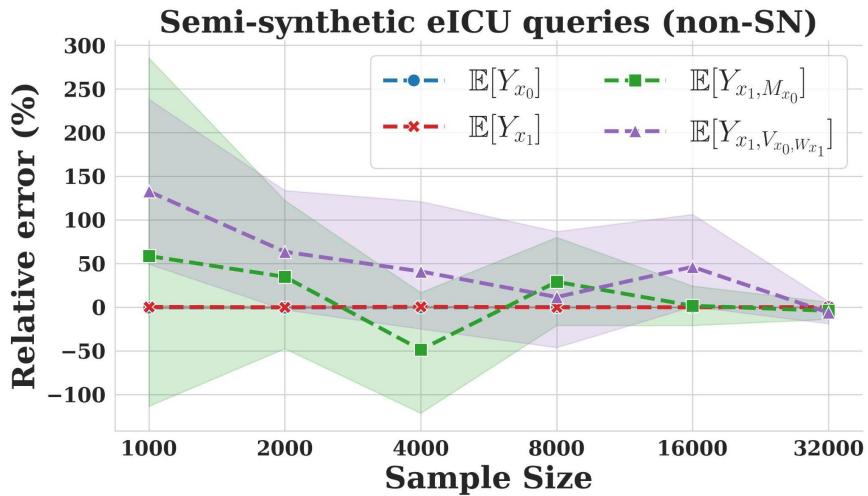
# Critical care data on invasive ventilation-related outcomes



EWA to summarize measurement trajectories



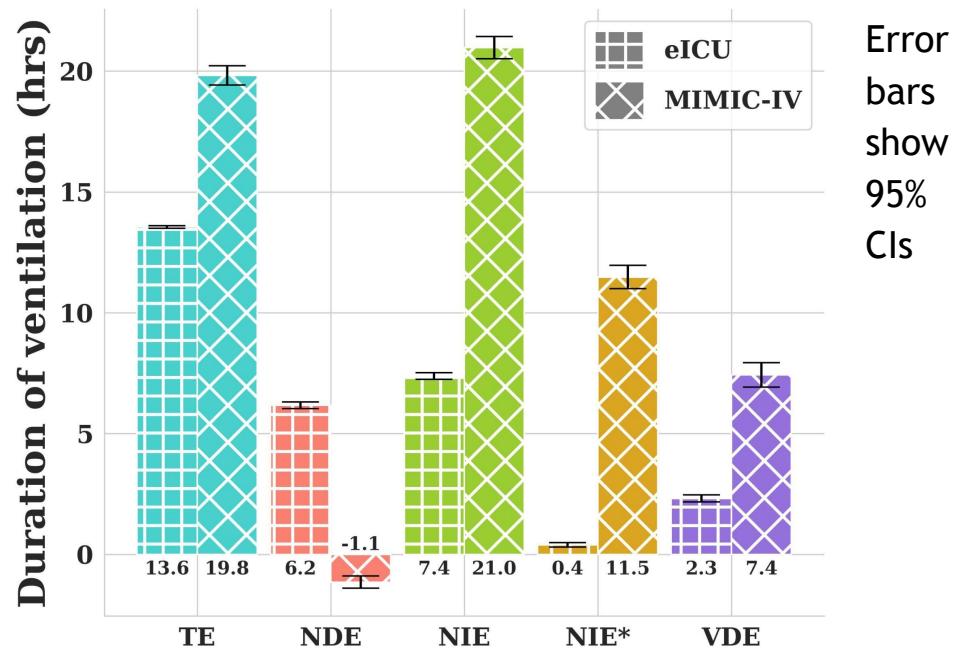
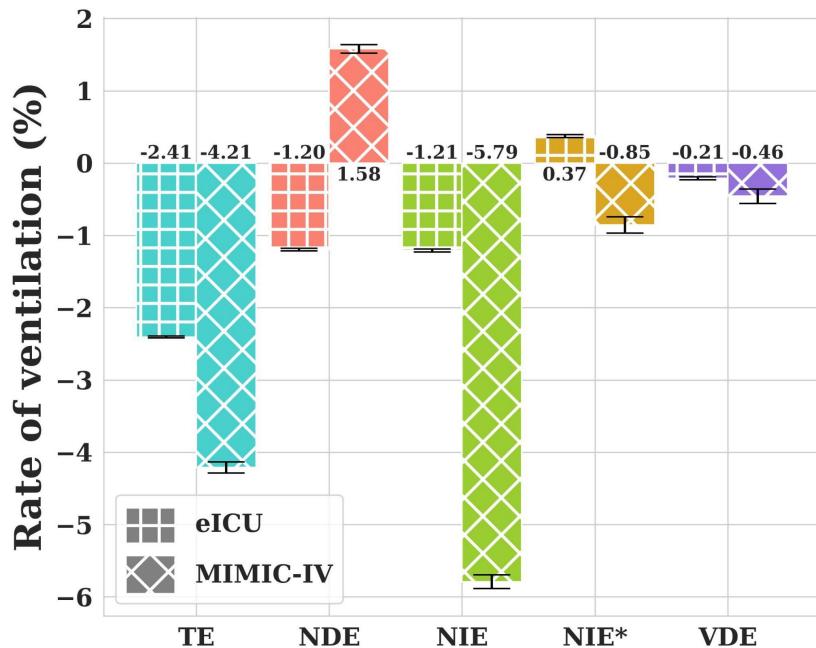
# Semi-synthetic experiments



The self-normalized estimator (right) has smaller variance compared to the canonical doubly robust estimator (left)

# Real-world experiments

Positive effect = more frequent or longer treatment for Black patients vs. White patients



Ventilation rate VDE is negligible (95% CI eICU [-0.23, -0.19], MIMIC-IV [-0.56, -0.36]), but more pronounced for ventilation duration (95% CI eICU [2.2, 2.5], MIMIC-IV [6.9, 7.9])

## Conclusions and future work

- We examine heterogeneity by race in a clinically actionable healthcare process using path-specific causal analysis with theoretical guarantees
- Bias primarily manifests in ventilation duration, not in treatment initiation
- Future work could model temporality explicitly



**In-person poster session @ San Diego**

Thu 4 Dec, 11am-2pm PST, Exhibit Hall C/D/E

# References

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