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NEURAL INFORMATION
PROCESSING SYSTEMS

Rethinking Fourier Transform from A Basis Functions Perspective for Long-term Time Series Forecasting

Presented by Runze Yang

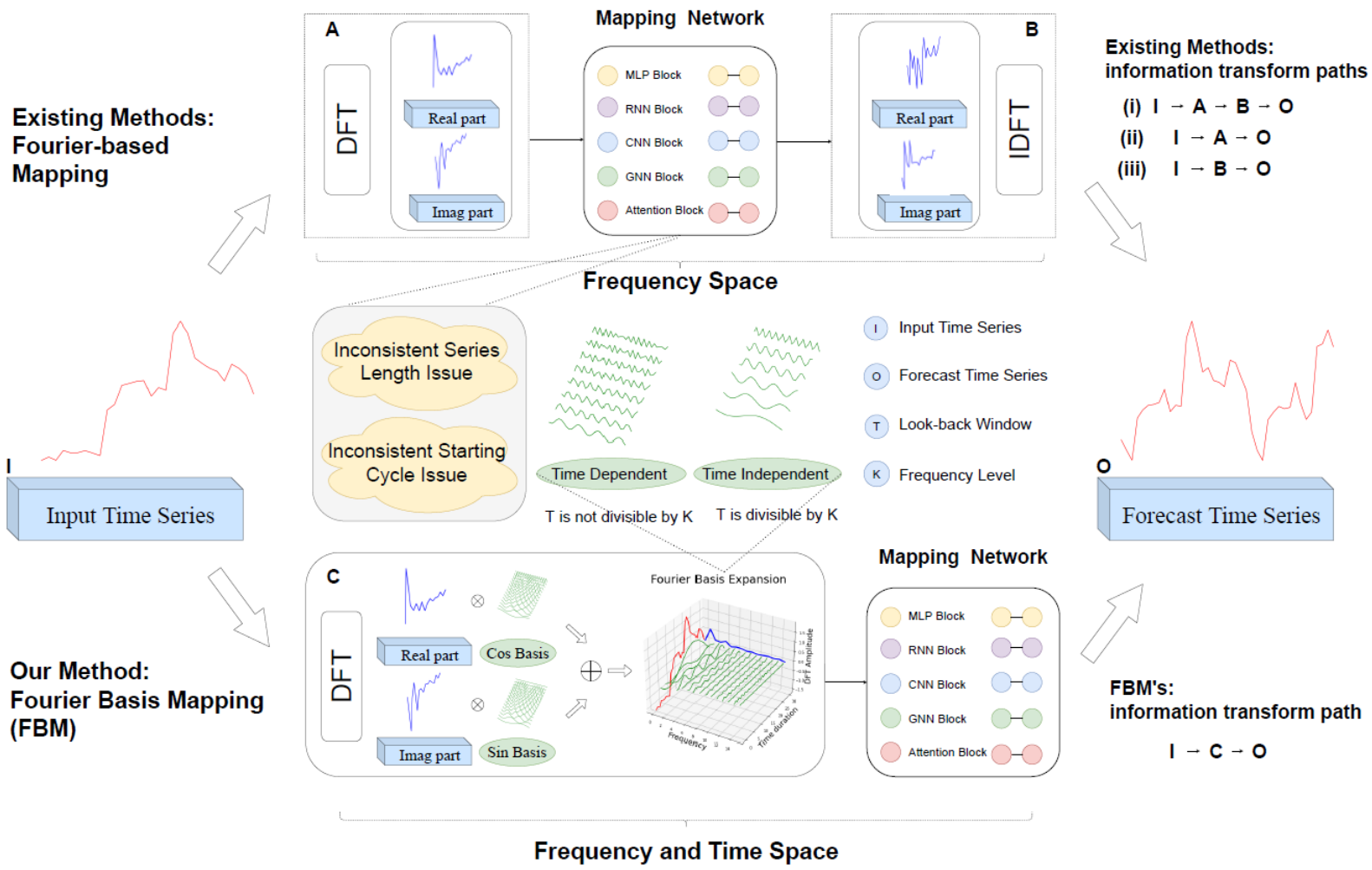
Introduction

- **Background**: Fourier Transform and Long-term Time Series Forecasting.
- **Question**: *Do existing studies precisely interpret frequency coefficients and sufficiently consider the time-frequency relationships?*
- **A New Perspective**: The real and imaginary parts of the frequency components can be viewed as the coefficients of cosine and sine basis functions at tiered frequency levels, respectively.
- **Research Issues**: Inconsistent Starting Cycles and Inconsistent Series Length Issues.
- **Fourier Basis Mapping**:

(i) embeds the discrete Fourier transform with basis functions

(ii) can enable plug-and play in various types of neural networks for better performance.

Existing Methods VS Our Method



Existing methods map from:

(i) Frequency to Frequency
(ii) Frequency to Time
(ii) Time to Frequency

Fourier basis functions :

(i) Time Dependent
(ii) Time Independent

Issues:

(i) Inconsistent Series Length Issue
(ii) Inconsistent Starting Cycle Issue

Our method maps from:

Time-frequency to Time

Mathematical Formula

- The Inverse Discrete Fourier Transform in terms of the basis functions:

$$\mathbf{H}(k) = DFT(\mathbf{X}) = \sum_{n=0}^{T-1} \mathbf{X}[n] \exp\left(-i\frac{2\pi kn}{T}\right), \qquad k = 0, 1, \ldots, T-1,$$

$$\mathbf{X}[n] = \frac{1}{T}\sum_{k=0}^{\frac{T}{2}} \left(\mathbf{a_k}\cos\left(\frac{2\pi kn}{T}\right) - \mathbf{b_k}\sin\left(\frac{2\pi kn}{T}\right)\right), \qquad n = 0, 1, \ldots T-1,$$

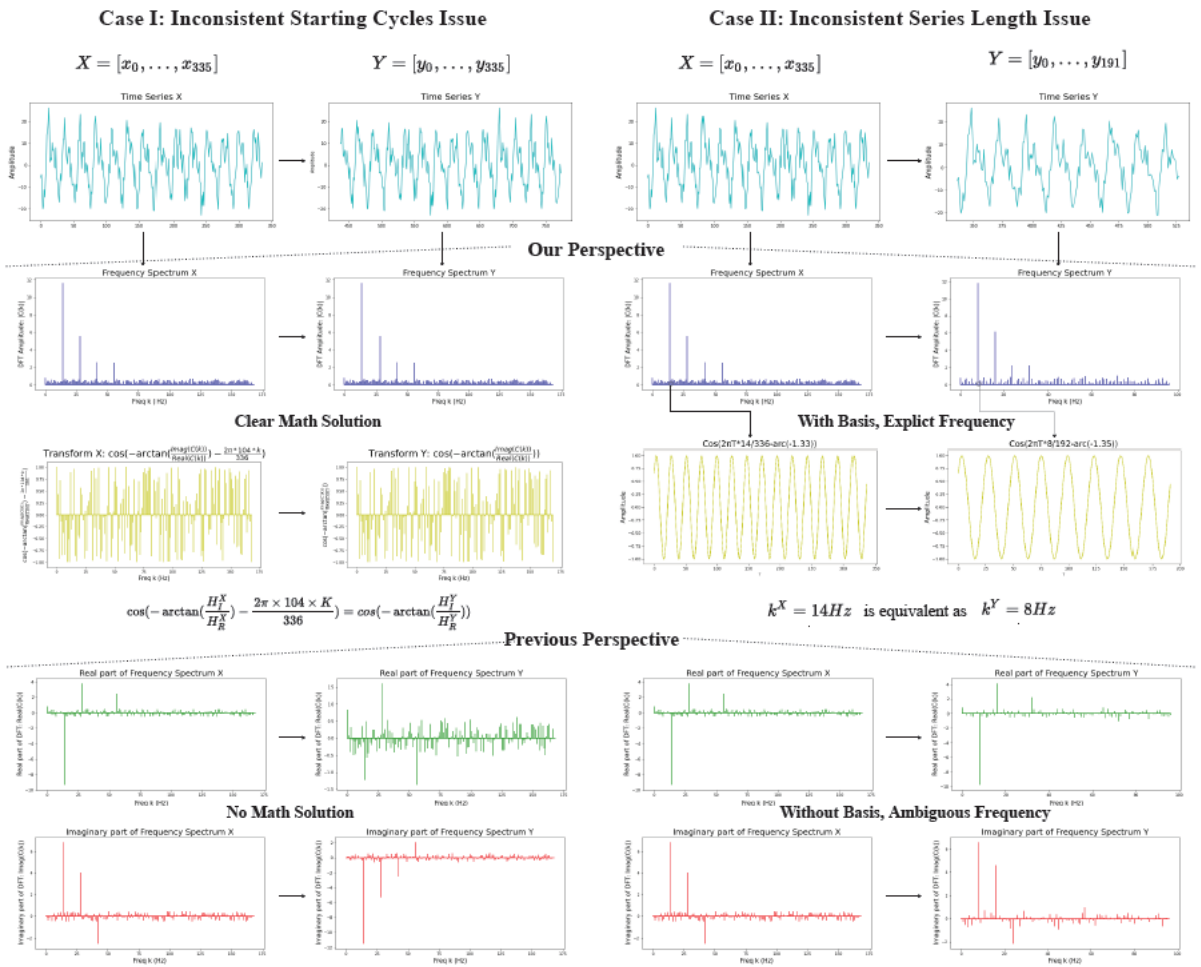
$$\mathbf{a_k} = \begin{cases} \mathbf{H_R}[k], & k = 0, \frac{T}{2} \\ 2 \cdot \mathbf{H_R}[k], & k = 1, \ldots, \frac{T}{2} - 1 \end{cases} \qquad \mathbf{b_k} = \begin{cases} \mathbf{H_I}[k] = 0, & k = 0, \frac{T}{2} \\ 2 \cdot \mathbf{H_I}[k], & k = 1, \ldots, \frac{T}{2} - 1. \end{cases}$$

$H_R[K]$ and $H_I[K]$ represent the real and imaginary parts of frequency components $H[K]$ respectively, where K refers to the frequency level, defined by $H[K] = H_R[K] + \mathrm{i}\, H_I[K]$

- The relationships between the real and imaginary components :

$$\mathbf{Z}(t) = A\cos(wt) + B\sin(wt) = R\cos(wt - \phi),$$
$$R = \sqrt{A^2 + B^2}, \quad \phi = arctan(B, A).$$

Inconsistent Starting Cycles and Inconsistent Series Length Issues



Case I: X and Y have the same frequency and series length but different starting cycles.

Case II: X and Y have almost the same frequency and starting cycles but different series lengths.

Fourier Basis Mapping (FBM)

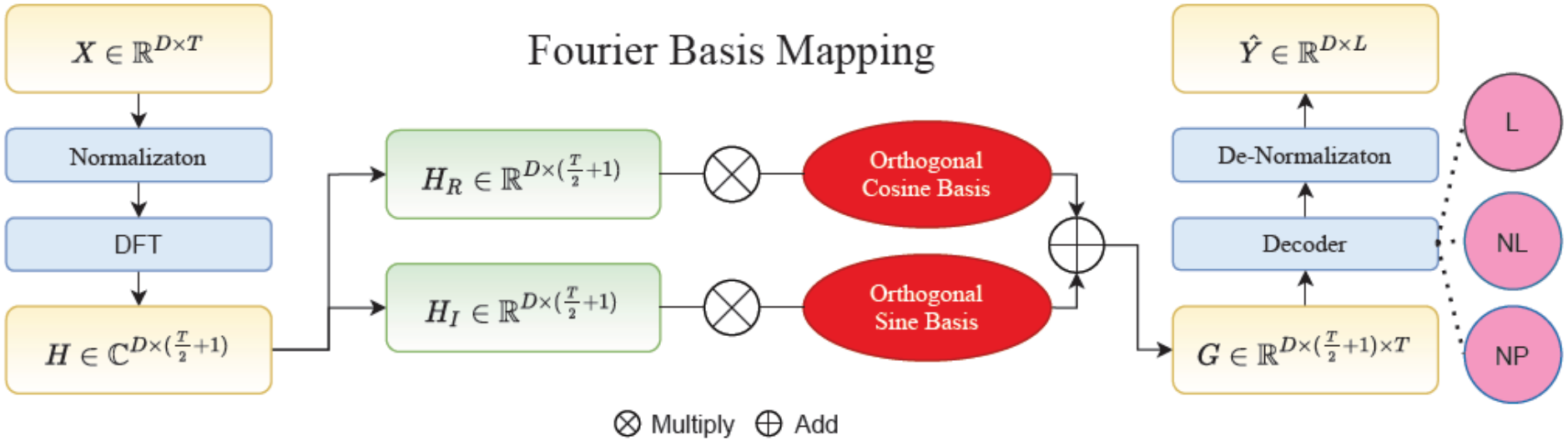


Figure 3: Architecture of Fourier Basis Mapping (FBM): with three feature-output mapping methods, denoted as L - vanilla linear network, NL - nonlinear MLP, and NP - nonlinear PatchTST.

Experiment Results

Method		FBM-L		FBM-NL		FBM-NP		NLinear		PatchTST		iTransformer		TimeMixer		N-BEATS		CrossGNN		FITS		FreTS	
Error		MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
ETTh1	96	0.366	0.390	0.368	0.395	0.367	0.395	0.391	0.416	0.374	0.399	0.399	0.417	0.385	0.408	0.387	0.410	0.376	0.400	0.368	0.392	0.404	0.423
	192	0.403	0.411	0.408	0.418	0.407	0.416	0.421	0.426	0.417	0.422	0.436	0.440	0.429	0.432	0.428	0.434	0.419	0.427	0.404	0.412	0.461	0.460
	336	0.418	0.420	0.425	0.430	0.433	0.438	0.435	0.435	0.431	0.436	0.446	0.451	0.456	0.450	0.448	0.447	0.439	0.442	0.419	0.435	0.488	0.480
	720	0.414	0.438	0.456	0.466	0.439	0.459	0.443	0.457	0.445	0.463	0.502	0.503	0.457	0.462	0.466	0.471	0.447	0.465	0.431	0.458	0.566	0.553
ETTh2	96	0.271	0.331	0.287	0.343	0.280	0.340	0.283	0.342	0.276	0.338	0.303	0.362	0.276	0.339	0.303	0.363	0.283	0.344	0.276	0.338	0.327	0.388
	192	0.332	0.373	0.351	0.386	0.342	0.382	0.350	0.387	0.341	0.378	0.372	0.403	0.340	0.381	0.364	0.402	0.342	0.387	0.336	0.377	0.428	0.450
	336	0.321	0.376	0.352	0.394	0.354	0.401	0.344	0.395	0.332	0.385	0.401	0.424	0.362	0.404	0.360	0.407	0.361	0.408	0.324	0.379	0.499	0.497
	720	0.369	0.412	0.397	0.432	0.386	0.424	0.395	0.436	0.379	0.420	0.420	0.446	0.398	0.433	0.428	0.465	0.423	0.460	0.373	0.416	0.727	0.637
ETTm1	96	0.301	0.343	0.286	0.339	0.293	0.346	0.307	0.349	0.295	0.344	0.309	0.361	0.303	0.350	0.324	0.367	0.300	0.343	0.305	0.347	0.326	0.373
	192	0.337	0.364	0.324	0.365	0.334	0.368	0.347	0.374	0.333	0.370	0.345	0.383	0.356	0.385	0.363	0.388	0.335	0.369	0.338	0.366	0.359	0.392
	336	0.371	0.384	0.359	0.385	0.371	0.389	0.377	0.390	0.363	0.394	0.380	0.401	0.366	0.392	0.400	0.408	0.375	0.390	0.372	0.386	0.389	0.408
	720	0.425	0.415	0.422	0.424	0.426	0.420	0.436	0.425	0.421	0.420	0.448	0.442	0.435	0.434	0.468	0.448	0.429	0.420	0.427	0.416	0.445	0.441
ETTm2	96	0.164	0.252	0.165	0.254	0.167	0.258	0.169	0.259	0.173	0.261	0.180	0.272	0.174	0.258	0.168	0.259	0.164	0.252	0.167	0.256	0.202	0.288
	192	0.219	0.290	0.225	0.296	0.224	0.296	0.223	0.294	0.255	0.306	0.239	0.311	0.238	0.300	0.225	0.301	0.220	0.294	0.222	0.293	0.250	0.322
	336	0.271	0.325	0.276	0.331	0.277	0.331	0.277	0.331	0.285	0.336	0.389	0.341	0.272	0.327	0.282	0.336	0.276	0.330	0.277	0.329	0.328	0.368
	720	0.364	0.381	0.365	0.386	0.367	0.386	0.371	0.387	0.365	0.386	0.374	0.392	0.368	0.389	0.376	0.394	0.372	0.390	0.366	0.382	0.431	0.436
Electricity	96	0.142	0.237	0.132	0.227	0.133	0.227	0.143	0.239	0.133	0.227	0.137	0.232	0.134	0.230	0.144	0.240	0.147	0.246	0.145	0.242	0.145	0.245
	192	0.155	0.248	0.149	0.243	0.149	0.242	0.157	0.250	0.151	0.244	0.156	0.249	0.153	0.245	0.158	0.252	0.161	0.258	0.158	0.253	0.158	0.255
	336	0.172	0.265	0.167	0.261	0.167	0.261	0.174	0.267	0.167	0.261	0.171	0.266	0.172	0.267	0.175	0.269	0.178	0.274	0.174	0.269	0.178	0.275
	720	0.212	0.297	0.207	0.295	0.208	0.295	0.214	0.299	0.210	0.297	0.195	0.288	0.212	0.298	0.217	0.304	0.214	0.299	0.213	0.301	0.220	0.315
Traffic	96	0.421	0.281	0.384	0.264	0.373	0.253	0.425	0.288	0.381	0.257	0.376	0.263	0.381	0.261	0.429	0.295	0.428	0.291	0.421	0.282	0.434	0.313
	192	0.434	0.286	0.399	0.269	0.396	0.266	0.438	0.291	0.402	0.270	0.396	0.274	0.408	0.273	0.441	0.299	0.441	0.295	0.435	0.288	0.471	0.311
	336	0.447	0.292	0.419	0.282	0.411	0.276	0.452	0.300	0.422	0.283	0.407	0.283	0.434	0.297	0.455	0.307	0.455	0.302	0.448	0.293	0.493	0.321
	720	0.477	0.309	0.448	0.297	0.442	0.291	0.482	0.317	0.454	0.296	0.449	0.305	0.469	0.319	0.486	0.326	0.486	0.318	0.478	0.310	0.535	0.339
Weather	96	0.159	0.207	0.152	0.199	0.156	0.204	0.176	0.226	0.156	0.206	0.162	0.211	0.158	0.204	0.186	0.238	0.163	0.227	0.149	0.198	0.159	0.218
	192	0.203	0.247	0.194	0.242	0.198	0.245	0.220	0.262	0.200	0.246	0.204	0.249	0.197	0.246	0.227	0.275	0.205	0.261	0.196	0.244	0.207	0.270
	336	0.252	0.285	0.244	0.282	0.248	0.285	0.265	0.296	0.252	0.285	0.248	0.285	0.242	0.281	0.274	0.307	0.250	0.295	0.245	0.283	0.252	0.299
	720	0.319	0.335	0.317	0.334	0.319	0.337	0.332	0.345	0.321	0.336	0.322	0.335	0.319	0.335	0.342	0.361	0.320	0.347	0.321	0.338	0.319	0.342
Exchange	96	0.093	0.211	0.104	0.226	0.096	0.196	0.098	0.219	0.104	0.227	0.128	0.254	0.119	0.247	0.147	0.274	0.093	0.211	0.109	0.235	0.209	0.350
	192	0.195	0.309	0.210	0.326	0.196	0.312	0.203	0.316	0.210	0.325	0.241	0.353	0.238	0.354	0.312	0.406	0.188	0.305	0.229	0.350	0.346	0.437
	336	0.347	0.421	0.398	0.460	0.353	0.425	0.356	0.426	0.366	0.435	0.393	0.459	0.417	0.472	0.522	0.532	0.363	0.430	0.400	0.463	0.634	0.583
	720	0.965	0.732	1.040	0.762	0.970	0.734	0.965	0.733	1.026	0.757	1.00	0.763	1.074	0.790	1.412	0.907	0.931	0.722	1.095	0.781	2.418	1.233
Average		0.323	0.339	0.325	0.344	0.321	0.340	0.333	0.349	0.326	0.344	0.341	0.353	0.335	0.352	0.366	0.371	0.330	0.350	0.331	0.347	0.431	0.406

Comparison:

1. FBM-L vs Linear network
2. FBM-NL vs MLP-based networks
3. FBM-NP VS Transformer-based networks
4. FBM variants vs. Fourier-based networks

THANK YOU!