FIDE: Frequency-Inflated Conditional Diffusion Model for Extreme-Aware Time Series Generation

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Generative Model

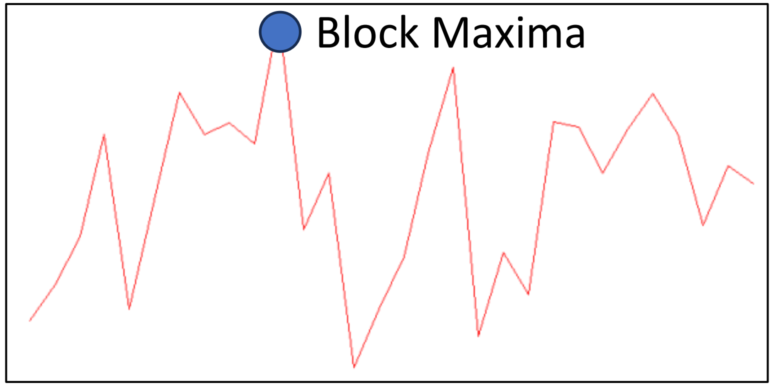
Computer Vision

Natural Language Processing

Time Series

Introduction

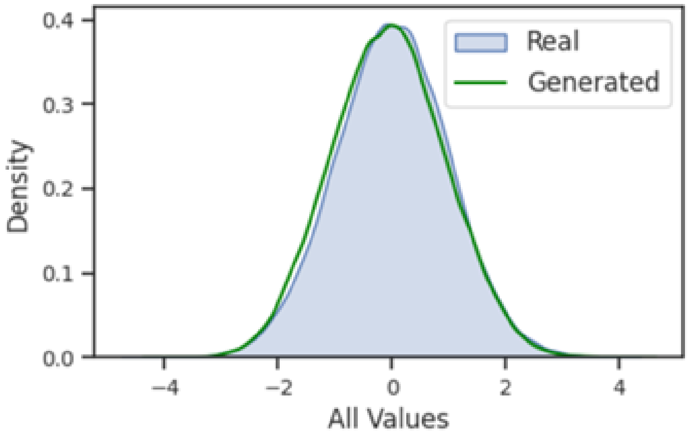
How to apply generative modeling for time series with a special focus on block maxima



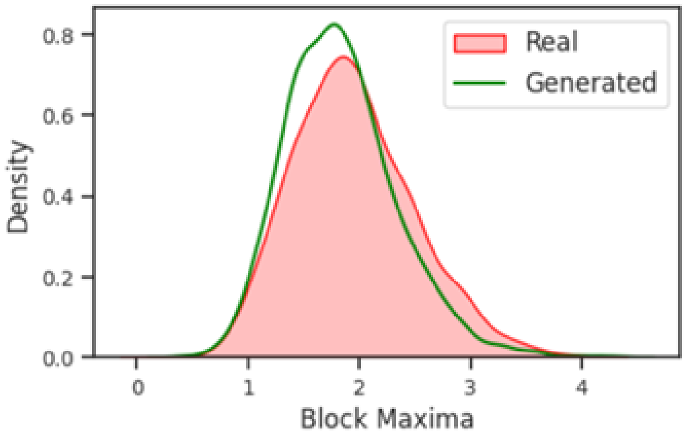
Source: Bloomberg; The New York Times

Challenge

Existing models struggle to capture the block maxima distribution



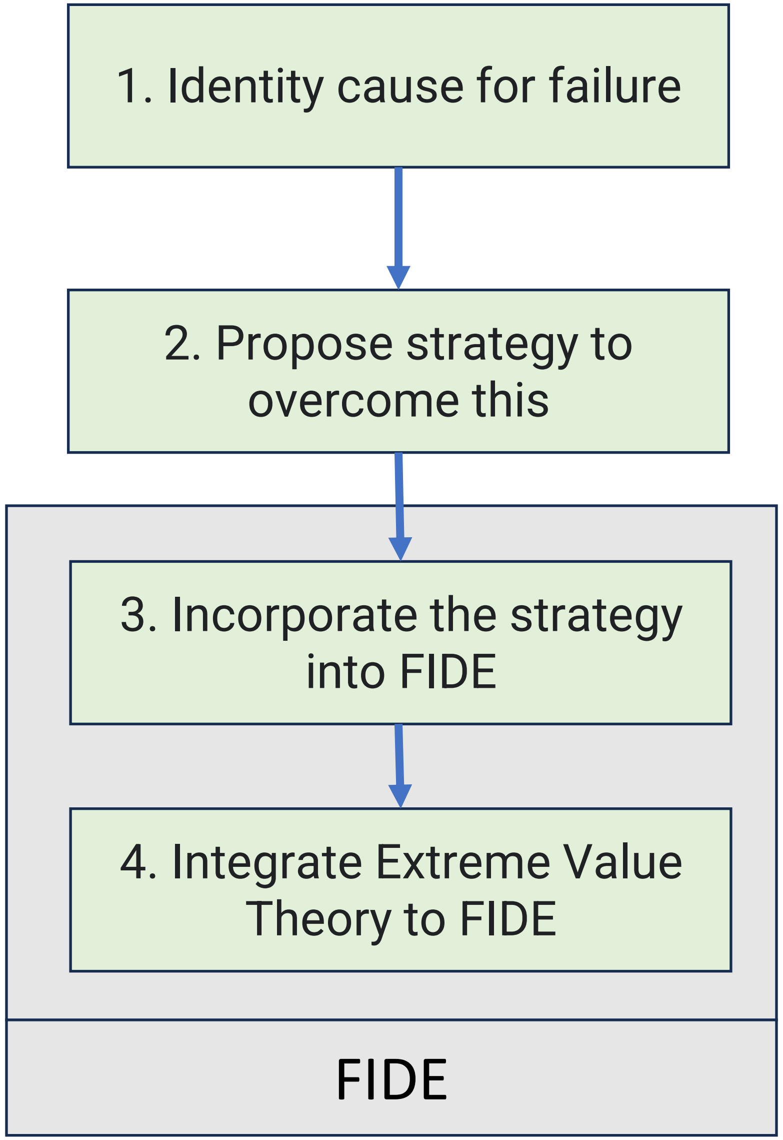
(a) All Values Distribution



(b) Block Maxima Distribution

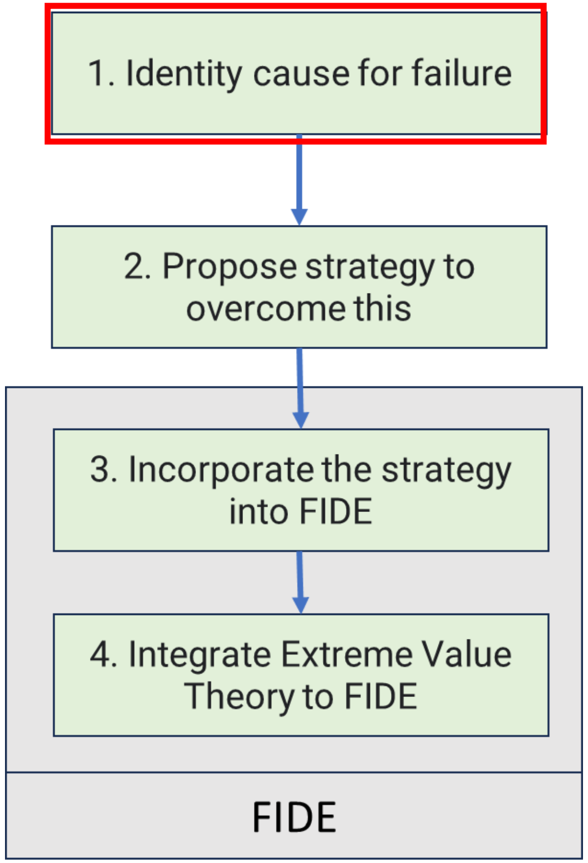
Comparing the distributions of all values and block maxima values for real and generated samples using DDPM when applied to the synthetic AR(1) data

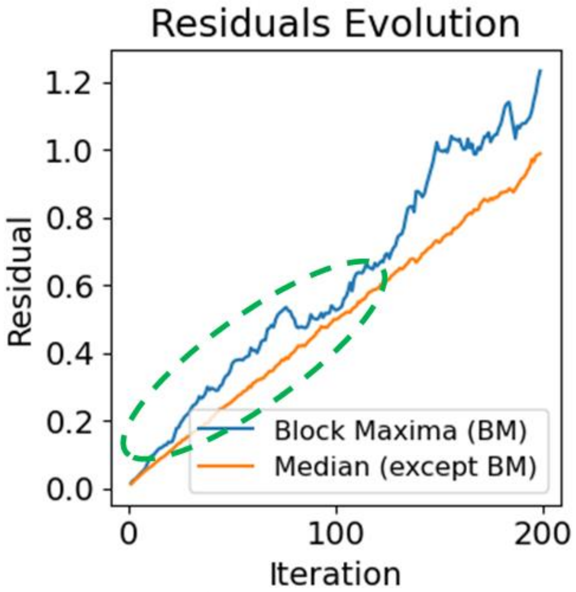
Contributions



The Rapid Dissipation of Block Maxima

In diffusion models, residual of block maxima dissipates faster than the residual of non-block maxima during the forward process

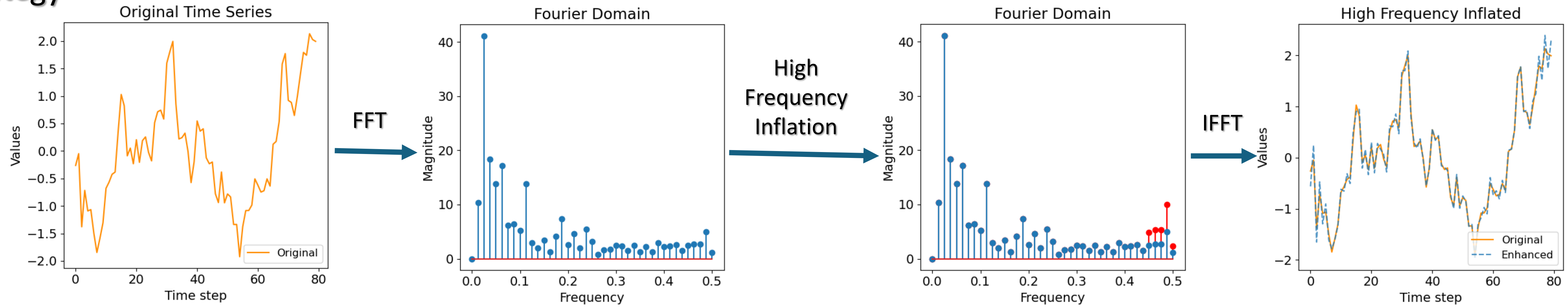




Residual = |Original – Perturbed|

High Frequency Components Inflation

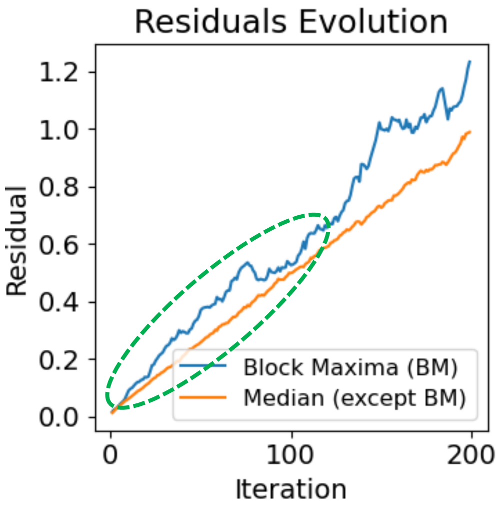
Strategy



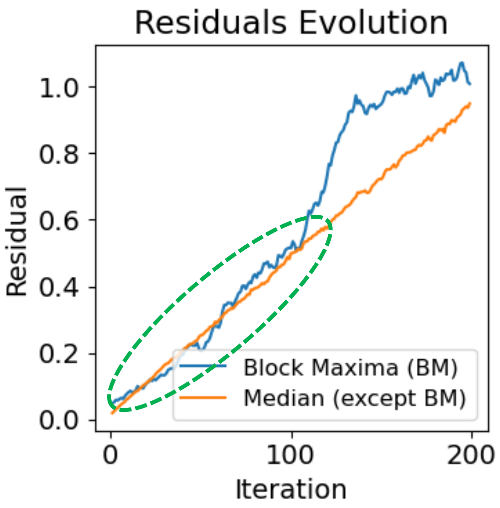
Effects

$$\frac{\lim_{k\to k_{\max}} |f_n^k|^2}{\lim_{k\to 0} |f_n^k|^2} = \delta \ll 1$$

Without Frequency Inflation



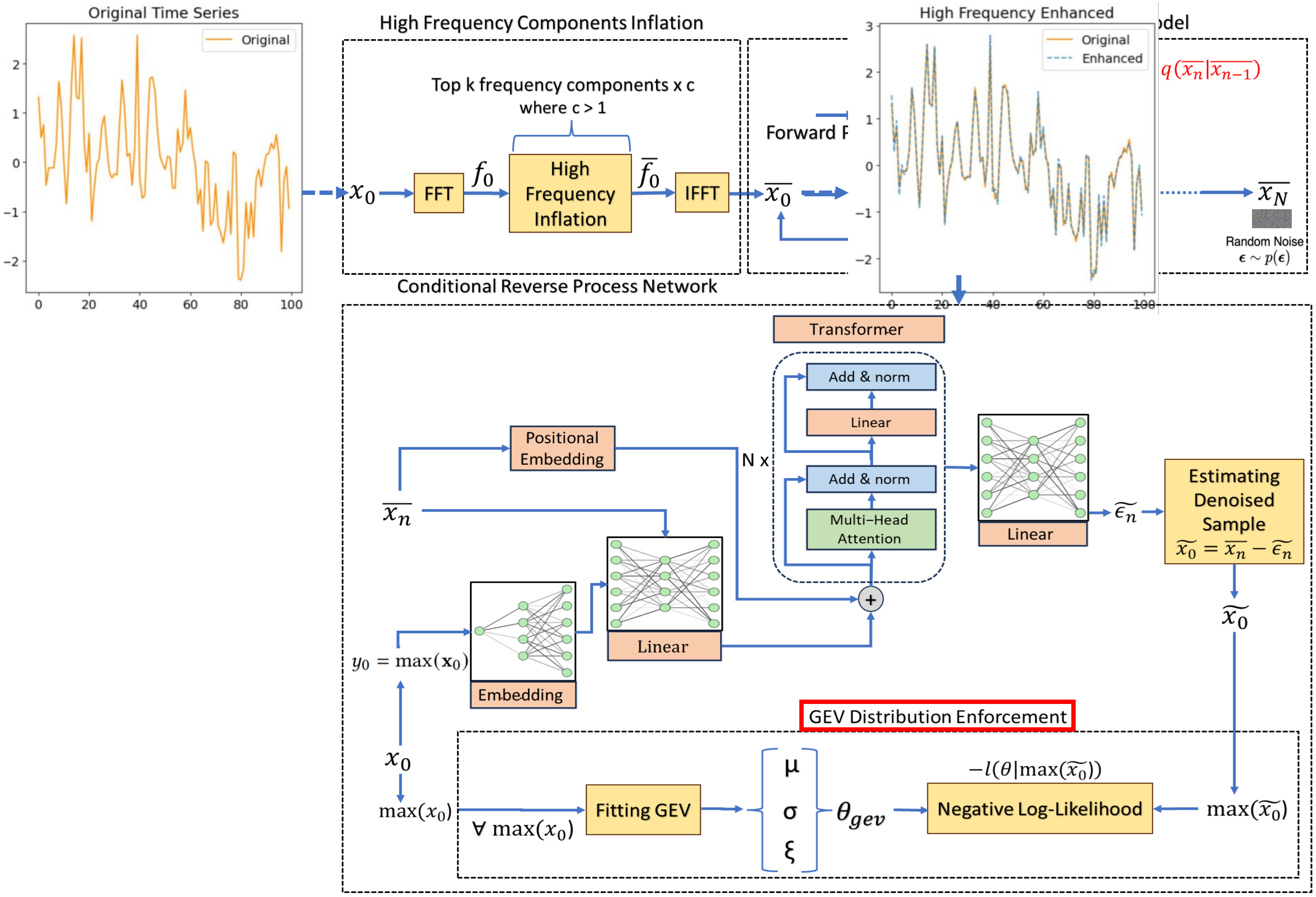
After Frequency Inflation



$$\frac{\lim_{k\to k_{\max}} |\overline{f_n^k}|^2}{\lim_{k\to 0} |\overline{f_n^k}|^2} = \delta \cdot \gamma$$

Take Home Message: with the proposed strategy, the high-frequency components including abrupt block maxima will be preserved by a factor of Υ compared to the previous case

FIDE Framework



$$\mathcal{L}_{\text{FIDE}} = \mathcal{L}_{\text{DDPM}} - \lambda \log \mathcal{L}_{\text{GEV}}(\mu, \sigma, \xi)$$

Experimental Evaluation

Datasets

- AR2: synthetic time series data using autoregressive model of order 2
- Financial Data (Stocks): daily historical Google stocks data from 2004 to 2019
- Energy Data (Appliance Energy): The UCI Appliances energy prediction dataset
- Weather/Climate Data (Daily Minimum Temperature): daily minimum temperatures in Melbourne, Australia, from 1981 to 1990
- Medical Data (ECG5000: Congestive Heart Failure): 20-hour long (5,000 heartbeats) electrocardiogram (ECG) obtained from the Physionet database

Experimental Evaluation

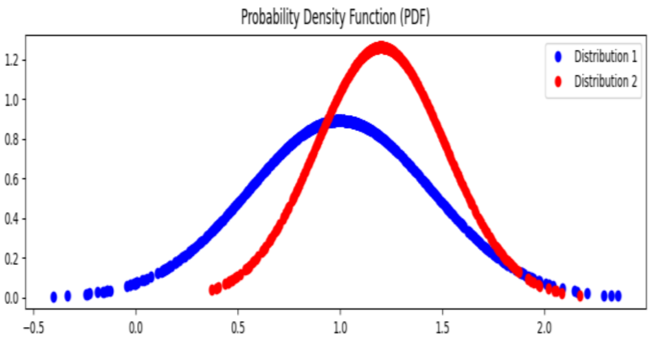


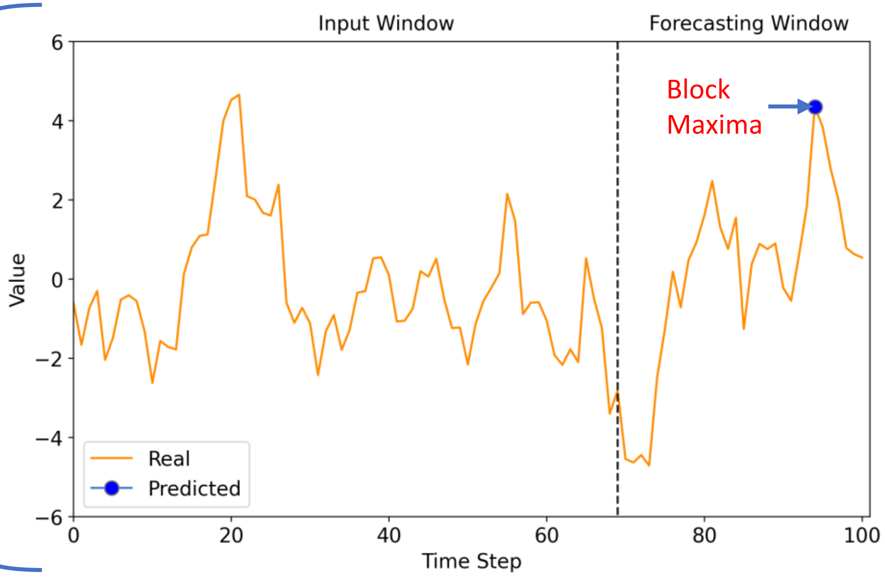
Table 1 Comparison of generated samples' block maxima distribution metrics and predictive score using the various methods. **Bold** and Underlined entries denote the best and second-best result

Metrics	Methods	AR1	Stock	Energy	Temperature	ECG
JS Divergence	beta-VAE	0.0211±0.0187	0.1105±0.0188	0.0722±0.0095	0.0140±0.0125	0.1210±0.0214
	c-beta-VAE	0.0190±0.0125	0.1011±0.0152	0.0710±0.0088	0.0109±0.0098	0.1120±0.0352
	TimeVAE	0.0015±0.0003	0.1054±0.0071	0.0795±0.0085	0.0096±0.0002	0.0985±0.0078
	TimeGAN	0.0840±0.0109	0.1411±0.1585	0.0950±0.0089	0.0112±0.0012	0.1620±0.0221
	cGAN	0.0690±0.0091	0.1211±0.0205	0.0890±0.0093	0.0091±0.0008	0.1440±0.0211
	RealNVP	0.0754±0.0121	0.1185±0.0108	0.0905±0.0084	0.0089±0.0007	0.1411±0.0116
	Fourier-Flows	0.0612±0.0045	0.1108±0.0195	0.0820±0.0044	0.0078±0.0010	0.1398±0.0202
	DDPM	0.0010±0.0007	0.0912±0.0062	0.0752±0.0082	0.0082±0.0009	0.1041±0.0122
	Diffusion-TS	0.0011±0.0008	0.0854±0.0045	0.0712±0.0071	0.0077±0.0008	0.1005±0.0108
	FIDE (Ours)	**0.0004±0.0001**	**0.0700±0.0061**	**0.0680±0.0092**	**0.0007±0.0001**	**0.0930±0.0082**
KL Divergence	beta-VAE	0.0110±0.0024	0.1947±0.0184	0.1210±0.0146	0.0410±0.0128	0.2020±0.0048
	c-beta-VAE	0.0091±0.0012	0.1744±0.0105	0.1160±0.0174	0.0360±0.0114	0.1880±0.0079
	TimeVAE	0.0105±0.0007	0.2514±0.0152	0.1625±0.0095	0.0490±0.0006	0.2254±0.0068
	TimeGAN	0.1920±0.0156	0.2425±0.0251	0.1590±0.0198	0.0550±0.0145	0.2540±0.0254
	cGAN	0.1240±0.0122	0.2101±0.0115	0.1510±0.0211	0.0490±0.0125	0.2210±0.0184
	RealNVP	0.1298±0.0215	0.2295±0.0154	0.1605±0.0310	0.0512±0.0108	0.2305±0.0145
	Fourier-Flows	0.1235±0.0104	0.2045±0.0255	0.1458±0.0345	0.0505±0.0136	0.2254±0.0141
	DDPM	0.0062±0.0008	0.1915±0.0125	0.1120±0.0108	0.0326±0.0090	0.1905±0.0094
	Diffusion-TS	0.0054±0.0007	0.1889±0.0108	0.1089±0.0115	0.0311±0.0078	0.1894±0.0081
	FIDE (Ours)	**0.0030±0.0009**	**0.1504±0.0128**	**0.0950±0.0098**	**0.0029±0.0008**	**0.1810±0.0084**
CRPS	beta-VAE	0.1247±0.0189	0.3149±0.0348	0.2410±0.0298	0.1554±0.0214	0.3059±0.0454
	c-beta-VAE	0.1154±0.0151	0.2698±0.0214	0.2574±0.0241	0.1420±0.0311	0.3150±0.0414
	TimeVAE	0.1511±0.0081	0.2547±0.0155	0.2853±0.1082	0.1847±0.0071	0.3252±0.0204
	TimeGAN	0.1858±0.0214	0.2825±0.0418	0.2685±0.0284	0.2110±0.0287	0.3240±0.0401
	cGAN	0.1224±0.0157	0.2689±0.0301	0.2385±0.0187	0.1990±0.0214	0.2985±0.0311
	RealNVP	0.1325±0.0144	0.2545±0.0258	0.2541±0.0214	0.2014±0.0354	0.2824±0.0425
	Fourier-Flows	0.1305±0.0254	0.2589±0.0214	0.2415±0.0211	0.1975±0.0251	0.2884±0.0215
	DDPM	0.0422±0.0084	0.2422±0.0187	0.2199±0.0874	0.1516±0.0211	0.2488±0.0388
	Diffusion-TS	0.0398±0.0092	0.2358±0.0211	0.2125±0.0454	0.1525±0.0315	0.2415±0.0451
	FIDE (Ours)	**0.0310±0.0098**	**0.2115±0.0152**	**0.2085±0.0985**	**0.0517±0.0082**	**0.2345±0.0204**

Generated data using FIDE outperformed baseline methods in capturing the distribution of block maxima

Experimental Evaluation

Predictive Task



Trained on → Generated Data → Tested on → Original Data

Predictive Score = RMSE on Original Data (Yoon et al., 2019)

Table 1 Comparison of generated samples' predictive score using the various methods. **Bold** and <u>Underlined</u> entries denote the best and second-best result

Metrics	Methods	AR1	Stock	Energy	Temperature	ECG
Predictive Score	beta-VAE	0.6350±0.0201	0.9528±0.0314	0.7410±0.0187	0.6814±0.0108	0.9420±0.0142
	c-beta-VAE	0.6240±0.0145	0.9226±0.0165	0.7317±0.0163	0.6718±0.0025	0.9310±0.0214
	TimeVAE	0.6150±0.0104	0.9140±0.0218	0.7325±0.0195	0.6723±0.0036	0.9150±0.0112
	TimeGAN	**0.6050±0.0104**	0.8950±0.0198	<u>0.7280±0.0187</u>	0.6718±0.0047	**0.8960±0.0084**
	cGAN	0.6120±0.0014	0.9354±0.0210	0.7310±0.0147	0.6847±0.0041	0.9220±0.0191
	RealNVP	0.6884±0.0011	0.9988±0.0354	0.7898±0.0254	0.7852±0.0017	0.9730±0.0215
	Fourier-Flows	0.6925±0.0031	0.9844±0.0241	0.7955±0.0088	0.7871±0.0021	0.9655±0.0221
	DDPM	0.6148±0.0081	0.8997±0.0111	0.7350±0.0102	<u>0.6708±0.0098</u>	0.9121±0.0121
	Diffusion-TS	0.6105±0.0045	<u>0.8912±0.0105</u>	0.7355±0.0084	<u>0.6708±0.0108</u>	0.9089±0.0095
	FIDE (Ours)	<u>0.6081±0.0098</u>	**0.8871±0.0104**	**0.7240±0.0087**	**0.6694±0.0082**	<u>0.9040±0.0112</u>

FIDE accurately replicates the temporal characteristics of the original data for predictive modeling task

Conclusion

- FIDE enhances diffusion models to capture extreme events by
  - preserving block maxima through high-frequency components
  - leveraging GEV distribution
- Superiority of the framework confirmed through experiments on real-world and synthetic data.