Randomized Sparse Matrix Compression for Large-Scale Constrained Optimization in Cancer Radiotherapy

Shima Adeli, **Mojtaba Tefagh**, Gourav Jhanwar, Masoud Zarepisheh

NeurIPS 2024

Motivation and Problem Statement

- Radiotherapy treats over half of all cancer patients and requires solving large-scale constrained optimization problems.
- Goal: Optimize radiation beamlet intensities x to achieve desired dose distribution $d = Ax$.
- Challenge: The dose influence matrix A is large and dense, making optimization computationally intensive.

Original Problem (A is large and dense)

$$\text{Minimize } f(Ax, x) \quad \text{subject to } g(Ax, x) \leq 0$$

Surrogate Problem (A $\approx$ S, S is sparse)

$$\text{Minimize } f(Sx, x) \quad \text{subject to } g(Sx, x) \leq 0$$

Algorithm and Theoretical Insights

- Proposed **Randomized Minor-value Rectification (RMR)** algorithm which retains significant matrix elements while redistributing smaller values via random sampling.

Feasibility Gap (Theorem 3.6 from main article)

An optimal point of the surrogate problem violates each constraint of the original problem by no more than $(19 + 5 \log m)\epsilon\|x\|_2$ with a probability of at least 0.95.

Sub-optimality Gap (Theorem 3.9 from main article)

An optimal point of the surrogate problem is a near-optimal solution to the original problem with a probability of at least 0.95, and the sub-optimality gap $O((19 + 5 \log m)\,\epsilon\, \max(\|x_A\|_2, \|x_S\|_2))$.

Experimental Results

- Experiments on real-world lung and prostate cancer data.
- **RMR** reduced computational time significantly while maintaining treatment quality.
- Achieved 96-98% sparsification with clinically acceptable optimality gaps.
- Outperformed existing techniques in preserving treatment quality.

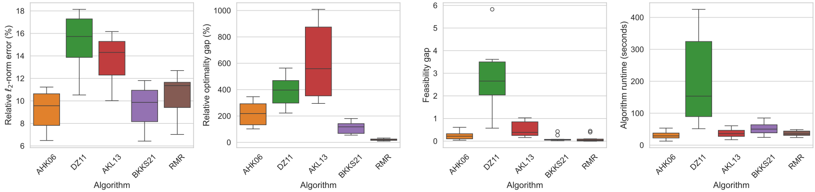

Figure: Comparison of RMR with existing methods for ten lung patients.

Conclusion

- New application: Optimizing radiotherapy treatment planning.
- New algorithm: **RMR** with strong mathematical properties.
- Theoretical guarantees ensure minimal impact on optimization constraints and objectives.
- **RMR** provides fast sparsification with accurate approximation, enabling rapid, high-quality treatment planning.
- Potential to improve cancer patient outcomes.

Resources and Code

- Code and data:
  `https://github.com/PortPy-Project/CompressRTP`