Maximizing utility in multi-agent environments by anticipating the behavior of other learners

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Agents in strategic environments have to make sequential decisions over a time horizon

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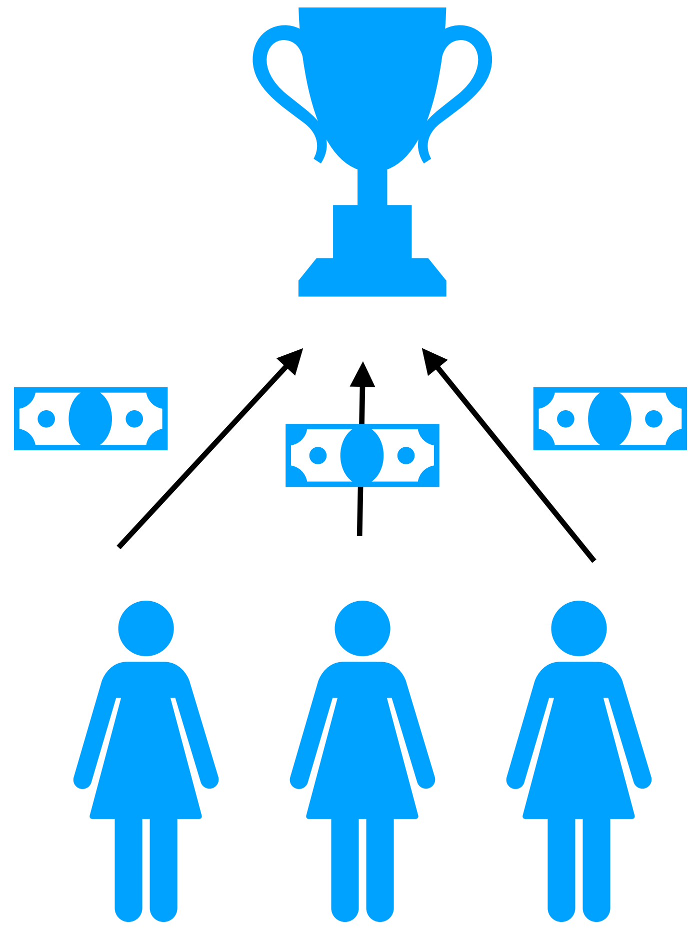
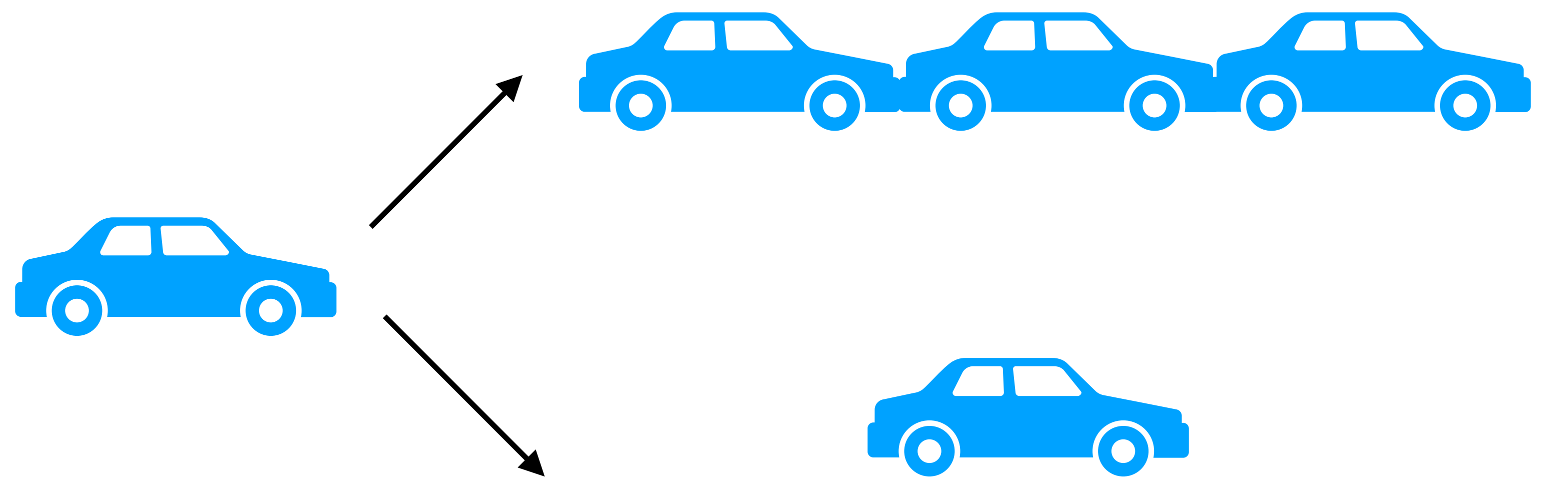
Examples include

- Repeated Auctions
- Congestion games
- Network routing games

Etc.





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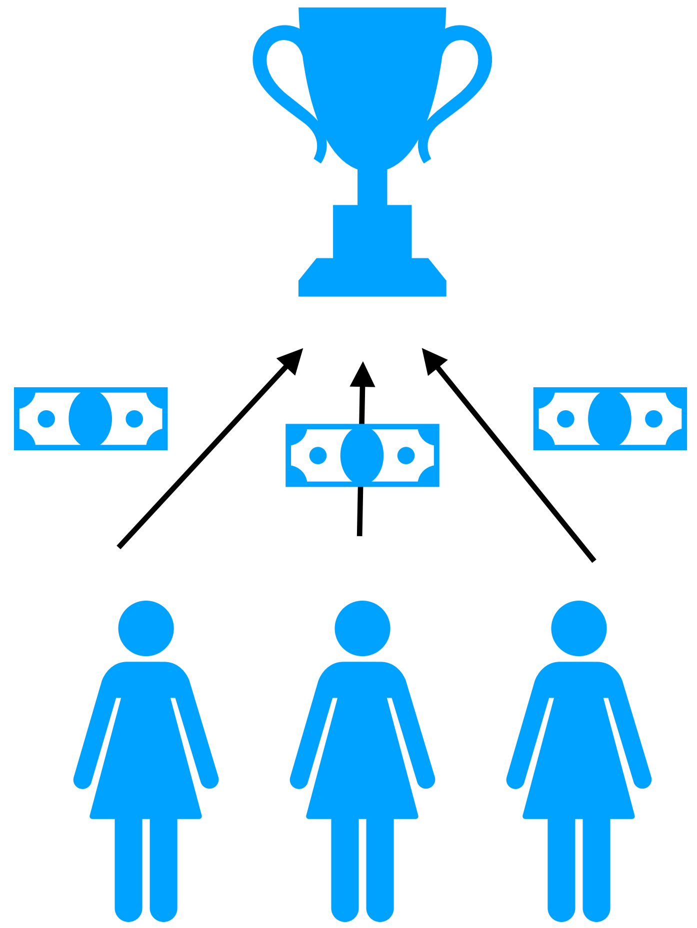

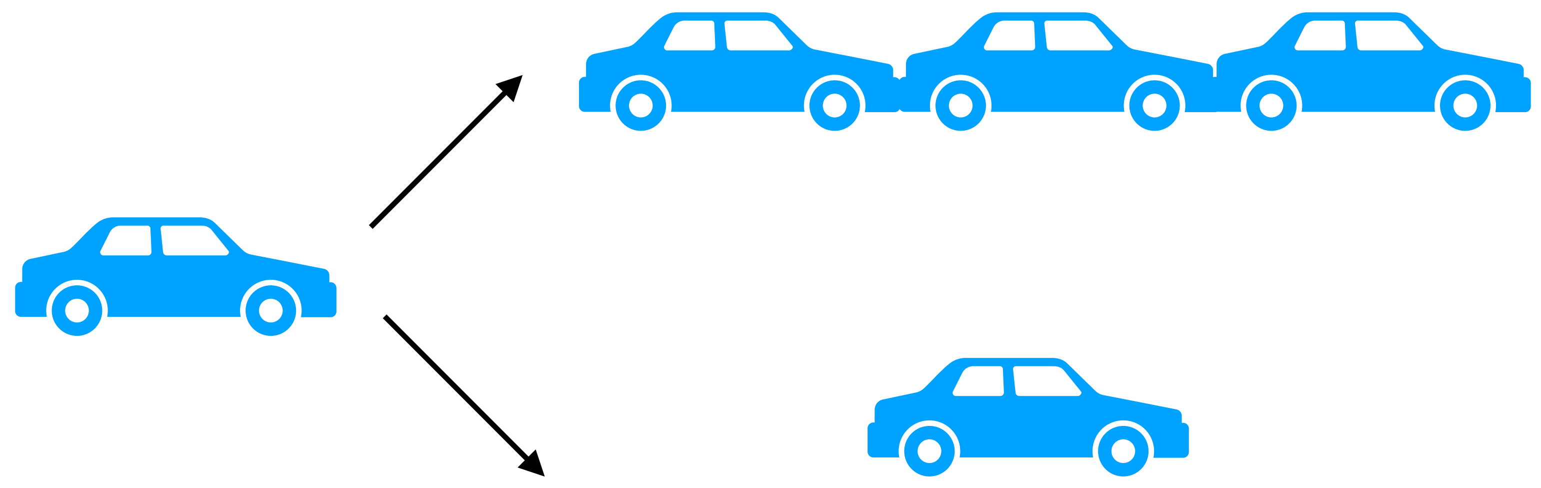

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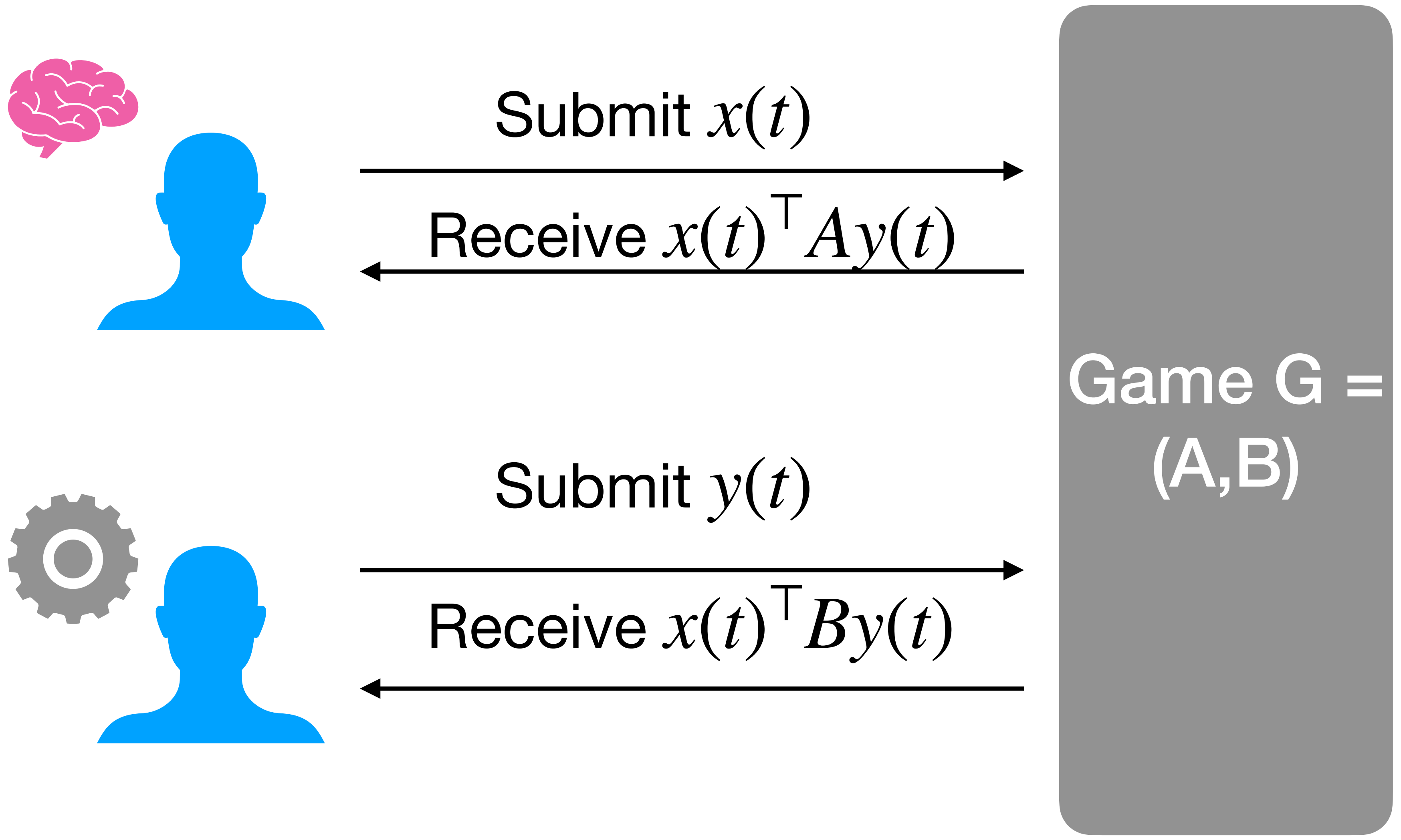
A lot of times agents use famous learning algorithms to determine what action to take.

Motivating question: Can strategic agents take advantage of these algorithms?

Setting

- Two player, repeated, normal form game played for time T.
- One player is called the *learner* and uses an algorithm throughout the game.
- Other player is called *optimizer,* knows the *learner's* algorithm and tries to take advantage of that to maximize their own utility.
- *Optimizer* and *learner* have n and m actions from action spaces $\mathscr{A}$ and $\mathscr{B}$ and utility matrices A, B respectively.



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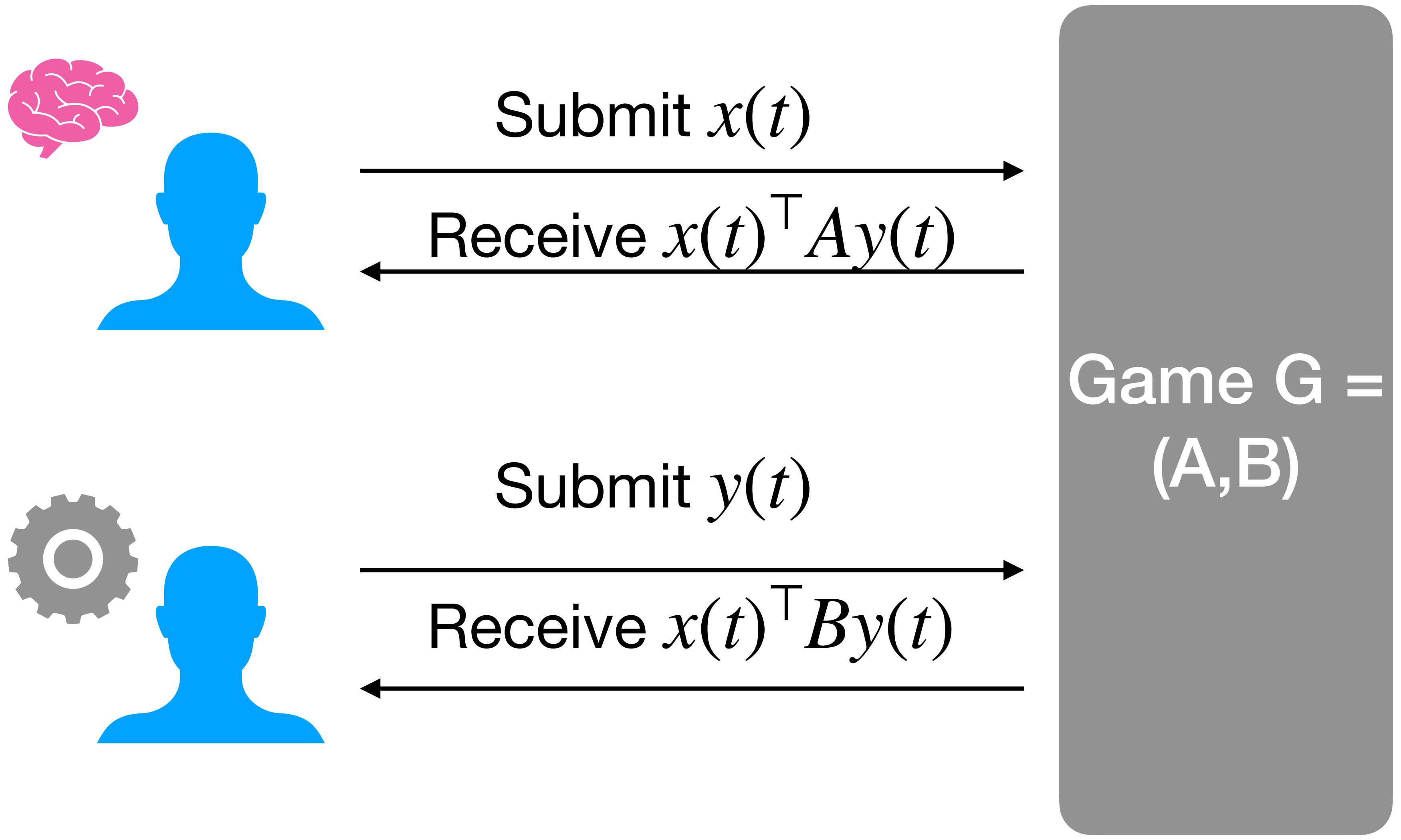

Questions we address

Against specific learning algorithms …

- In zero-sum games (where $A + B = 0$), what should the *optimizer* do to maximize their own utility?
- In general-sum games (where $A + B \neq 0$), is the best play for the *optimizer* efficiently computable?

Maximizing utility in zero-sum games

Continuous time setting

Optimizer strategy: any $x : [0,T] \to \Delta(\mathscr{A})$

Learner strategy: $y : [0,T] \to \Delta(\mathscr{B})$

, where:

$$y_i(t) = \frac{exp(\eta \int_0^t x(s)^\top Be_i ds)}{\sum_{i=1}^m exp(\eta \int_0^t x(s)^\top Be_i ds}$$

a.k.a. replicator dynamics, the continuous time analog
of MWU.

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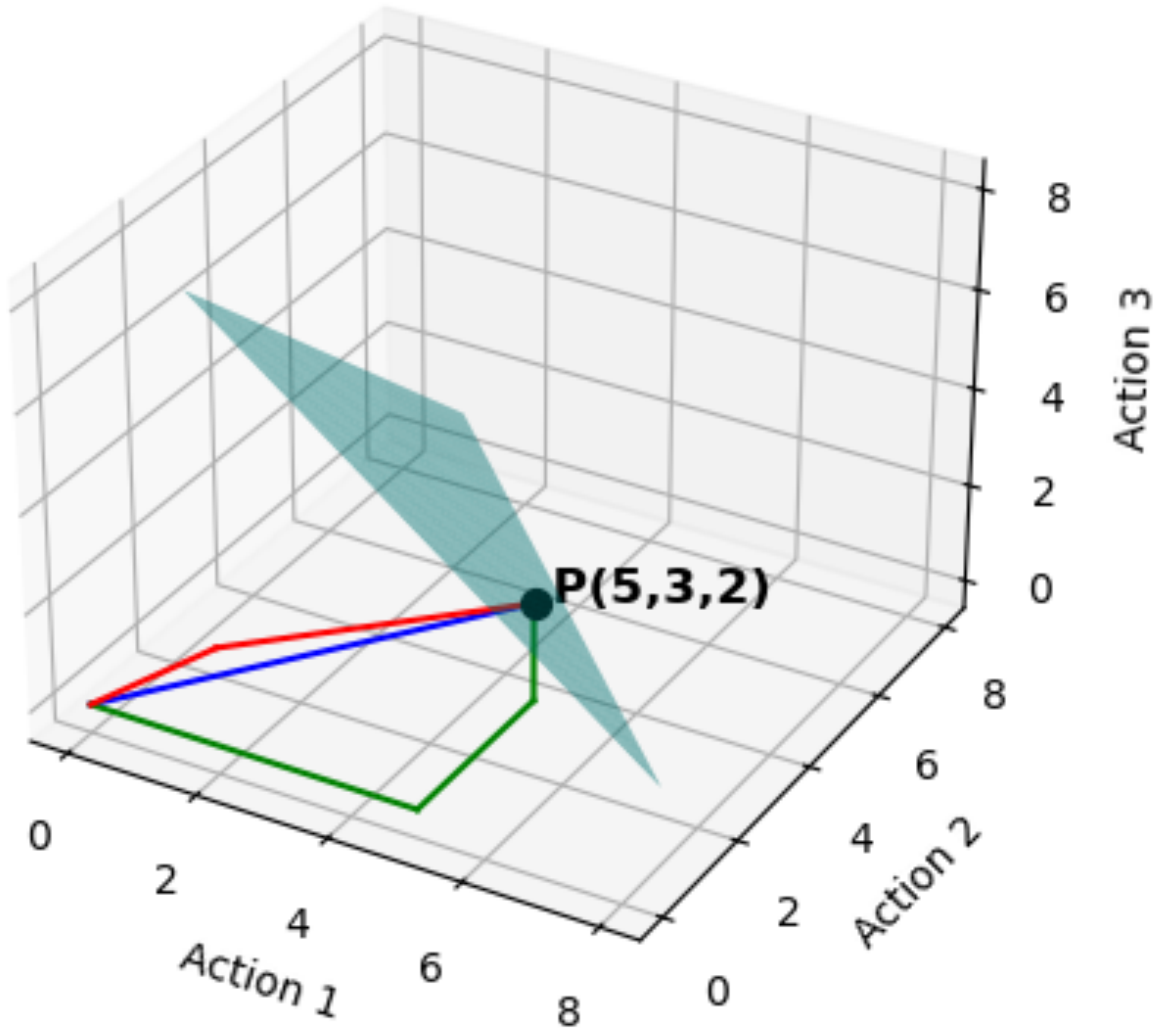


Maximizing utility in zero-sum games

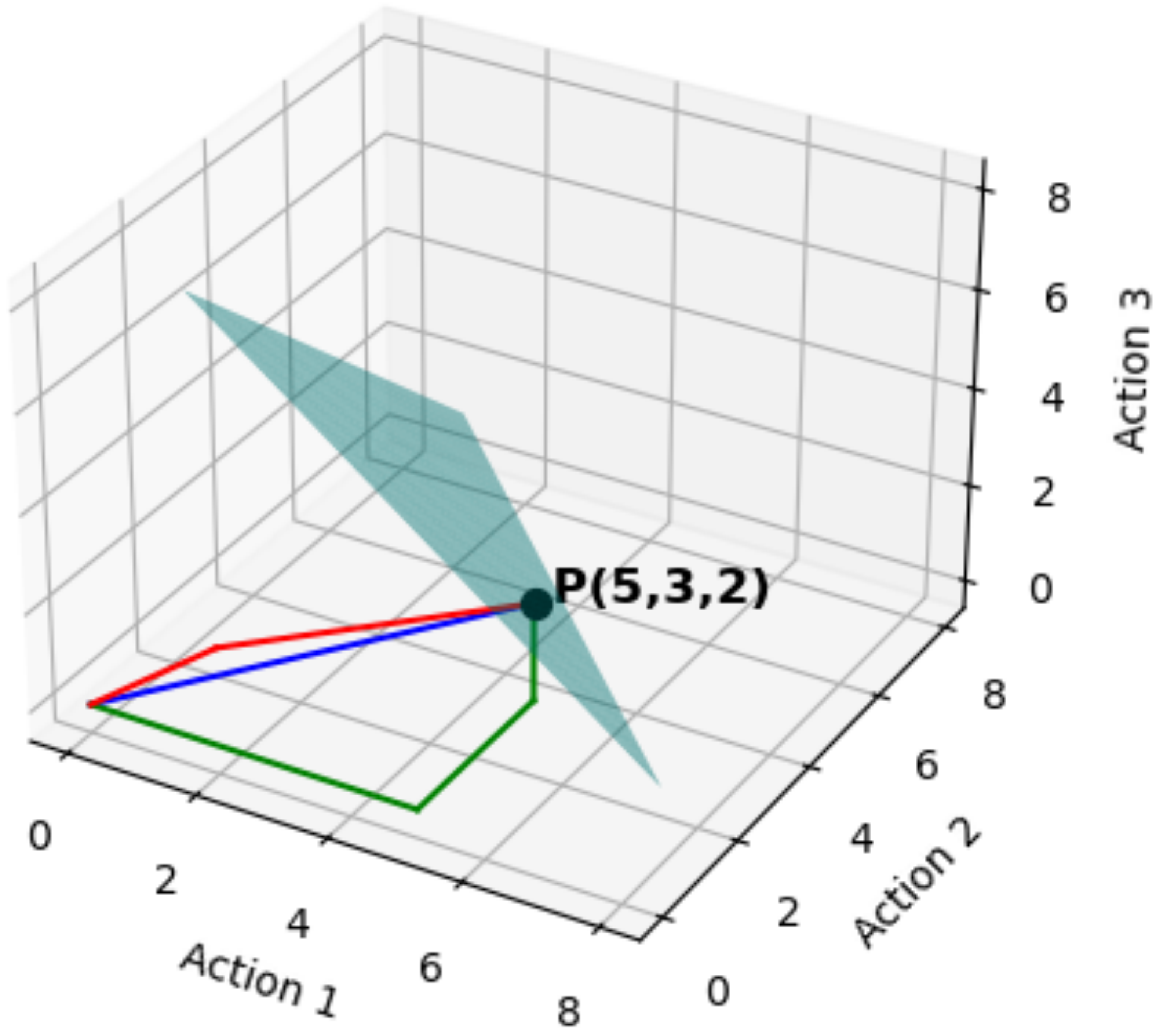
<u>Continuous time setting</u>

<u>Theorem 1:</u> The rewards of the optimizer depend only on the total time played each action.

<u>Corollary 1:</u> Optimal rewards can be achieved by a constant strategy i.e.

$$x(t) = x^*, x^* \in \Delta(\mathscr{A}), \forall t \in [0,T]$$

Moreover, this strategy can be efficiently computed in polynomial time.



Maximizing utility in zero-sum games

Discrete time setting

Optimizer strategy: any $x : \{1,\ldots,T\} \to \Delta(\mathscr{A})$

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$$y_i(t) = \frac{exp(\eta \sum_{s=1}^{t-1} x(s)^\top Be_i)}{\sum_{i=1}^{m} exp(\eta \sum_{s=1}^{t-1} x(s)^\top Be_i}$$

a.k.a. MWU or Hedge.

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$R_{cont}(A, B, T)$: optimal rewards for the optimizer in the continuous game.

$R_{disc}(A, B, T)$: optimal rewards for the optimizer in the discrete game.

Theorem 2: The following are true:

1. $R_{cont}(A, -A, T) \leq R_{disc}(A, -A, T) \leq R_{cont}(A, -A, T) + \frac{\eta T}{2}$

2. There are classes of games for which $R_{disc}(A, -A, T) = R_{cont}(A, -A, T) + \Omega(\eta T)$

Computational Barrier in general-sum games

Learner is purely best responding to the history:

$$y(t) = \text{arg max}_{y \in \Delta(\mathscr{B})} \sum_{s=1}^{t-1} x(s)^\top B y$$

a.k.a. fictitious play or MWU with $\eta \to \infty$.

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<u>**OCDP**</u> instance defined by (A, B, n, m, k, T):

- A, B matrices and n, m number of actions for *learner* and *optimizer respectively*.
- T total rounds of the game.

Instance is 'YES' if the optimizer can achieve total reward more than k and 'NO' otherwise.

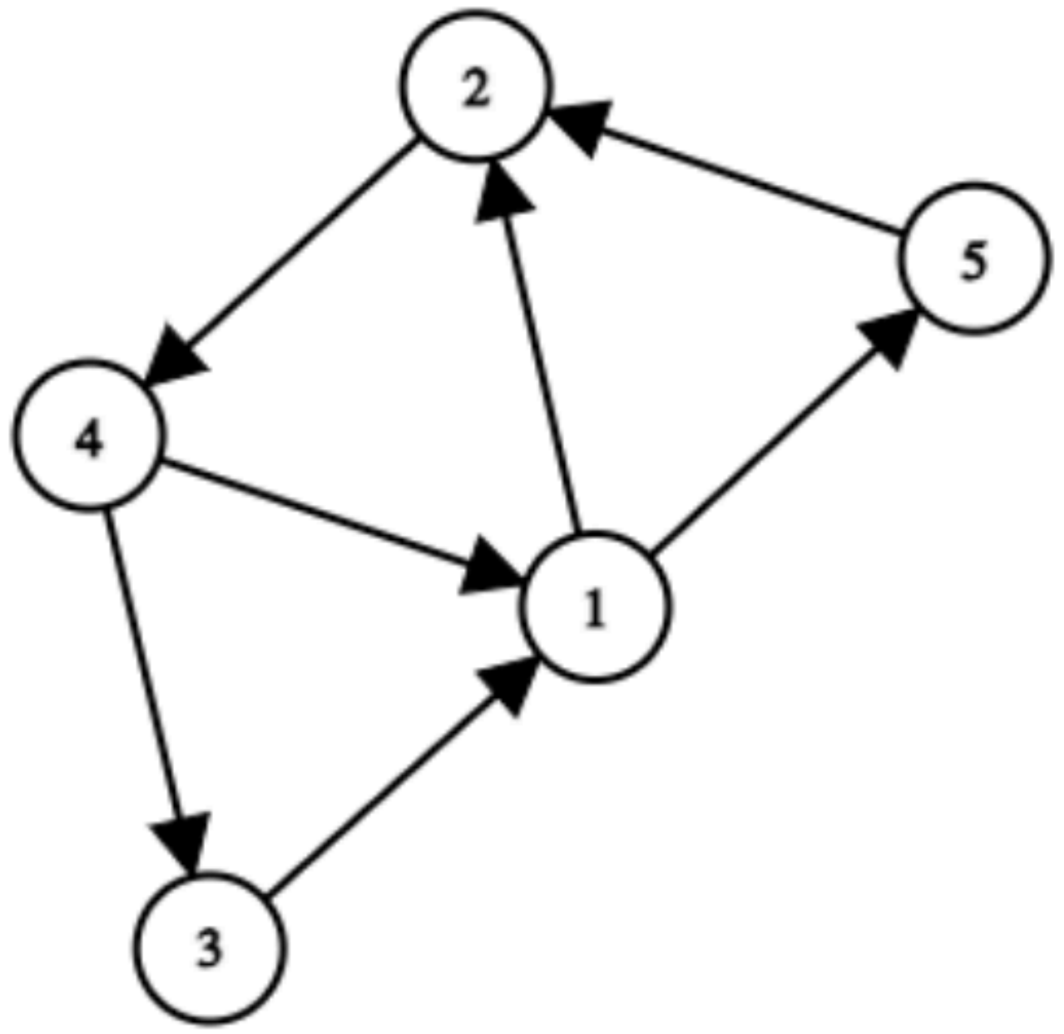
Computational Barrier in general-sum games

Theorem 2: OCDP is NP hard.

Proof sketch: Reduction from Hamiltonian Cycle.

Hamiltonian Cycle instance

$G = (V, E)$



OCDP instance

$T = k = |V| + 1$

A =

	v_1	v_2	v_3	v_4	v_5	v_{in_1}	v_{in_2}	v_{in_3}	v_{in_4}	v_{in_5}
e_1	1	0	0	0	0	0	0	0	0	0
e_2	0	0	0	0	1	0	0	0	0	0
e_3	1	0	0	0	0	0	0	0	0	0
e_4	0	1	0	0	0	0	0	0	0	0
e_5	0	0	0	1	0	0	0	0	0	0
e_6	0	0	0	1	0	0	0	0	0	0
e_7	0	0	1	0	0	0	0	0	0	0

B =

	v_1	v_2	v_3	v_4	v_5	v_{in_1}	v_{in_2}	v_{in_3}	v_{in_4}	v_{in_5}
e_1	-1	0	0	0	1	0.85	0	0	0	0
e_2	0	1	0	0	-4	0	0	0	0	0.85
e_3	-1	1	0	0	0	0.85	0	0	0	0
e_4	0	-4	0	1	0	0	0.85	0	0	0
e_5	1	0	0	-4	0	0	0	0	0.85	0
e_6	0	0	1	-4	0	0	0	0	0.85	0
e_7	1	0	-4	0	0	0	0	0.85	0	0

Summary

In short, our results:

1. In zero sum games, we show exactly how the optimizer should play against a MWU learner.

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Thank you!