Approximately Pareto-optimal Solutions for Bi-Objective k-Clustering Problems

Anna Arutyunova, **Jan Eube**, Heiko Röglin,
Melanie Schmidt, Sarah Sturm, Julian Wargalla

Clustering

Given: Metric space $(\mathcal{P}, d)$ with $|\mathcal{P}| = n$.

k-**clustering:** Choose k centers $C \subset \mathcal{P}$ and assignment $\sigma\colon \mathcal{P} \to C$ optimizing

k-center: $\mathsf{rad}(C, \sigma) = \max_{p \in P} d(p, \sigma(p))$ **k-means:** $\mathsf{mean}(C, \sigma) = \sum_{p \in P} d(p, \sigma(p))$ **k-separation:** $\mathsf{sep}(C, \sigma) = \min_{\substack{p,q \in P \\ \sigma(p) \neq \sigma(q)}} d(p, q)$

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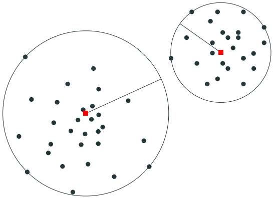

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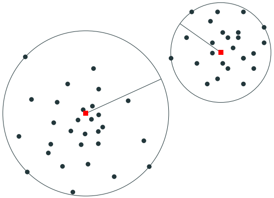

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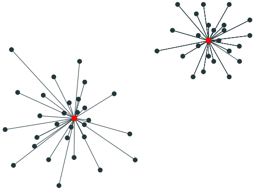

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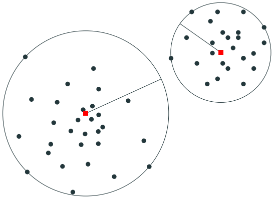

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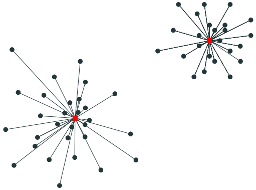

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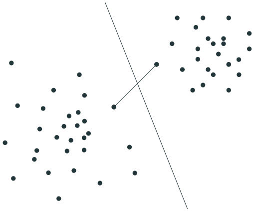

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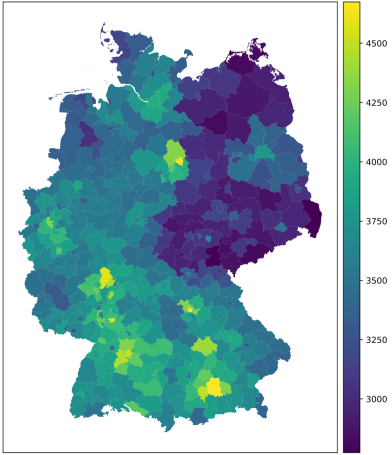
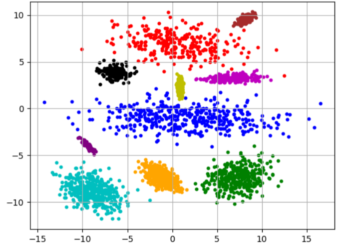
What if we want to consider multiple metrics?



Average income of the German districts in Euro[1]

[1]Source: Bundesagentur für Arbeit

What if we want to consider multiple objective functions?



The ground truth.[2]

[2]Data Set generated by Julia Handl and Joshua D. Knowles, 2007

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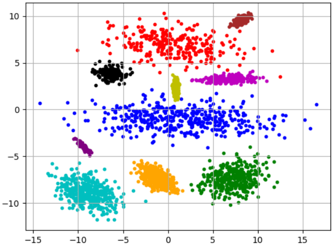

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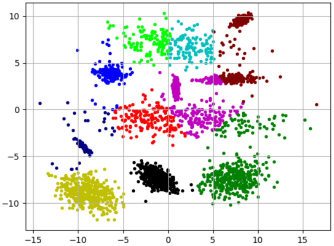

Clustering by k-means++

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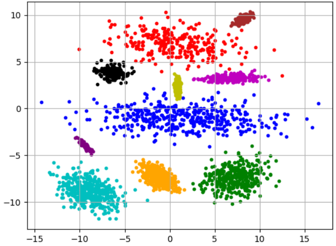

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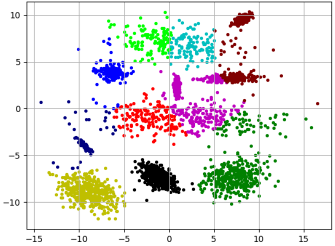
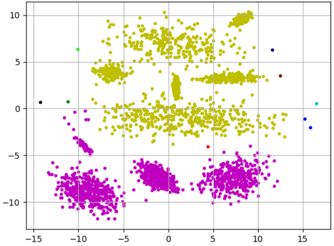


Clustering by *k*-means++



Optimum *k*-separation clustering

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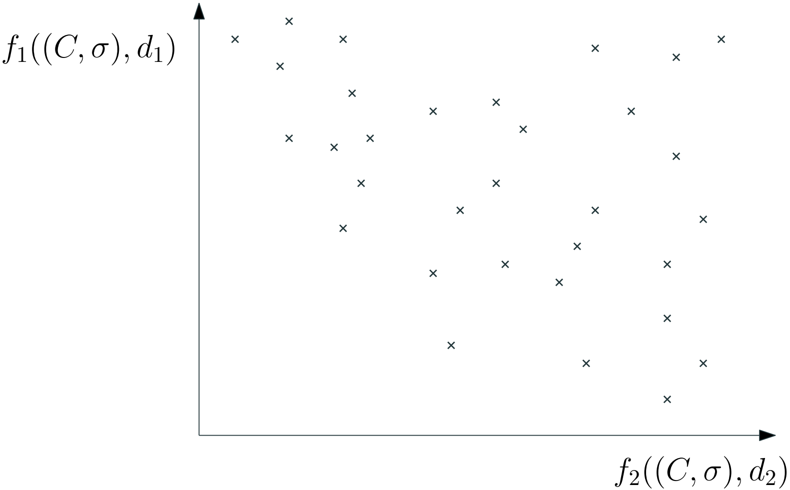
Pareto Sets



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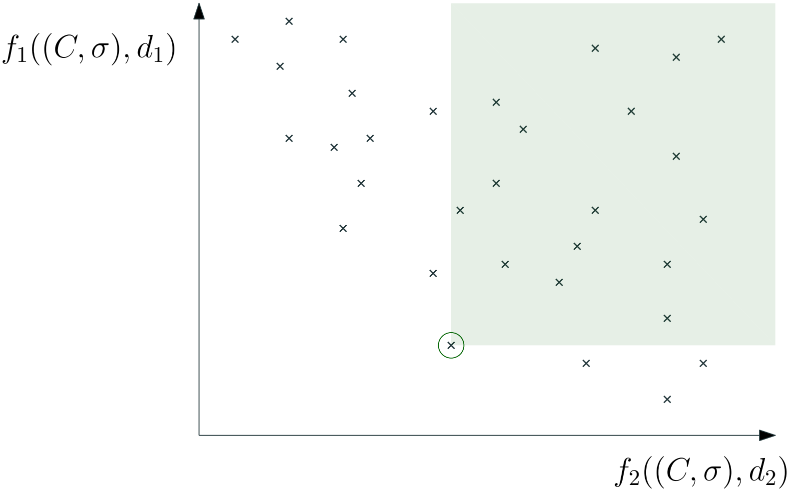

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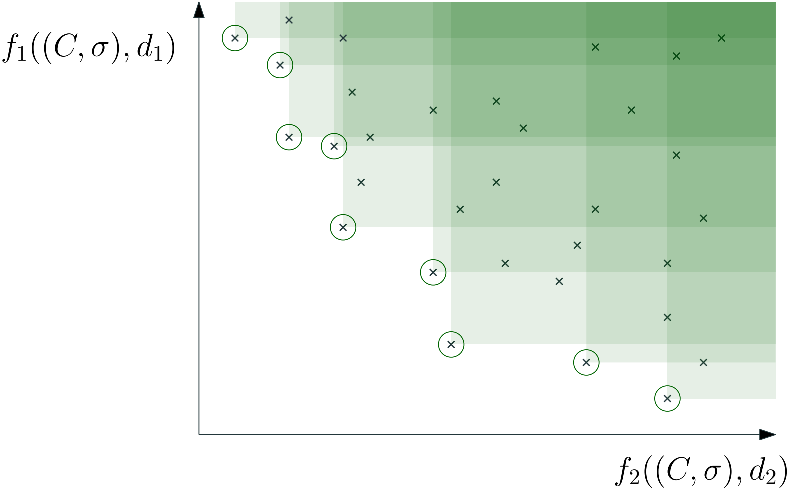

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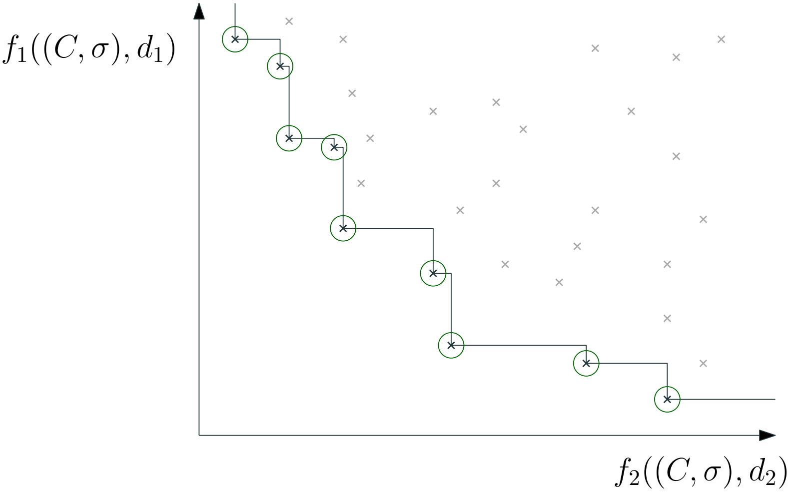
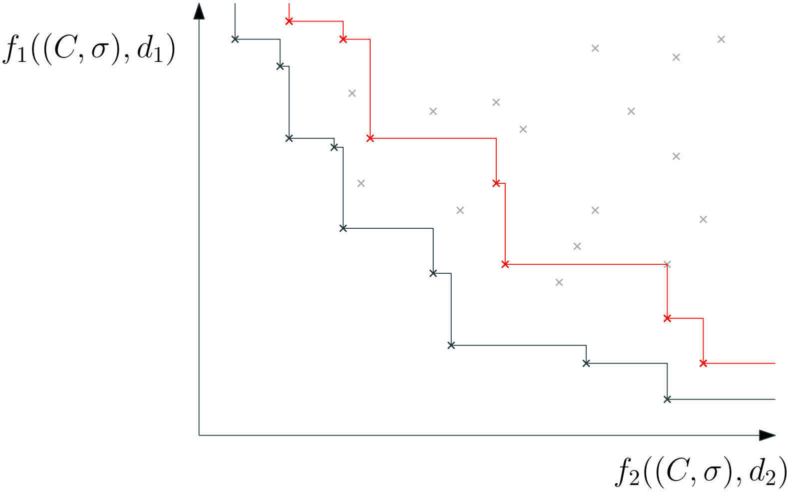
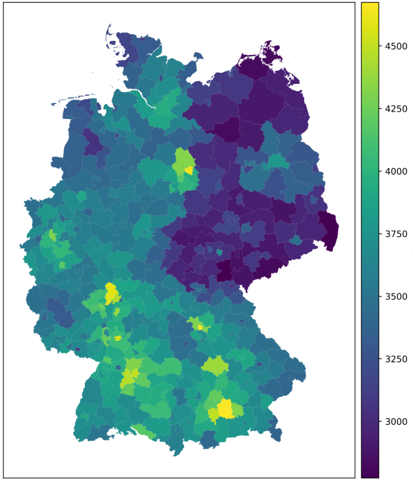


Approximate Pareto Set



Experimental Evaluation



Average income of the German districts in Euro

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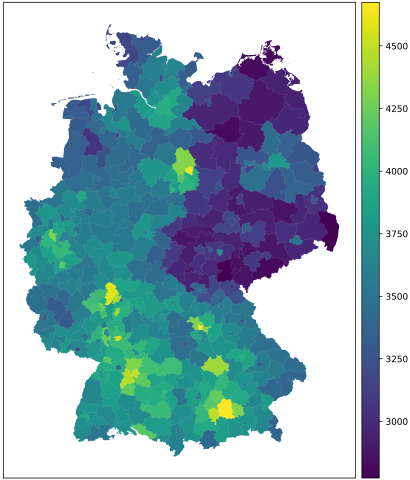

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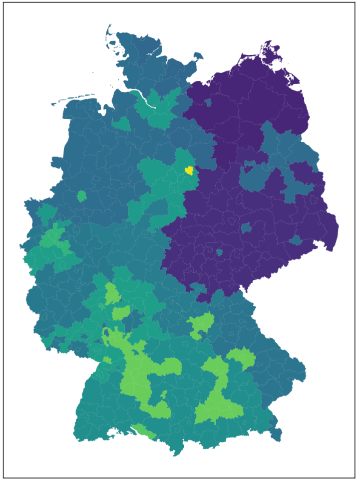
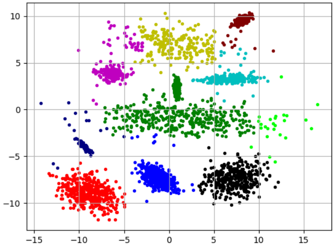


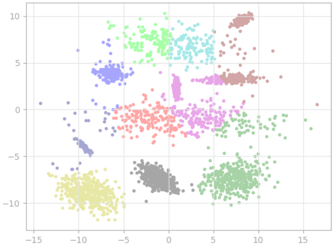
A good Pareto solution

Experimental Evaluation

Optimum *k*-separation clustering



The best Pareto Solution



Clustering by *k*-means++

Theoretical Results

	k-center	k-diameter	k-separation	k-median	k-means
k-center	(2,2)	(2,2)	(2, 1)	$(9,\ 6+\epsilon)$[1]	$(9{,}54+\epsilon)$
k-diameter	-	(2,2)	(2, 1)	$(18{,}6+\epsilon)$	$(18{,}54+\epsilon)$
k-separation	-	-	(1, 1)	$(1{,}2+\delta_1)$[2]	$(1,\ 4+4\sqrt{\delta_2}+\delta_2)$[2]
k-median	-	-	-	$2.675+\epsilon$[3]	$9+\epsilon$[3]
k-means	-	-	-	-	$9+\epsilon$[3]

[1] Alamdari & Shmoys, 2017 also provided a $(4, 8)$ approximation algorithm

[2] δ_1 and δ_2 refer to the currently best single-objective approximation factors respectively.

[3] Calculates only an approximate convex Pareto Set, the approximation is the same for both objectives

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Thank you for your attention!