



Bridging The Gap between Low-rank and Orthogonal Adaptation via Householder Reflection Adaptation

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▶ LoRA hypothesizes that the weights have a low "intrinsic rank" during adaptation.



OFT: The preservation of pretrained knowledge



> OFT emphasizes the retention of pre-trained knowledge during adaptation.





 \rightarrow Orthogonal fine-tuning (OFT)

Qiu, Zeju, et al. "Controlling text-to-image diffusion by orthogonal finetuning." NeurIPS 2023.





Householder Reflection: A simple orthogonal transform



Alston Householder



 $\boldsymbol{H} = \boldsymbol{I} - 2\boldsymbol{u}\boldsymbol{u}^{\top}, \ \boldsymbol{u} \in \mathbb{S}^{d-1}$

Householder Reflection Adaptation (HRA)



> Implement OFT by a chain of Householder reflections

$$\boldsymbol{z} = \boldsymbol{W} \underbrace{\left(\prod_{i=1}^{r} \boldsymbol{H}_{i}\right)}_{\boldsymbol{H}^{(r)}} \boldsymbol{x} = \boldsymbol{W} \left(\prod_{i=1}^{r} (\boldsymbol{I} - 2\boldsymbol{u}_{i}\boldsymbol{u}_{i}^{\top})\right) \boldsymbol{x}, \text{ with } \{\boldsymbol{u}_{i} \in \mathbb{S}^{d-1}\}_{i=1}^{r}.$$

$$\boldsymbol{W} \times \boldsymbol{I} - 2\boldsymbol{u}_{1}\boldsymbol{u}_{1}^{\top} \times \boldsymbol{I} - 2\boldsymbol{u}_{2}\boldsymbol{u}_{2}^{\top} \dots \boldsymbol{I} - 2\boldsymbol{u}_{r}\boldsymbol{u}_{r}^{\top}$$

$$\boldsymbol{W} \times \boldsymbol{I} - 2\boldsymbol{u}_{1}\boldsymbol{u}_{1}^{\top} \times \boldsymbol{I} - 2\boldsymbol{u}_{2}\boldsymbol{u}_{2}^{\top} \dots \boldsymbol{I} - 2\boldsymbol{u}_{r}\boldsymbol{u}_{r}^{\top}$$

 $\blacktriangleright \text{ Implement } \boldsymbol{W}\boldsymbol{H}^{(r)} \text{ with low complexity } (\mathcal{O}(d(r+d_{\mathsf{out}})) \text{ for } \boldsymbol{W} \in \mathbb{R}^{d_{\mathsf{out}} \times d})$ 1) $\boldsymbol{x}^{(j+1)} = \boldsymbol{x}^{(j)} - 2\langle \boldsymbol{u}_{r-j}, \boldsymbol{x}^{(j)} \rangle \boldsymbol{u}_{r-j}, \text{ for } j = 0, ..., r-1.$ 2) $\boldsymbol{z} = \boldsymbol{W}\boldsymbol{x}^{(r)}.$



Connections to LoRA: HRA is an adaptive LoRA

➤ Reformulation of the HR chain:

(11)

$$oldsymbol{H}^{(r)} = \prod_{i=1}^r (oldsymbol{I} - 2oldsymbol{u}_ioldsymbol{u}_i^ op) = oldsymbol{I} + oldsymbol{U}_roldsymbol{\Gamma}_roldsymbol{U}_r^ op,$$

 $\succ \Gamma_r = [\gamma_{ij}] \in \mathbb{R}^{r \times r}$ is a upper-triangular matrix, and its upper-triangular element is

$$\boldsymbol{\Gamma}_1 = -2, \ \boldsymbol{\Gamma}_r = \begin{bmatrix} \boldsymbol{\Gamma}_{r-1} & -2\boldsymbol{\Gamma}_{r-1}\boldsymbol{U}_{r-1}^\top \boldsymbol{u}_r \\ \boldsymbol{0}_{r-1}^\top & -2 \end{bmatrix}$$

> HRA is equivalent to an adaptive LoRA, making Range(W) unchanged. $WH^{(r)} = W + \underbrace{WU_r\Gamma_r}_r U_r^\top$.

 $A_{W,U}$

Orthogonality: The key of balancing expressiveness and regularity





$$\min_{\{\boldsymbol{U}_{r}^{(l)}\}_{l=1}^{L}} \mathsf{Loss}(\mathcal{D}; \{\boldsymbol{U}_{r}^{(l)}\}_{l=1}^{L}) + \lambda \underbrace{\sum_{l=1}^{L} \|\boldsymbol{I}_{r} - (\boldsymbol{U}_{r}^{(l)})^{\top} \boldsymbol{U}_{r}^{(l)}\|_{F}^{2}}_{\mathsf{Orthogonal regularizer}},$$

 $\succ \lambda \in [0,\infty)$: Normalization,

> $\lambda = \infty$: (Modified) Gram-Schmidt (GS) Orthogonalization.



Experiments: NLP tasks

Table: Results (%) of various methods on GLUE development set	et.
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Method	#Param (M)	MNLI	SST-2	CoLA	QQP	QNLI	RTE	MRPC	STS-B	All
Full Fine-tune	184	89.90	95.63	69.19	92.40	94.03	83.75	89.46	91.60	88.25
BitFit	0.10	89.37	94.84	66.96	88.41	92.24	78.70	87.75	91.35	86.20
H-Adapter	1.22	90.13	95.53	68.64	91.91	94.11	84.48	89.95	91.48	88.28
P-Adapter	1.18	90.33	95.61	68.77	92.04	94.29	85.20	89.46	91.54	88.41
LoRA $_{r=8}$	1.33	90.65	94.95	69.82	91.99	93.87	85.20	89.95	91.60	88.50
AdaLoRA	1.27	90.76	96.10	71.45	<u>92.23</u>	<u>94.55</u>	88.09	90.69	91.84	89.46
OFT $b=16$	0.79	90.33	96.33	73.91	92.10	94.07	87.36	92.16	<u>91.91</u>	89.77
BOFT $_{b=8}^{m=2}$	0.75	90.25	96.44	72.95	92.10	94.23	<u>88.81</u>	92.40	91.92	89.89
HRA $_{r=8, \lambda=0}$	0.66	<u>90.70</u>	96.45	73.70	91.29	94.66	88.45	93.69	91.86	90.10
HRA $_{r=8, \lambda=10^{-5}}$	0.66	90.43	96.79	71.91	91.02	94.44	89.53	94.10	91.74	90.00
HRA $_{r=8, \lambda=\infty}$	0.66	90.52	95.87	70.71	90.71	94.12	87.00	92.59	91.54	89.13

Experiments: Controllable text-to-image generation



a [V] teapot on top of a purple rug in a forest



a [V] teapot floating on top of water



Original Images











LoRA OFT

HRA_{7,0}

 $HRA_{7,10^{-3}} HRA_{7,\infty}$

Figure: Qualitative results on subject-driven generation.

Experiments: Controllable text-to-image generation

(M









Thank you for listening!

The code is available at https://github.com/DaShenZi721/HRA and PEFT!

B HRA Public

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Bridging The Gap between Low-rank and Orthogonal Adaptation via Householder Reflection Adaptation (HRA)

HRA is a simple but effective adapter-based fine-tuning method by leveraging Householder reflections. This method harnesses the advantages of both strategies, reducing parameters and computation costs while penalizing the loss of pre-training knowledge. It consistently achieves better performance with fewer trainable parameters and outperforms state-ofthe-art adapters across different models, including large language models (LLMs) and conditional image generators.

The abstract from the paper is:

"While following different technical routes, both low-rank and orthogonal adaptation techniques can efficiently adapt large-scale pre-training models in specific tasks or domains based on a small piece of trainable parameters. In this study, we bridge the gap between these Bridging The Gap between Lowrank and Orthogonal Adaptation via Householder Reflection

HRAConfig

HRAMode