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NEURAL INFORMATION PROCESSING SYSTEMS

Spectral Learning of Shared Dynamics between Generalized-Linear Processes

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Generalized-linear dynamical models are useful for modeling time-series

Latent dynamical process

$$x_{k+1} = A x_k + w_k$$

Gaussian observations

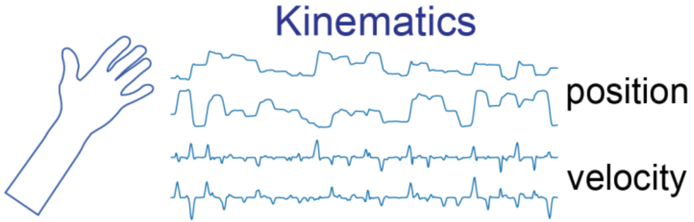
$$r_k = C_r x_k + v_k + b$$

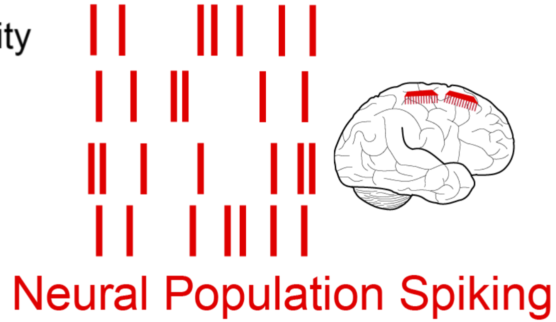
Poisson observations

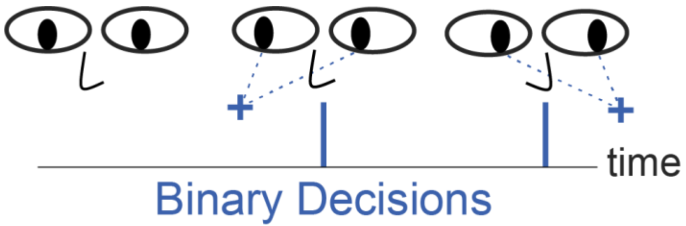
$$y_k | r_k \sim Poisson(\exp(r_k))$$

Bernoulli observations

$$y_k | r_k \sim Bern(\text{sigmoid}(r_k))$$



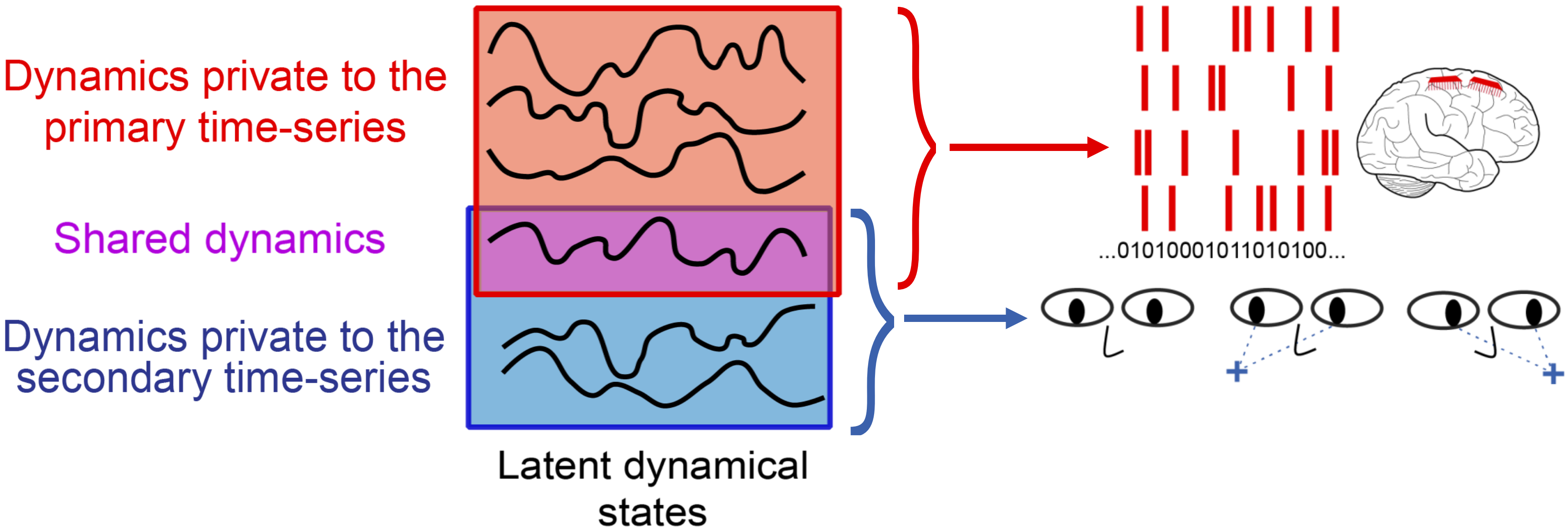




Most existing GLDM models and their associated learning algorithms only model a single source.

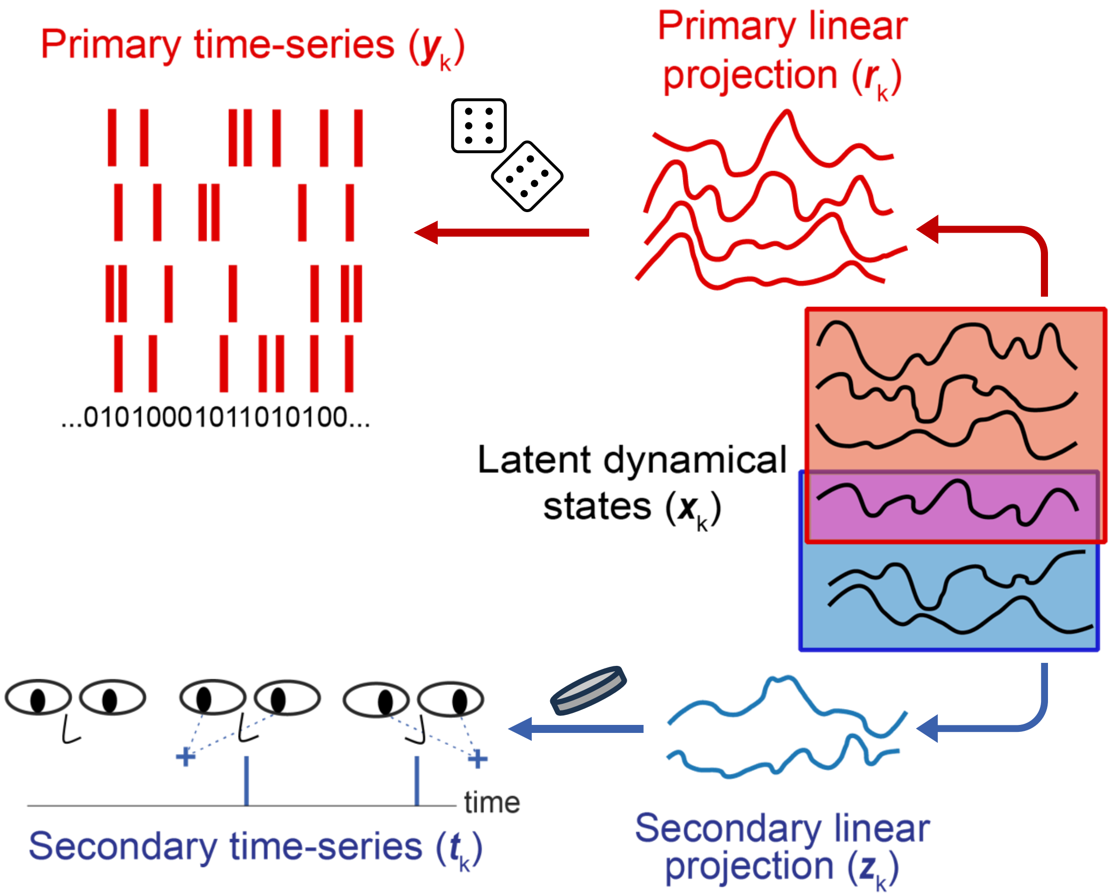
Some applications require joint modeling of two generalized-linear time-series.

Joint GL time-series have shared and private dynamics



PGLDM (prioritized generalized-linear dynamical modeling): a multi-step, subspace identification algorithm for explicitly modeling shared vs private dynamics between two generalized-linear time-series

Partition dynamics using a block-structure latent model definition



Block-structured Latent Dynamical Model

$$\begin{bmatrix} x_{k+1}^{(1)} \\ x_{k+1}^{(2)} \\ x_{k+1}^{(3)} \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + w_k$$

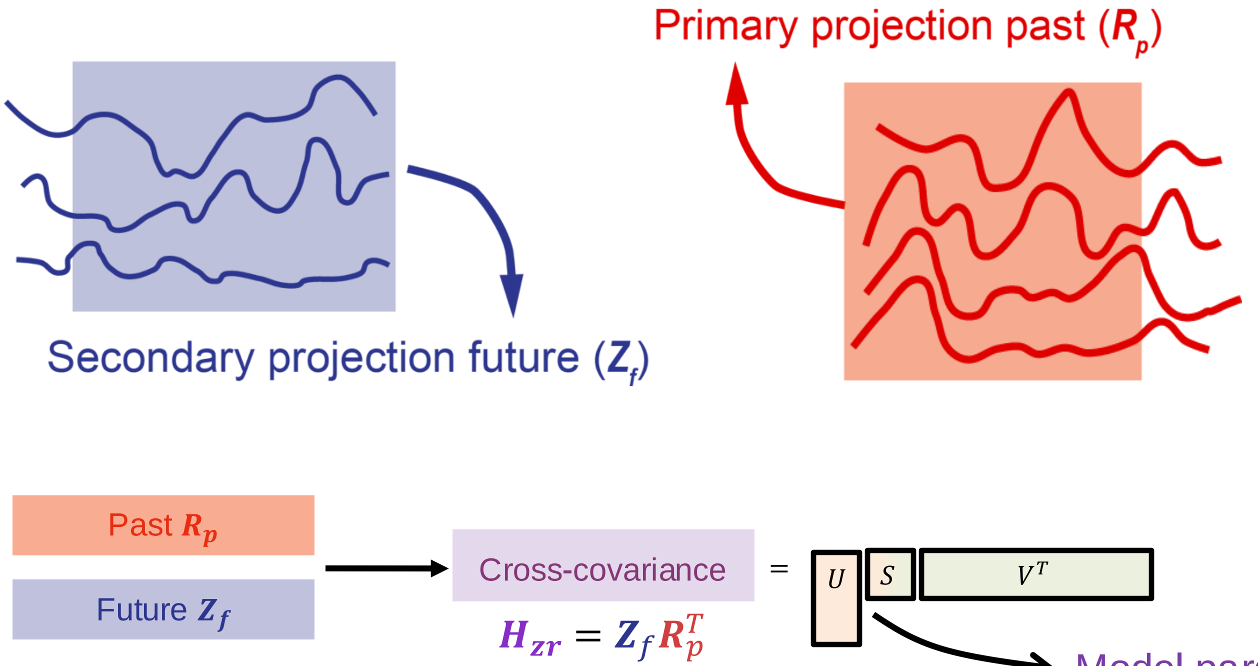
$$r_k = \begin{bmatrix} C_r^{(1)} & C_r^{(2)} & 0 \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + v_k + b$$

$$z_k = \begin{bmatrix} C_z^{(1)} & 0 & C_z^{(3)} \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + \epsilon_k + d$$

$$y_k \mid r_k \sim P_{y|r}(y_k;\ g(r_k))$$

$$t_k \mid z_k \sim P_{t|z}(t_k;\ h(z_k))$$

PGLDM multi-stage learning dissociates shared from private dynamics



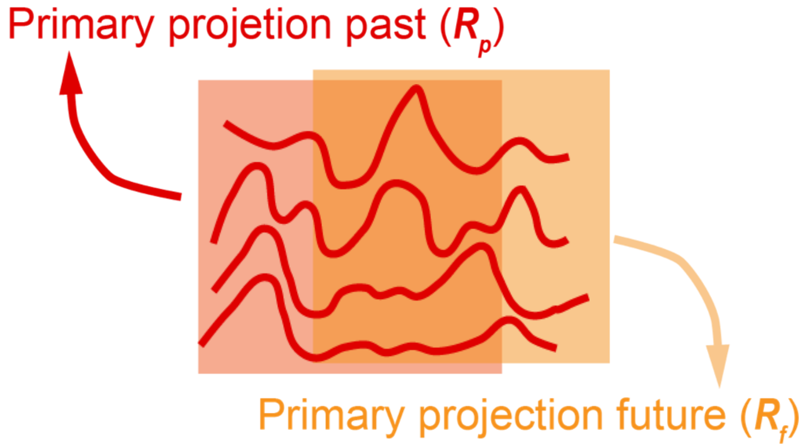
Model parameters: A_{11}, $C_r^{(1)}$, $C_z^{(1)}$, b, d

$$\begin{cases} \begin{bmatrix} x_{k+1}^{(1)} \\ x_{k+1}^{(2)} \\ x_{k+1}^{(3)} \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + w_k \\ r_k = \begin{bmatrix} C_r^{(1)} & C_r^{(2)} & 0 \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + v_k + b \\ z_k = \begin{bmatrix} C_z^{(1)} & 0 & C_z^{(3)} \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + \epsilon_k + d \\ y_k \mid r_k \sim P_{y|r}(y_k;\, g(r_k)) \\ t_k \mid z_k \sim P_{t|z}(t_k;\, h(z_k)) \end{cases}$$

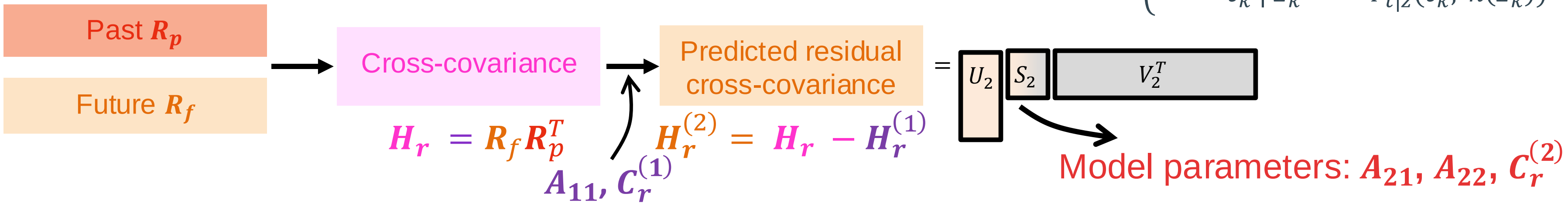
Stage 1. Given shared latent dimensionality n_1, secondary projection z future, and primary projection r past: learn the **shared dynamics**.

PGLDM multi-stage learning dissociates shared from private dynamics

Stage 2. Given private latent dimensionality n_2 and primary projection $\boldsymbol{r}$ future and past: learn the **dynamics private to $\boldsymbol{r}$**.



$$\begin{cases} \begin{bmatrix} x_{k+1}^{(1)} \\ x_{k+1}^{(2)} \\ x_{k+1}^{(3)} \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + w_k \\ r_k = \begin{bmatrix} C_r^{(1)} & C_r^{(2)} & 0 \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + v_k + b \\ z_k = \begin{bmatrix} C_z^{(1)} & 0 & C_z^{(3)} \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + \epsilon_k + d \\ y_k \mid r_k \sim P_{y|r}(y_k;\, g(r_k)) \\ t_k \mid z_k \sim P_{t|z}(t_k;\, h(z_k)) \end{cases}$$



Stage 3. Given private latent dimensionality n_3 and secondary projection $\boldsymbol{z}$ future and past: learn the **dynamics private to $\boldsymbol{z}$, $\{A_{33}, C_z^{(3)}\}$**.

Extending to the generalized-linear case using moment conversion

Algorithm relies on cross-covariances between future and past intermediate linear observations $\boldsymbol{r}$ and $\boldsymbol{z}$

$\boldsymbol{r}$ and $\boldsymbol{z}$ are latent in the generalized-linear case

Instead, primary and secondary generalized-linear time-series $\boldsymbol{y}$ and $\boldsymbol{t}$ (potentially discrete) are observed

How can future-past cross-covariances $\boldsymbol{H}_{zr} = \boldsymbol{Z}_f \boldsymbol{R}_p^T$ and $\boldsymbol{H}_r = \boldsymbol{R}_f \boldsymbol{R}_p^T$ be computed?

$$\begin{cases} \begin{bmatrix} \boldsymbol{x}_{k+1}^{(1)} \\ \boldsymbol{x}_{k+1}^{(2)} \\ \boldsymbol{x}_{k+1}^{(3)} \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{k}^{(1)} \\ \boldsymbol{x}_{k}^{(2)} \\ \boldsymbol{x}_{k}^{(3)} \end{bmatrix} + \boldsymbol{w}_k \\ \boldsymbol{r}_k = \begin{bmatrix} C_r^{(1)} & C_r^{(2)} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{k}^{(1)} \\ \boldsymbol{x}_{k}^{(2)} \\ \boldsymbol{x}_{k}^{(3)} \end{bmatrix} + \boldsymbol{v}_k + \boldsymbol{b} \\ \boldsymbol{z}_k = \begin{bmatrix} C_z^{(1)} & 0 & C_z^{(3)} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{k}^{(1)} \\ \boldsymbol{x}_{k}^{(2)} \\ \boldsymbol{x}_{k}^{(3)} \end{bmatrix} + \boldsymbol{\epsilon}_k + \boldsymbol{d} \\ \boldsymbol{y}_k \mid \boldsymbol{r}_k \sim P_{y|r}(\boldsymbol{y}_k;\ g(\boldsymbol{r}_k)) \\ \boldsymbol{t}_k \mid \boldsymbol{z}_k \sim P_{t|z}(\boldsymbol{t}_k;\ h(\boldsymbol{z}_k)) \end{cases}$$

Empirically computable future-past cross-covariances (ex: $\boldsymbol{Y}_f \boldsymbol{Y}_p^T$) → **Transformation of Moments** → Latent linear future-past cross-covariances (ex: $\boldsymbol{R}_f \boldsymbol{R}_p^T$)

Buesing et al., NeurIPS, 2012

Shared dynamics identified more accurately in simulations

PGLDM more accurately identified shared modes across most generalized-linear observation pairs

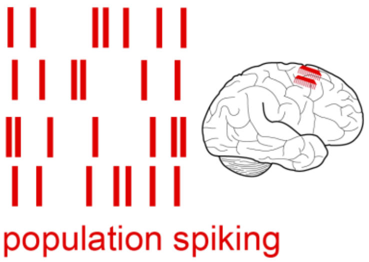
	Primary time-series (r_k or y_k) / Secondary time-series (z_k or t_k)			
Method Name	Gaussian/Gaus.	Poisson/Gaus.	Pois./Pois.	Bernoulli/Gaus.
PGLDM (stage 1)	-2.757 ± 0.070	**-2.707± 0.09**	**-1.969 ± 0.07**	**-2.864 ± 0.072**
Laplace-EM	-1.320 ± 0.09	-1.083 ± 0.11	-1.088 ± 0.11	-1.027 ± 0.067
PSID (stage 1)	**-2.985 ± 0.102**	X	X	X
Covariance SSID	-1.467 ± 0.080	X	X	X
PLDSID	X	-1.319 ± 0.132	-1.203 ± 0.112	X
bestLDS	X	X	X	-1.209 ± 0.117

Normalied eigenvalue error $\frac{||\hat{M} - M||_2}{||M||_2}$

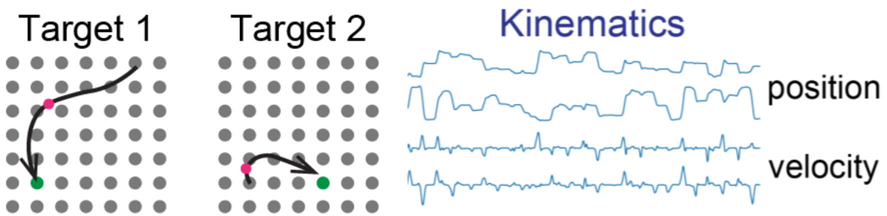
$M \triangleq [\lambda_{z_1} \cdots \lambda_{z_m}]$

Modeling shared dynamics improves secondary time-series decoding performance

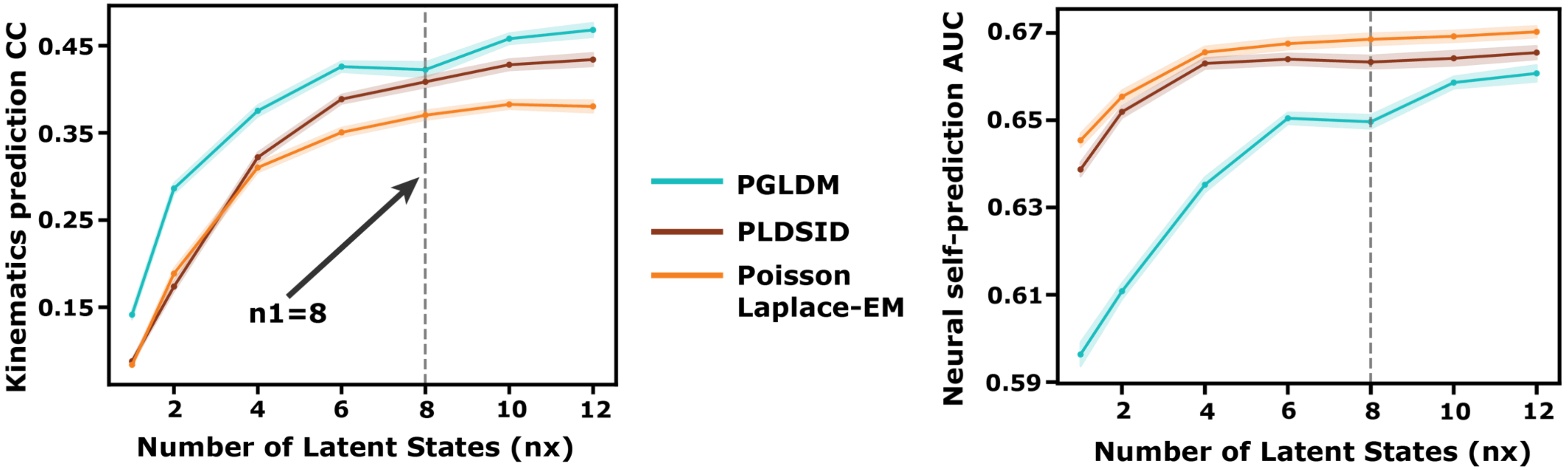
Poisson primary time-series $\boldsymbol{y}_k$



Gaussian secondary time-series $\boldsymbol{z}_k$



O'Doherty et al., doi: 10.5281/zenodo.583331



Conclusions

- We developed and validated PGLDM, a novel analytical subspace identification algorithm for explicitly modeling shared vs private dynamics between two generalized-linear time-series
- Because PGLDM more accurately dissociated shared from private dynamics, models learned by PGLDM more accurately decoded a secondary time-series from a primary time-series using lower-dimensional latent models
- We also demonstrated that PGLDM can be used with any combination of generalized-linear observation pairs -- as long as there exists computationally tractable moment conversion equations

Acknowledgments

Shanechi Lab

Thank you for your attention!