

Problem & Motivation

- Quantify and analyze structures of individual Gaussians in 3DGS
- Improve needle-like artifacts and surface geometry of the reconstruction results.
- Adopt **Effective rank** for analyzing the Gaussian structure and regularization

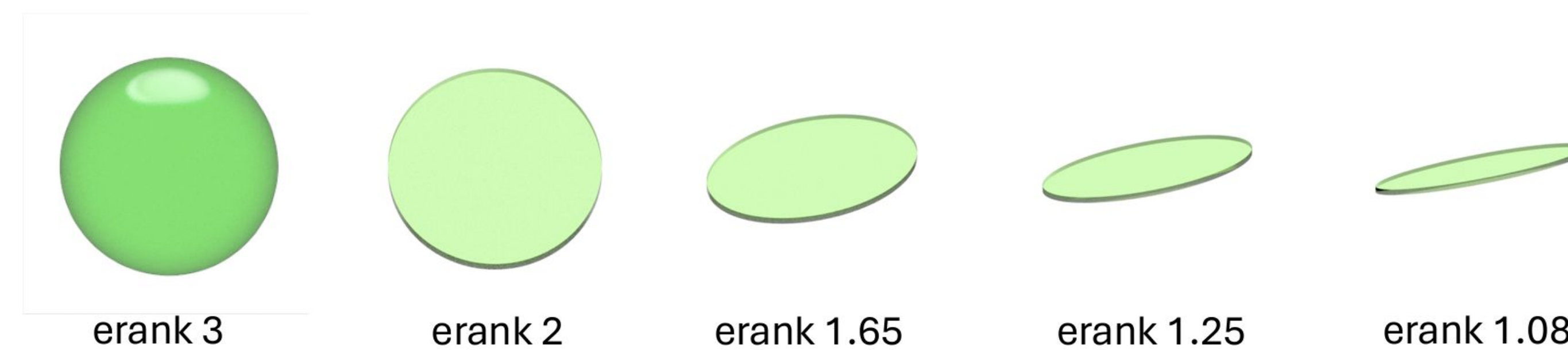
Contribution

- Gaussian Structure Analysis:** We analyze Gaussian primitive structures and their training dynamics via effective rank, finding convergence to anisotropic forms with one dominant variance.
- Optimal Gaussian Structure:** We identify that disk-like Gaussians are preferred structure.
- Effective Rank Regularization:** We introduce a regularization method to reduce needle-like artifacts in 3DGS, enhancing geometry reconstruction.
- Compatibility:** Our method is a modular add-on for 3DGS variants, improving geometry without sacrificing visual quality.

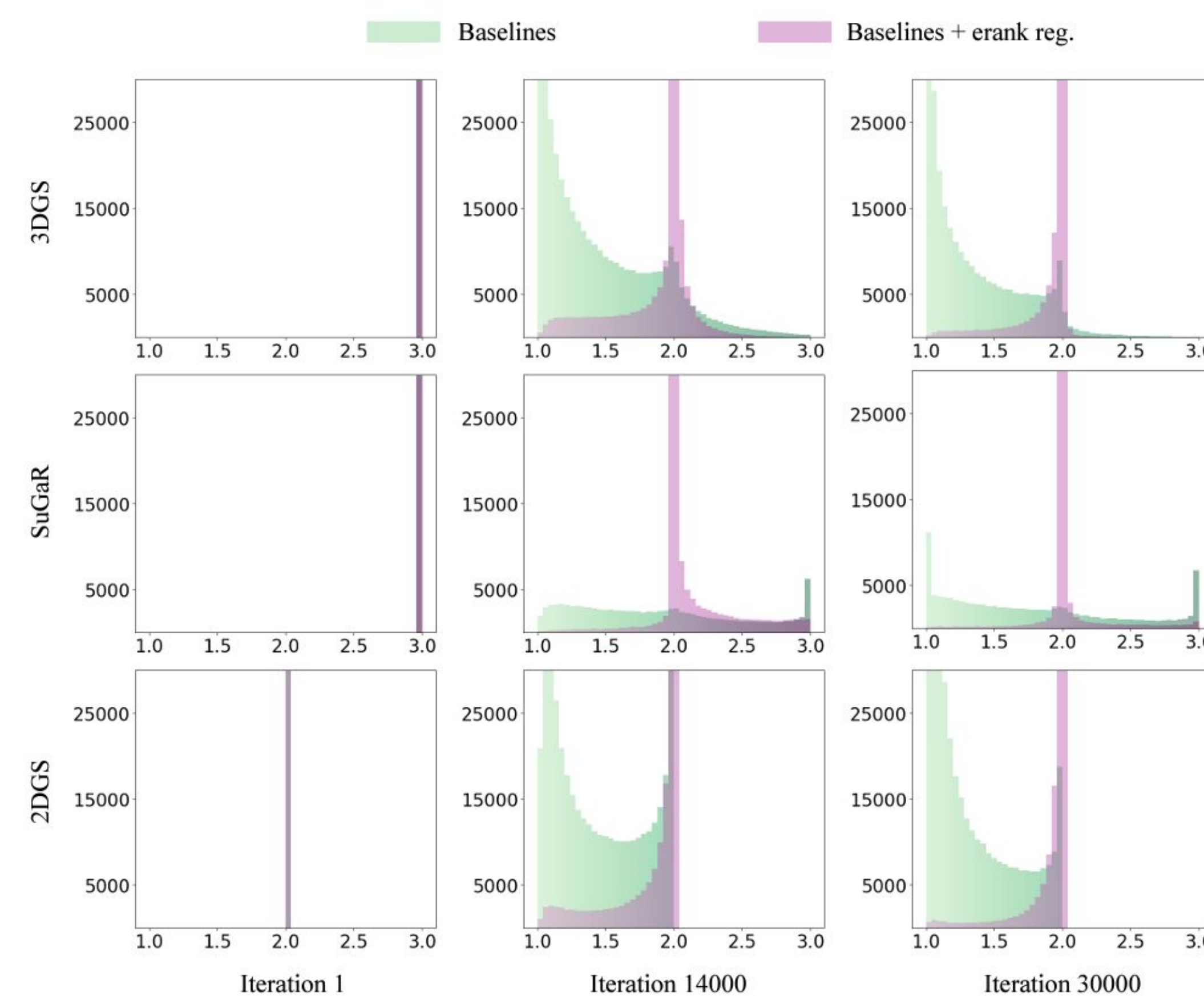
Effective Rank

Definition 1 (Effective rank). The effective rank of the matrix \mathbf{A} is concisely defined as $\text{erank}(\mathbf{A}) = \exp\{H(q_1, q_2, \dots, q_L)\}$, where $H(q_1, q_2, \dots, q_L)$ is the Shannon entropy given by

$$H(q_1, q_2, \dots, q_L) = -\sum_{i=1}^L q_i \log q_i.$$



Effective Rank Analysis



Effective Rank Regularization

$$\mathcal{L}_{\text{erank}} = \sum_k \lambda_{\text{erank}} \max(-\log(\text{erank}(\mathcal{G}_k) - 1 + \epsilon), 0) + s_3$$

- Directly leverage the effective rank for regularization
 - logarithmic term suitable for loss function
- Spherical Gaussian:** $\text{erank}(\mathcal{G}) \approx 3$
 - Biased, inadequate for representing surfaces
- Needle-like Gaussian:** $\text{erank}(\mathcal{G}) \approx 2$
 - Artifacts, holes, Inadequate for representing surfaces
- Disk-like Gaussians** for unbiased & efficient surface representation

Results

Method	24	37	40	55	63	65	69	83	97	105	106	110	114	118	122	Mean	Std.	PSNR
3DGS	2.14	1.53	2.08	1.68	3.49	2.21	1.43	2.07	2.22	1.75	1.79	2.55	1.53	1.52	1.50	1.96	0.52	32.82
3DGS+e	0.85	0.77	0.88	0.51	1.21	1.45	0.96	1.30	2.09	0.72	0.86	1.45	0.87	0.94	0.66	1.03	0.39	33.09
SuGaR	1.47	1.33	1.13	0.61	2.25	1.71	1.15	1.63	1.62	1.07	0.79	2.45	0.98	0.88	0.79	1.33	0.52	31.59
SuGaR+e	0.86	0.78	0.89	0.53	1.28	1.45	0.87	1.31	1.60	0.72	0.86	1.45	0.87	0.94	0.66	1.00	0.33	31.76
2DGS	0.48	0.91	0.39	0.39	1.01	0.83	0.81	1.36	1.27	0.76	0.70	1.40	0.40	0.76	0.52	0.80	0.33	32.43
2DGS+e	0.46	0.86	0.39	0.40	0.96	0.84	0.81	1.29	1.19	0.72	0.70	1.32	0.40	0.75	0.50	0.77	0.30	32.57
GOF	0.50	0.82	0.37	0.37	1.12	0.78	0.73	1.18	1.29	0.71	0.77	0.90	0.44	0.69	0.49	0.74	0.28	32.88
GOF+e	0.45	0.66	0.32	0.42	0.97	0.78	0.64	1.13	1.22	0.64	0.62	0.70	0.40	0.53	0.48	0.66	0.26	33.01

