DMesh: A Differentiable Mesh Representation

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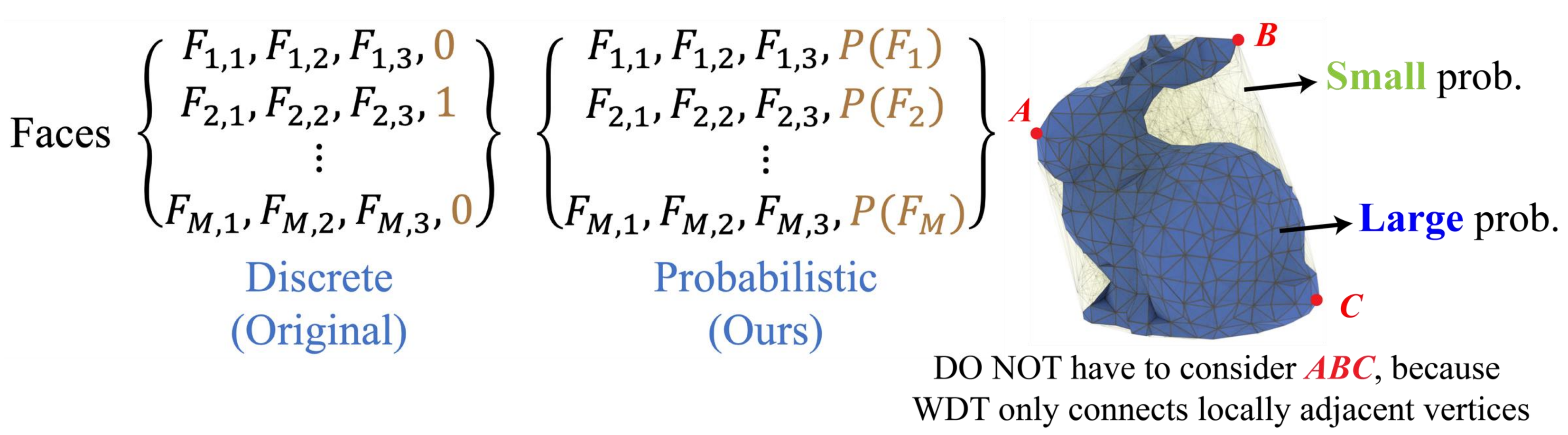
Background

A mesh is defined with a set of vertices and faces, which illustrate the connectivity relationship between the vertices. This definition poses two challenges in adopting mesh into the current ML pipeline:

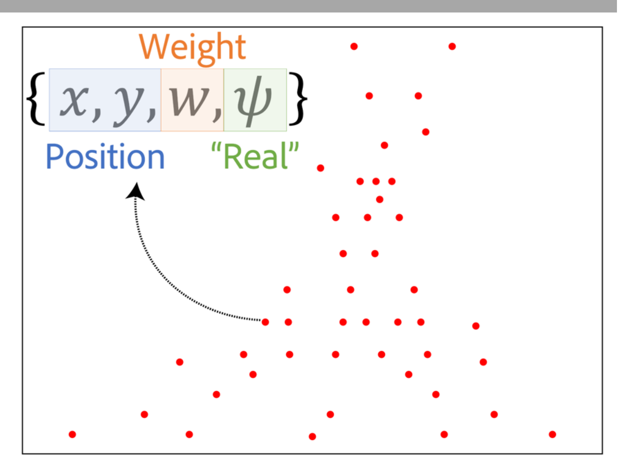
1) Discrete connectivity (Exist: 1, Not Exist: 0)
2) Exponential growth of possible connectivity

DMesh addresses these challenges by proposing
1) Probabilistic connectivity
2) Linear growth of possible connectivity based on WDT (Weighted Delaunay Triangulation)

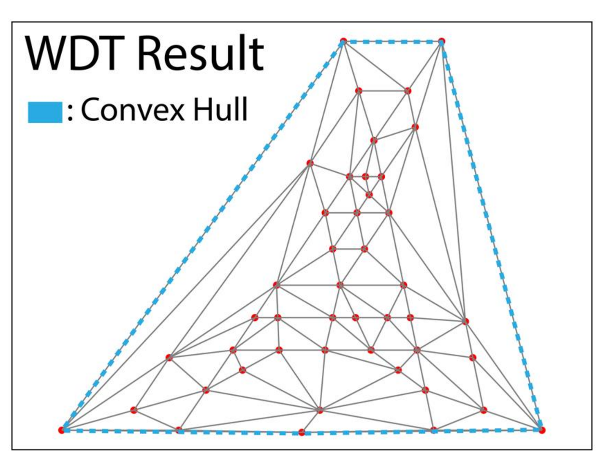


Approach

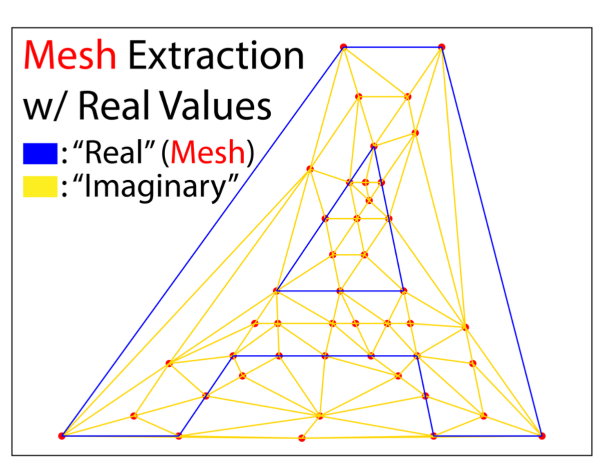
1. We define **DMesh** with a set of (featured) points. Each point carries position, weight, and "real" value. Weight represents the importance of the point and used in WDT process. "Real" value is used to select desirable faces from WDT. *(Faces are edges in 2D case)*



2. We compute WDT of the points. The WDT result solely depends on the point positions and weights. WDT tessellates the convex hull of the points, and the points with small weights are discarded in this process. Note that the *faces do not intersect with each other*. Also note that *we do not need every face to recover the target shape* ("A").



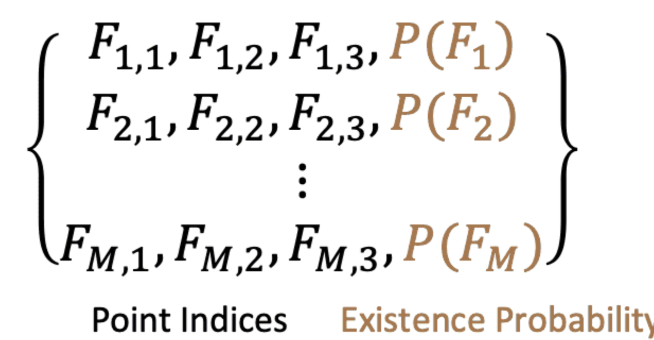
3. We select desirable faces in WDT to recover the target shape ("A"). We use "real" values to select the faces. If every point on a face has "real" value of 1, the face is "real". Otherwise, the face is "imaginary". *We define the final mesh with the "real" faces* and discard "imaginary" faces. *Note that the final mesh solely depends on the point features.*



Formulation

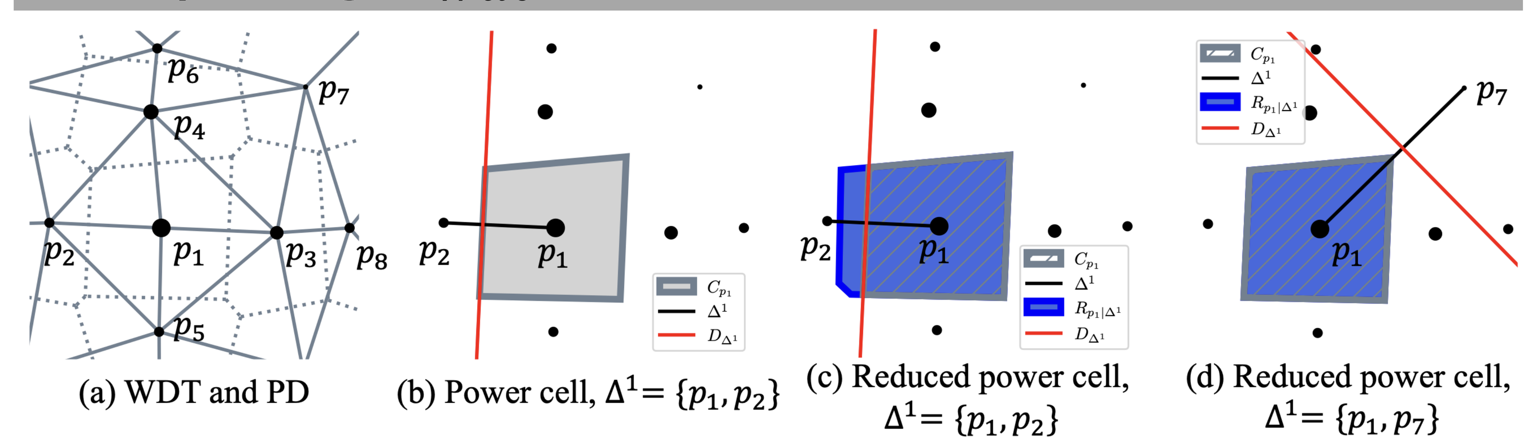
SUM UP: Two conditions for a face to be included in the mesh:
(a) The face should be in WDT
(b) All of the points on the face should have "real" value of 1
(i.e. the minimum "real" value should be 1)

For a given face F, we compute its probability to satisfy each condition:
(a) $\Lambda_{wdt}(F)$ and (b) $\Lambda_{real}(F)$.



Then, the final probability of F, $P(F)$, becomes $\Lambda_{wdt}(F) \times \Lambda_{real}(F)$. We can compute $\Lambda_{real}(F)$ efficiently with differentiable min. operator.

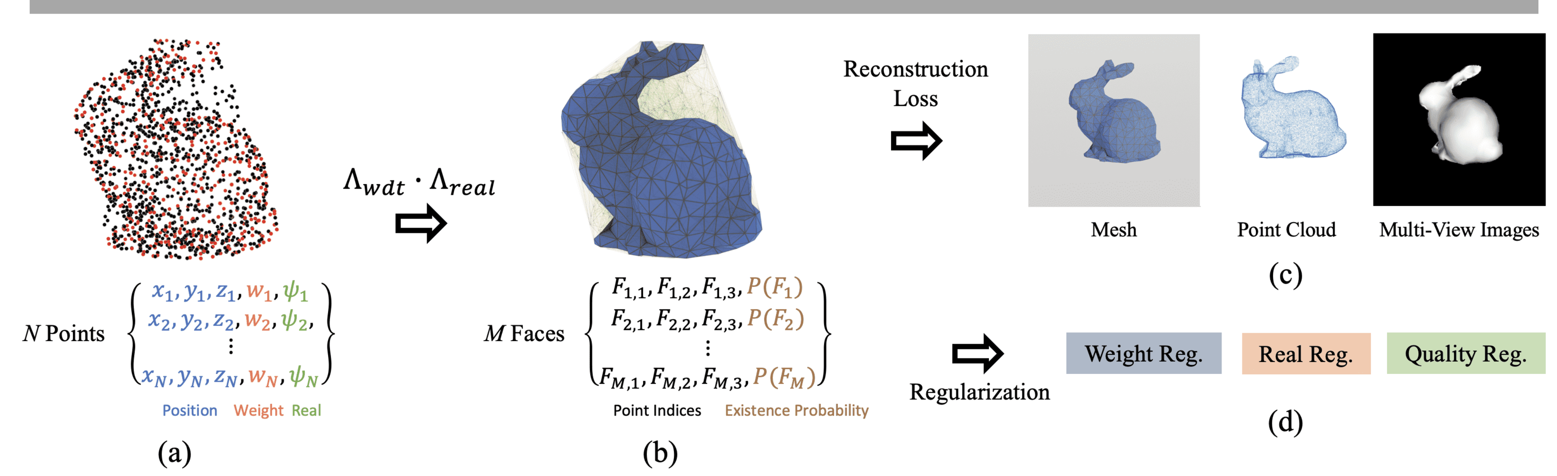
Computing $\Lambda_{wdt}(F)$



We use Power Diagram (PD), which is dual of WDT, to compute $\Lambda_{wdt}(F)$. For a given face F, its dual line goes through reduced power cells of the points on F if and only if F exists in WDT. We compute signed distance between the dual line of F and the reduced power cells to compute $\Lambda_{wdt}(F)$. We construct PD first for acceleration (Please see our paper for details!)

- The previous exhaustive approach[1] requires $O(n^2)$ cost ***(quadratic)***
- Our approach requires (approximately) $O(n)$ cost ***(linear)***

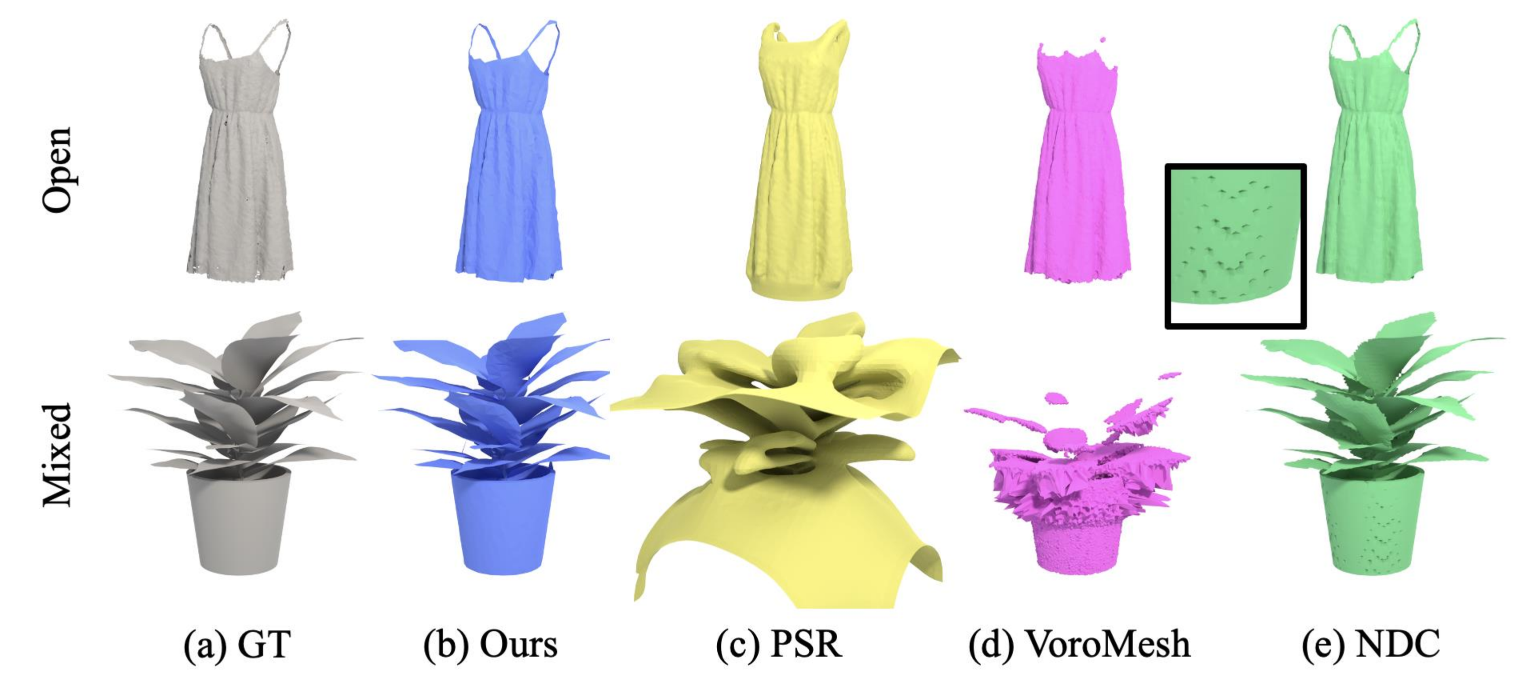
Reconstruction Task



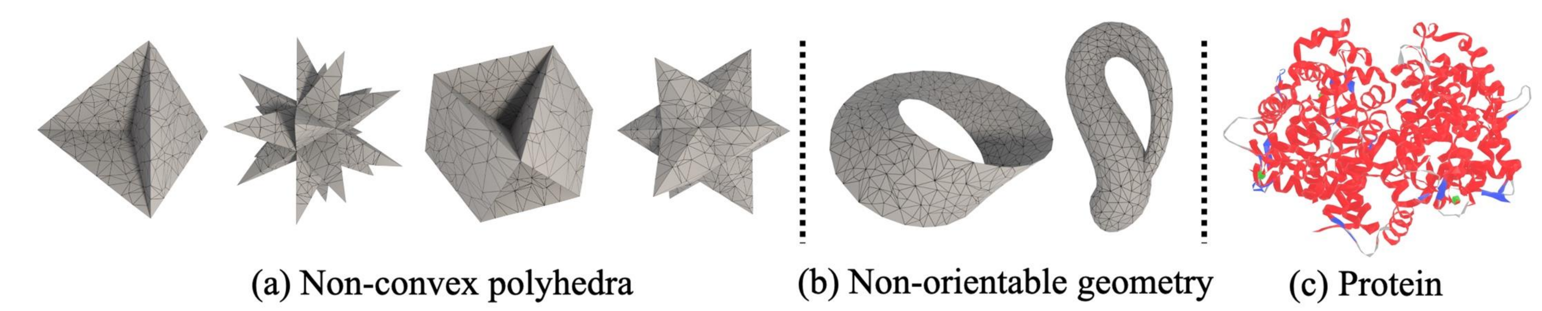
For the given point configuration, we evaluate face probabilities. Then, we compute **reconstruction loss** based on the given observations (e.g. ground truth mesh, point cloud, or multi-view images). We also compute additional **regularization losses** to produce mesh of desired quality. Then, we minimize the loss by optimizing only the point features. We introduce loss formulations for these probabilistic faces in our paper.

Experimental Results

Reconstruction from Point Clouds

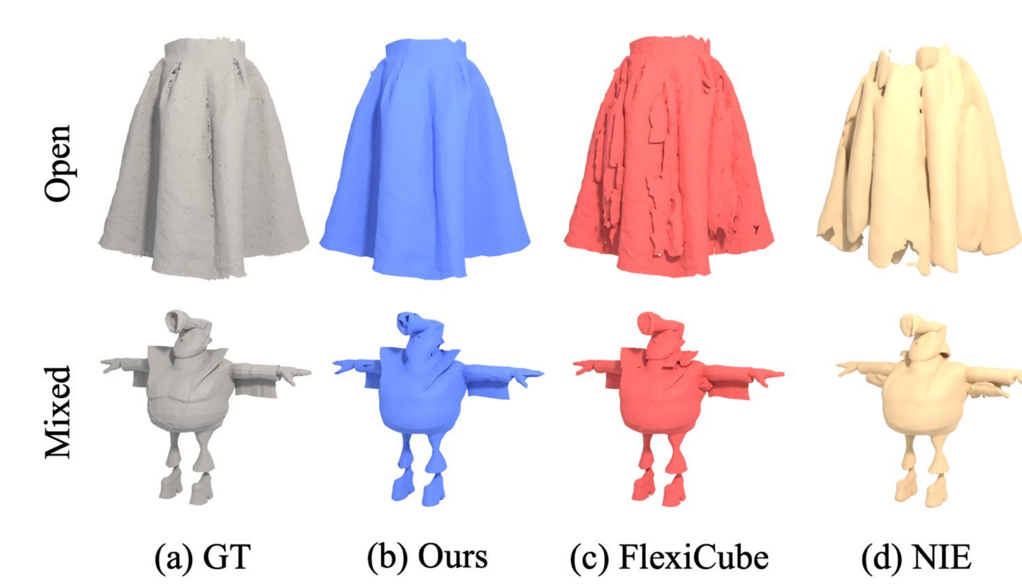


As **DMesh** is a versatile shape representation, it can reconstruct shapes of diverse topology better than the other methods from point clouds as shown above. For even harder cases, **DMesh** works well as shown below.

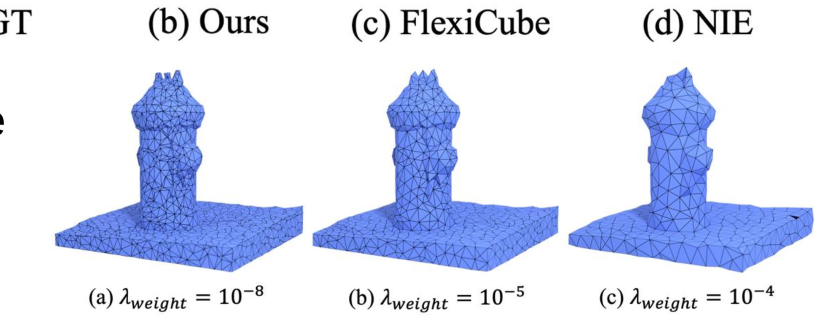


Reconstruction from Multi-View Images

DMesh can be used for reconstructing shapes from multi-view images, too. As shown on the right, it performs better than the other methods because of its better representation power.



As shown on the right, we can use regularization to control the mesh complexity during optimization.



Limitations

1. ***Computational Cost:*** Linear time complexity is still excessive when the number of points exceeds 30-50K.
2. ***Non-Manifoldness:*** There is no guarantee of non-manifoldness, because we do not have any topological assumption.

[1] Rakotosaona, Marie-Julie, et al. "Differentiable surface triangulation." *ACM Transactions on Graphics (TOG)* 40.6 (2021): 1-13.