## **Mind the Gap Between Prototypes and Images in Cross-domain Finetuning**

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# **Outline**

- Background
- Revisit Previous Adaptation Strategy
- Contrastive Prototype-Image Adaptation (CoPA)
- Summary

### Mind the Gap Between Prototypes and Images in **Cross-domain Finetuning**

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## **Preliminary: Cross-domain Few-shot Classification**

### **Few-shot classification**



### **Challenges** in CFC:

- **Varied** numbers of classes & shots in CFC tasks;
- **Distribution discrepancy** among datasets.

An example of conventional few-shot classification tasks

Phillip Lippe, Tutorial 16: Meta-Learning - Learning to Learn, UvA DL Notebooks v1.2 Documentation.

## **Preliminary: Prototypical Networks**

**Few-shot classification with prototypes**

• **Construct prototypes:**



$$
c_i = \frac{1}{|C_i|} \sum_{x \in C_i} x
$$

• **Calculate similarities/distances:**

$$
L = \frac{1}{|D_T|} \sum_{i=1}^{|D_T|} log(p(\hat{y} = y_i | x_i))
$$

$$
p(\hat{y} = y_i | x_i) = \frac{exp(-d(x, c_i))}{\sum_j exp(-d(x, c_j))}
$$

Snell et al., Prototypical networks for few-shot learning, NIPS 2017.

# **Previous Works**

**Finetuning a transformation on top of a universal pretrained backbone**

- **Pretraining:** Distill a universal backbone from several taskspecific backbones.
- **Meta-Test:** Adapting a simple transformation module on top of the pretrained backbone.



Li et al., Universal representation learning from multiple domains for few-shot classification, ICCV 2021.

## **Motivation**

### **An Implicit Assumption**



**Instance-level and prototype-level embeddings share the same representation transformation.** 

# **Motivation**

### **An intuition of prototypes**



**Text:** Abstract information of a set of image instances.



**Prototype:** Information commonly shared across of images in a class.

**Prototypes play the similar role (Higher level information) to the texts in multimodal frameworks.** 

Radford et al., Learning Transferable Visual Models From Natural Language Supervision, ICML 2021.



### There exist modality gaps between different modalities of data



 $-0.7$ 

 $-0.8$ 

 $-0.9$ 

 $-1.0$ 

 $\vec{\Delta} := \frac{1}{|\mathcal{D}_{\mathcal{T}}|} \sum_{i=1}^{|\mathcal{D}_{\mathcal{T}}|} \boldsymbol{z}_i - \frac{1}{N_C} \sum_{i=1}^{N_C} \boldsymbol{c}_j$ 

images

 $-0.6$ 

prototypes

 $-0.5$ 



**Larger gap facilitates better generalization performance**

Slightly enlarging the gap improves the generalization performance!



### **Potential Reasons:**

- **[Overfitting]** The prototype representations are generated directly from image instance representations. Thus, instances are naturally similar to prototypes;
- **[Alignment]** Representations of prototypes and image instances are not well aligned.

#### **The effect of the shared transformation**

- The shared transformation **narrows** the gap between prototype and image instance representations;
- The shared transformation fails to learn image representations that are well clustered for each class.









#### **Further Analyses**

The shared transformation tends to drop the discriminative information in gradients respectively for prototypes and images.

**Theorem 3.1.** Let the measure  $d(\cdot, \cdot)$  be the cosine similarity function. Given a set of normalized finite support data representation  $\mathcal{Z} = \{ (z_i, y_i) \}_{i=1}^n$ , where  $||z||_2 = 1$  for  $\forall z \in \mathcal{Z}$  and  $N_C$  classes are included, then we have a lower bound of the NCC-based loss in Eq.  $(1)$ :

$$
\mathcal{L}(\theta) \geq -\frac{1}{n}\sum_{i=1}^n \bm{z}_i^\top \bm{c}_c + \frac{\alpha}{n}\sum_{i=1}^n \sum_{\bm{z}'\in\mathcal{Z}} \bm{z}_i^\top \bm{z}',
$$

where z' is an independent copy of samples in Z, C, denotes sets of sample representations  $C_c =$  $\{z_i|y_i=c\}$ , and  $\alpha$  is a constant that satisfies  $0 \leq \alpha < 1/(N_C|\mathcal{C}_i|)$  for  $\forall j$ .

$$
\mathcal{L}(\Theta_{\mathrm{P}},\Theta_{\mathrm{I}}) = -\frac{1}{|\mathcal{D}_{\mathcal{T}}|}\mathrm{Tr}\left(f_{\phi^*}(\boldsymbol{X})\Theta_{\mathrm{I}}(\boldsymbol{Y}\boldsymbol{Y}^{\top}\boldsymbol{f}_{\phi^*}(\boldsymbol{X})\Theta_{\mathrm{P}})^{\top}\right) + \frac{\alpha}{|\mathcal{D}_{\mathcal{T}}|}\mathrm{Tr}\left(f_{\phi^*}(\boldsymbol{X})\Theta_{\mathrm{I}}\Theta_{\mathrm{I}}^{\top}\boldsymbol{f}_{\phi^*}(\boldsymbol{X})^{\top}\right),
$$

where  $X \in \mathbb{R}^{|\mathcal{D}_{\mathcal{T}}| \times d_{\text{out}}}$  and  $Y \in \mathbb{R}^{|\mathcal{D}_{\mathcal{T}}| \times N_C}$  respectively denote the support image instances and the corresponding one-hot labels,  $\Theta_{\rm P} \in \mathbb{R}^{d_{\rm out} \times d_{\rm out}}$  and  $\Theta_{\rm I} \in \mathbb{R}^{d_{\rm out} \times d_{\rm out}}$  denote the model parameters of linear transformation heads respectively for prototype and image instance embeddings,  $Tr(\cdot)$  denotes the matrix trace operation.  $YY^{\top} f_{\phi^*}(\mathbf{X}) \in \mathbb{R}^{|\mathcal{D}_{\tau}| \times d_{\text{out}}}$  denotes the prototypes which are expanded to the same size of instance embeddings. In this way, the gradients w.r.t.  $\Theta_P$  and  $\Theta_T$  are:

$$
\nabla_{\Theta_{\mathbf{P}}}\mathcal{L}(\Theta_{\mathbf{P}},\Theta_{\mathbf{I}}) = -\frac{1}{|\mathcal{D}_{\mathcal{T}}|} \Theta_{\mathbf{I}}^{\top} f_{\phi^*}(\boldsymbol{X})^{\top} Y Y^{\top} f_{\phi^*}(\boldsymbol{X}),
$$
  

$$
\nabla_{\Theta_{\mathbf{I}}} \mathcal{L}(\Theta_{\mathbf{P}},\Theta_{\mathbf{I}}) = -\frac{1}{|\mathcal{D}_{\mathcal{T}}|} \Theta_{\mathbf{P}}^{\top} f_{\phi^*}(\boldsymbol{X})^{\top} Y Y^{\top} f_{\phi^*}(\boldsymbol{X}) + \frac{2\alpha}{|\mathcal{D}_{\mathcal{T}}|} \Theta_{\mathbf{I}}^{\top} f_{\phi^*}(\boldsymbol{X})^{\top} f_{\phi^*}(\boldsymbol{X}).
$$

#### **Further Analyses**

The empirical results regarding the coefficient of the upper bound indicate that the shared transformation will shrink the gap since **the coefficient is consistently smaller than 1.0**.

**Theorem 3.2 (The shared transformation).** Consider a support data set  $\mathcal{D}_{\mathcal{T}} = \{ (x_i, y_i) \}_{i=1}^{|\mathcal{D}_{\mathcal{T}}|}$ composed of N<sub>C</sub> classes and a frozen pretrained backbone  $f_{\phi^*}: \mathbb{R}^{d_{\text{in}}} \to \mathbb{R}^d$  parameterized with the optimal parameters  $\phi^*$ . Let  $\Theta \in \mathbb{R}^{d \times d}$  be a shared linear transformation across the prototype and image instance embeddings. Then, we can obtain the image instance representations  $Z = \{z_i\}_{i=1}^{|\mathcal{D}_{\mathcal{T}}|} = \{f_{\phi^*}(x_i) \Theta\}_{i=1}^{|\mathcal{D}_{\mathcal{T}}|}$ , and the prototype representations  $\mathcal{C} = \{c_i\}_{i=1}^{N_C}$ , where  $c_i =$  $\frac{1}{|C_1|}\sum_{\mathbf{z}'\in C_1}\mathbf{z}'=\frac{1}{|C_1|}\sum_{\mathbf{z}'\in C_1}\int_{\phi^*}(\mathbf{z}')\Theta.$  Then we can obtain the bounds of the representation gap:

$$
m \left\|\Theta\right\|_{F}^{2} \left\|\vec{\Delta}_{\text{emb}}\right\|_{2}^{2} \leq \left\|\frac{1}{\left|\mathcal{D}_{\mathcal{T}}\right|} \sum_{\mathbf{z} \in \mathcal{Z}} \mathbf{z} - \frac{1}{N_{C}} \sum_{\mathbf{c} \in \mathcal{C}} \mathbf{c}\right\|_{2}^{2} \leq M \left\|\Theta\right\|_{F}^{2} \left\|\vec{\Delta}_{\text{emb}}\right\|_{2}^{2},
$$

where  $\vec{\Delta}_{emb} = \frac{1}{|\mathcal{D}_{\tau}|} \sum_{\bm{x} \in \mathcal{D}_{\tau}} f_{\phi^*}(\bm{x}) - \frac{1}{N_C} \sum_{b=1}^{N_C} \left( \frac{1}{|\mathcal{C}_b|} \sum_{\bm{x}' \in \mathcal{C}_b} f_{\phi^*}(\bm{x}') \right)$  denotes the gap between prototype and image embeddings,  $m = \min_{1 \leq i \leq d} \cos^2(\vec{\Delta}_{emb}, \Theta^i)$  denotes the minimum value of  $\cos^2(\vec{\Delta}_{emb}, \Theta^i)$ , and  $M = \max_{1 \leq j \leq d} \cos^2(\vec{\Delta}_{emb}, \Theta^j)$  denotes the maximum of  $\cos^2(\vec{\Delta}_{emb}, \Theta^j)$ .



#### The coefficient is consistently smaller than 1.0.

### **CoPA: Contrastive Prototype and Image Adaptation**

$$
\lim_{\theta_{\mathrm{P}},\theta_{\mathrm{I}}} \mathcal{L}(\theta_{\mathrm{P}},\theta_{\mathrm{I}}) := \mathcal{L}_{\mathrm{CE}}(\frac{1}{\tau} \bm{Z}_{\mathrm{I}} \bm{Z}_{\mathrm{P}}^{\top}, Y_{\mathrm{pseudo}}) + \mathcal{L}_{\mathrm{CE}}(\frac{1}{\tau} \bm{Z}_{\mathrm{P}} \bm{Z}_{\mathrm{I}}^{\top}, Y_{\mathrm{pseudo}})
$$

- The discriminative information in gradients are preserved in two different sets of parameters;
- The expanded prototypes indicate the cluster structure of the given support data set.



#### Algorithm 1 CoPA Algorithm.

**Input:** pre-trained backbone  $f_{\phi^*}$ , number of inner iterations n, learning rate  $\eta$ , linear transformation heads  $h_{\theta_{\rm p}}$  and  $h_{\theta_{\rm r}}$ , temperature coefficient  $\tau$ . **Output:** the optimal parameters for linear transformation heads  $\theta_{\rm P}^*$  and  $\theta_{\rm I}^*$ . # Sample a task **Sample** a new support data set  $\mathcal{D}_{\mathcal{T}} = \{X, Y\}$ ; **Generate** pseudo labels  $Y_{\text{pseudo}} = \{0, 1, ..., |\mathcal{D}_{\mathcal{T}}| - 1\};$ # Performing contrastive prototype-image adaptation for  $i=1$  to n do **Obtain** the prototype and instance representations:  $\mathbf{Z}_{\mathrm{P}} = h_{\theta_{\mathrm{P}}}(YY^{\top} f_{\phi^*}(\boldsymbol{X}));$  $Z_{I} = h_{\theta_{I}}(f_{\phi^{*}}(\boldsymbol{X}));$ **Compute** SCE loss  $\mathcal{L}(\hat{\theta}_P, \theta_I)$  in Eq. (3); **Update** parameters:  $\theta_{\rm P} \leftarrow \theta_{\rm P} - \eta \nabla_{\theta_{\rm P}} \mathcal{L}(\theta_{\rm P}, \theta_{\rm I});$  $\theta_{\rm I} \leftarrow \theta_{\rm I} - \eta \nabla_{\theta_{\rm I}} \mathcal{L}(\theta_{\rm P}, \theta_{\rm I});$ end for



 $0.000000$ 0  $\Lambda$  $0$  O O 1111 V  $\sqrt{1}$  $222$  $\mathfrak{D}$ 2 2  $\overline{\mathbf{r}}$  $222$ 3 Β, 3 - 3 33 -S -S 5 -5  $\leq$ ما 6 6 ת 88 88  $\mathcal{P}$  $\mathcal{L}$ 2999999999999999





Triantafillou et al., Meta-dataset: A dataset of datasets for learning to learn from few examples, ICLR 2020. Requeima et al. Fast and flexible multi-task classification using conditional neural adaptive processes. NeurIPS 2019.

**Main results: train on ImageNet only**

Table 2: Results on Meta-Dataset under the "train on ImageNet only" setting. Under the "train on ImageNet only" setting, only ImageNet is treated as "seen domain" while the remaining as "unseen domains". Mean accuracy and 95% confidence interval are reported.



<sup>1</sup> The results on URL, TSA, TA<sup>2</sup>-Net and our proposed methods are reproduced with 5 random seeds and reported as the average of the 5 reproduction. The ranks only consider the first 10 datasets and are calculated only with the methods in the table.

**Main results: train on all datasets**

Table 1: Results on Meta-Dataset under the "train on all datasets" setting. Under the "train on all datasets" setting, the first 8 datasets are treated as "seen domians" while the last 5 are treated as "unseen domains". Mean accuracy and 95% confidence interval are reported.



<sup>1</sup> For fairness, the results of URL, TSA, TA<sup>2</sup>-Net, and our proposed CoPA methods are reproduced with 5 random seeds, and we report the average of the 5 reproductions in the table. Particularly, although the reported performance of URL is lower than that in the original paper, the reproduction results are consistent with those reported on their project website. The ranks are calculated only with the first 10 datasets and only with the methods mentioned above.



(1) Fungi CoPA:  $||\vec{\Delta}|| = 1.75$ 

(k) Fungi URL:  $||\vec{\Delta}|| = 0.08$ 

#### **Further Analysis - Alignment**

$$
\mathcal{L}_{\text{SCE}} = -\frac{1}{|\mathcal{D}_{\mathcal{T}}|} \sum_{i=1}^{|\mathcal{D}_{\mathcal{T}}|} \log \frac{\exp(\boldsymbol{z}_i^{\top} \boldsymbol{c}_i)}{\sum_{j=1}^{|\mathcal{D}_{\mathcal{T}}|} \exp(\boldsymbol{z}_i^{\top} \boldsymbol{c}_j)} - \frac{1}{|\mathcal{D}_{\mathcal{T}}|} \sum_{i=1}^{|\mathcal{D}_{\mathcal{T}}|} \log \frac{\exp(\boldsymbol{c}_i^{\top} \boldsymbol{z}_i)}{\sum_{j=1}^{|\mathcal{D}_{\mathcal{T}}|} \exp(\boldsymbol{c}_i^{\top} \boldsymbol{z}_j)}.
$$
(4)

**Theorem 5.1.** Given a set of normalized finite support data representation  $\mathcal{Z} = \{ (z_i, y_i) \}_{i=1}^n$  and a set of normalized prototype representations  $C = \{ c_i \}_{i=1}^n$ , where  $||z||_2 = 1$  for  $\forall z \in \mathcal{Z}$  and  $||c||_2 = 1$ for  $\forall c \in C$ , then we are able to obtain a lower bound of SCE loss in Eq. (4):

$$
\mathcal{L}_{\text{SCE}} \geq -\frac{2}{n}\sum_{i=1}^n \bm{z}_i^\top \bm{c}_i + \boxed{\frac{2}{n}\sum_{i=1}^n\sum_{k=1}^{N_C}\frac{|\mathcal{C}_k|}{n}\bm{z}_i^\top \bm{c}_k},
$$

where  $C_k$  denotes the set of support data of the class k and  $N_C$  denotes the number of classes.

- The lower bound of SCE loss functions similarly to the NCC loss, which aims at maximizing the similarity between each sample and its corresponding prototype while minimizing the similarities between the sample and all prototypes. Minimizing the second term is equivalent to enlarging the gap.
- The similarities between images and prototypes are minimized with the weights calculated based on the size of the class set. Thus, the similarities between samples and the prototypes involving more samples will be significantly reduced.

# **Summary**

- **Empirically**, we find that there exists a gap, which resembles the modality gap, between prototype and image instance embeddings extracted from a frozen backbone. And the shared representation transformation tends to shrink the gap between prototype and image representations.
- **Theoretically**, we find that the shared transformation potentially drop the discriminative information in gradients and constrains learning representations where the gap is preserved.
- **Technically**, we propose a simple yet effective method, CoPA, to finetune two different transformations respectively for prototypes and image instances with SCE loss.
- **Empirically**, extensive experiments under several settings are conducted to verify the effectiveness of CoPA in improving generalization performance and demonstrate that CoPA can enlarge the gap between prototypes and image instances and learn a better image representation cluster for each class.



