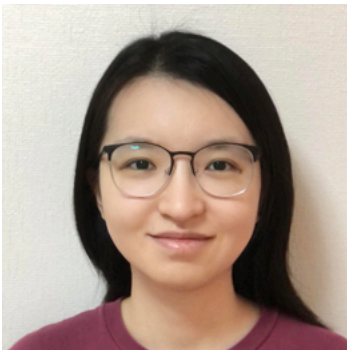


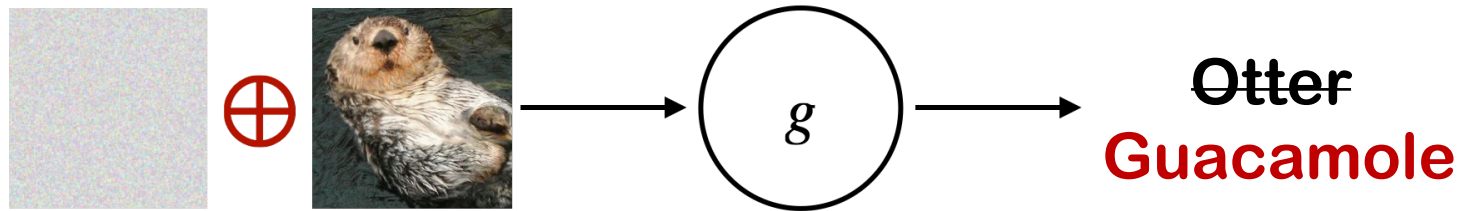
# Adaptive Randomized Smoothing: Certified Adversarial Robustness for Multi-Step Defences

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# Adversarial Examples

- Adversarial Examples (AE): test-time attacks to control model predictions with small crafted input perturbations.

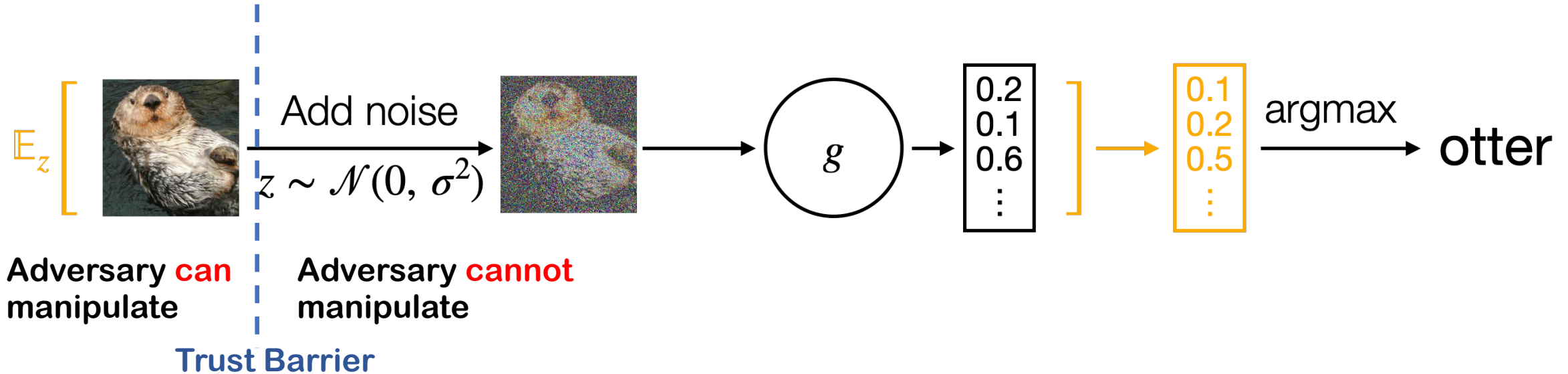


- The power of the adversary is determined by the maximum size of the attack:

$$\ell_2 \text{ attack: } \|\text{[gray square]}\|_2 \qquad \ell_\infty \text{ attack: } \|\text{[gray square]}\|_\infty$$

- **Randomized Smoothing can provide provable defenses against adversarial examples!**

# Randomized Smoothing (RS)



- Randomized Smoothing gives provable robustness by averaging over noisy predictions:
- **Theorem (Cohen et al. 2019):** with  $\mathbb{P}(f(X + z) = y_+) \geq \underline{p}_+ \geq \bar{p}_- \geq \max_{y_- \neq y_+} \mathbb{P}(f(X + z) = y_-)$  we have:

No  $\ell_2$  attack with  $\| \cdot \|_2 \leq r_X = \frac{\sigma}{2} (\Phi^{-1}(\underline{p}_+) - \Phi^{-1}(\bar{p}_-))$  **certificate**

# Limitations of RS

- Noise degrades accuracy.
- Difficulty scaling to high dimensional inputs for  $\ell_\infty$  threat models:

$$r_X \leq \| \square \|_2 \leq \sqrt{d} \| \square \|_\infty$$

- Does not support test-time adaptivity to adapt the accuracy/robustness trade-off to the input.

We use **Gaussian differential privacy** to address these shortcomings!

# Gaussian Differential Privacy

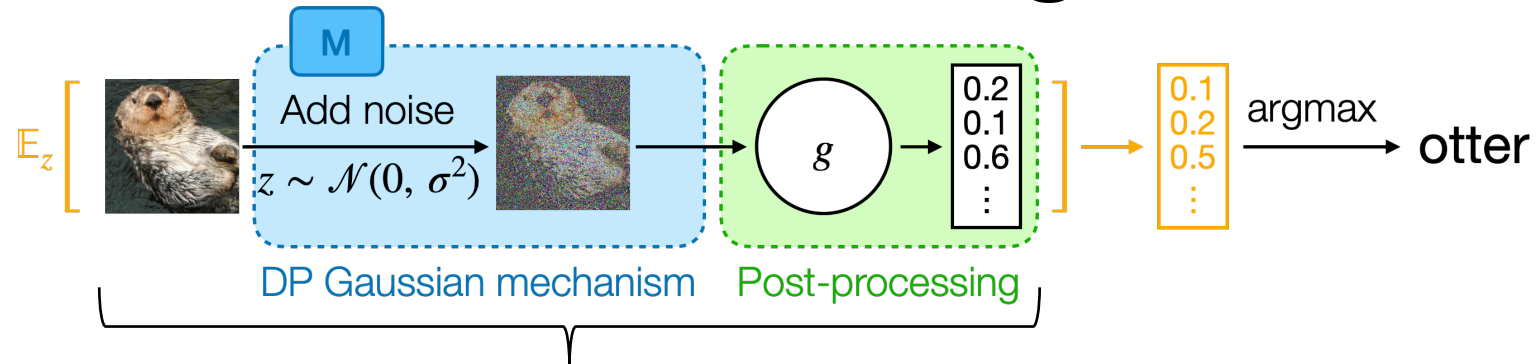
- We can frame privacy as a hypothesis test between  $\mathcal{H}_0: D$  and  $\mathcal{H}_1: D'$  (i.e. does  $x \in D$ ?). This enables a hypothesis test definition of DP.
- A tradeoff function  $f$  bounds the power of any statistical test of  $\mathcal{H}_0$  v.s.  $\mathcal{H}_1$ .

(Theorem 2.7 Dong et al. 2019) For a Gaussian mechanism  $\mathcal{M}(D) = \theta(D) + z, z \sim \mathcal{N}\left(0, \frac{r^2}{\mu^2}\right)$ , such that for any neighboring  $D, D', \theta(D) - \theta(D') \in B_2(r)$  (i.e., the  $\ell_2$  sensitivity of  $\theta$  is  $r$ ), we have that  $\mathcal{M}$  is  $G_\mu$ -DP with function  $f = G_\mu$  defined by :

$$G_\mu(\alpha) = \Phi(\Phi^{-1}(1 - \alpha) - \mu), \text{ for all } \alpha \in [0, 1]$$

- **Composition:** the composition of an  $G_{\mu_1}$ -DP Gaussian mechanism and an  $G_{\mu_2}$ -DP Gaussian mechanism is  $G_\mu$ -DP Gaussian mechanism with  $\mu = \sqrt{\mu_1^2 + \mu_2^2}$ .

# GDP and Randomized Smoothing



GDP under neighbouring definition  $D' = D + \delta, \|\delta\|_p \leq r$ .

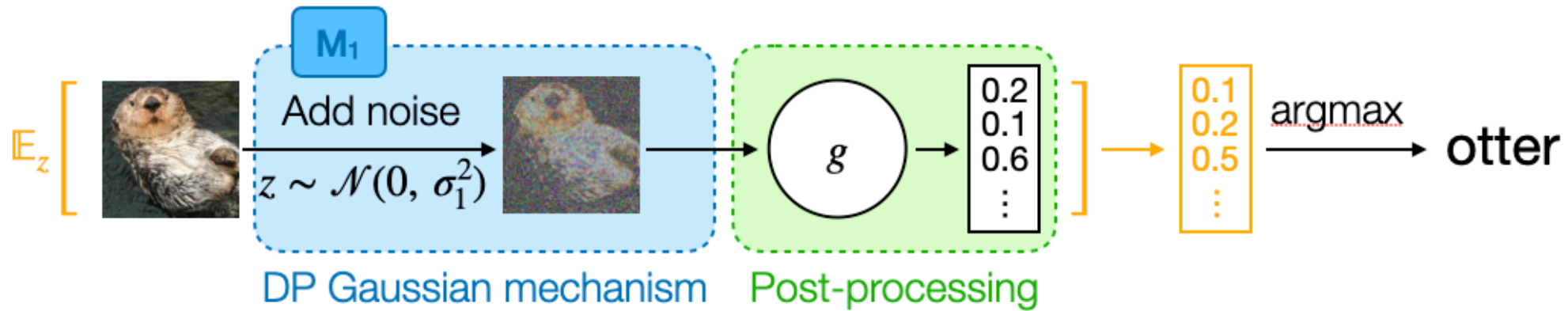
- We prove that our GDP randomized smoothing mechanism satisfies

$$f(1 - \underline{p}_+) \geq 1 - f(\overline{p}_-) \Rightarrow \forall \|\delta\|_p \leq r, M_S(D + \delta) = y_+$$

- Using this result and GDP, we prove that:

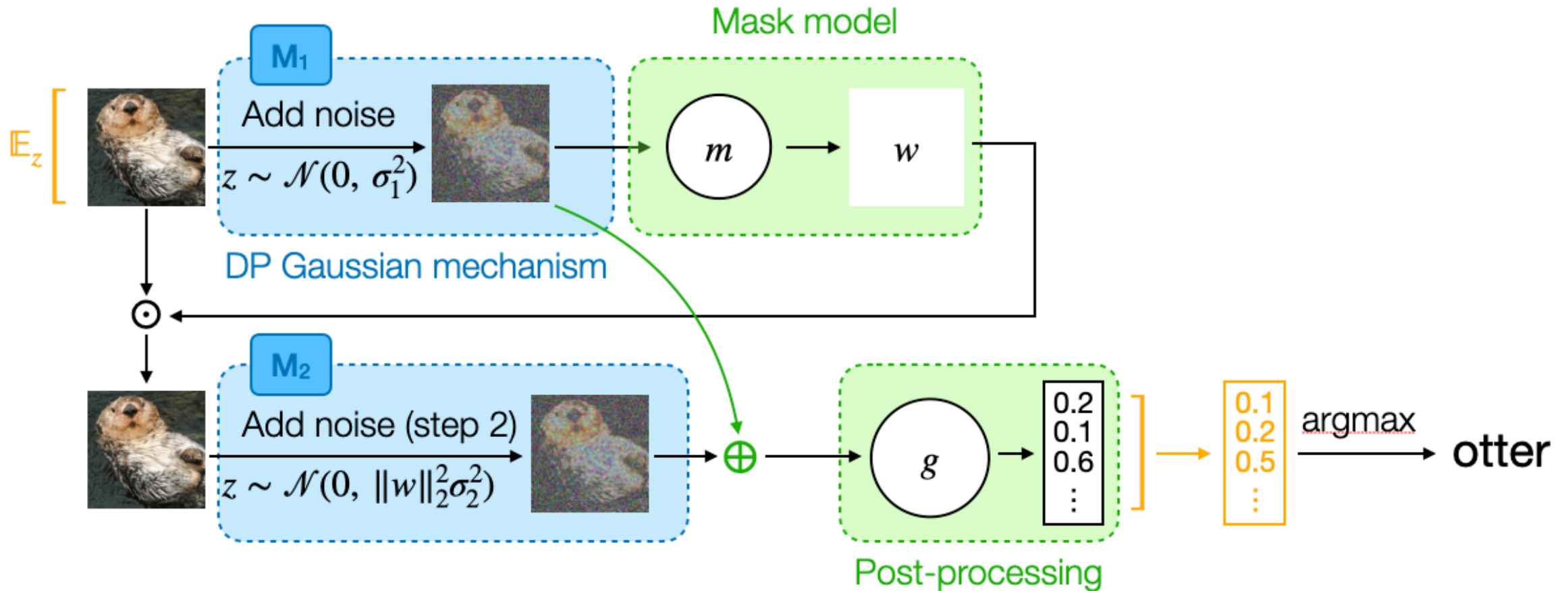
No  $\ell_2$  attack is possible with  $\| \blacksquare \|_2 \leq r_X = \frac{\sigma}{2} (\Phi^{-1}(\underline{p}_+) - \Phi^{-1}(\overline{p}_-))$

# Adaptive Randomized Smoothing for $\ell_\infty$



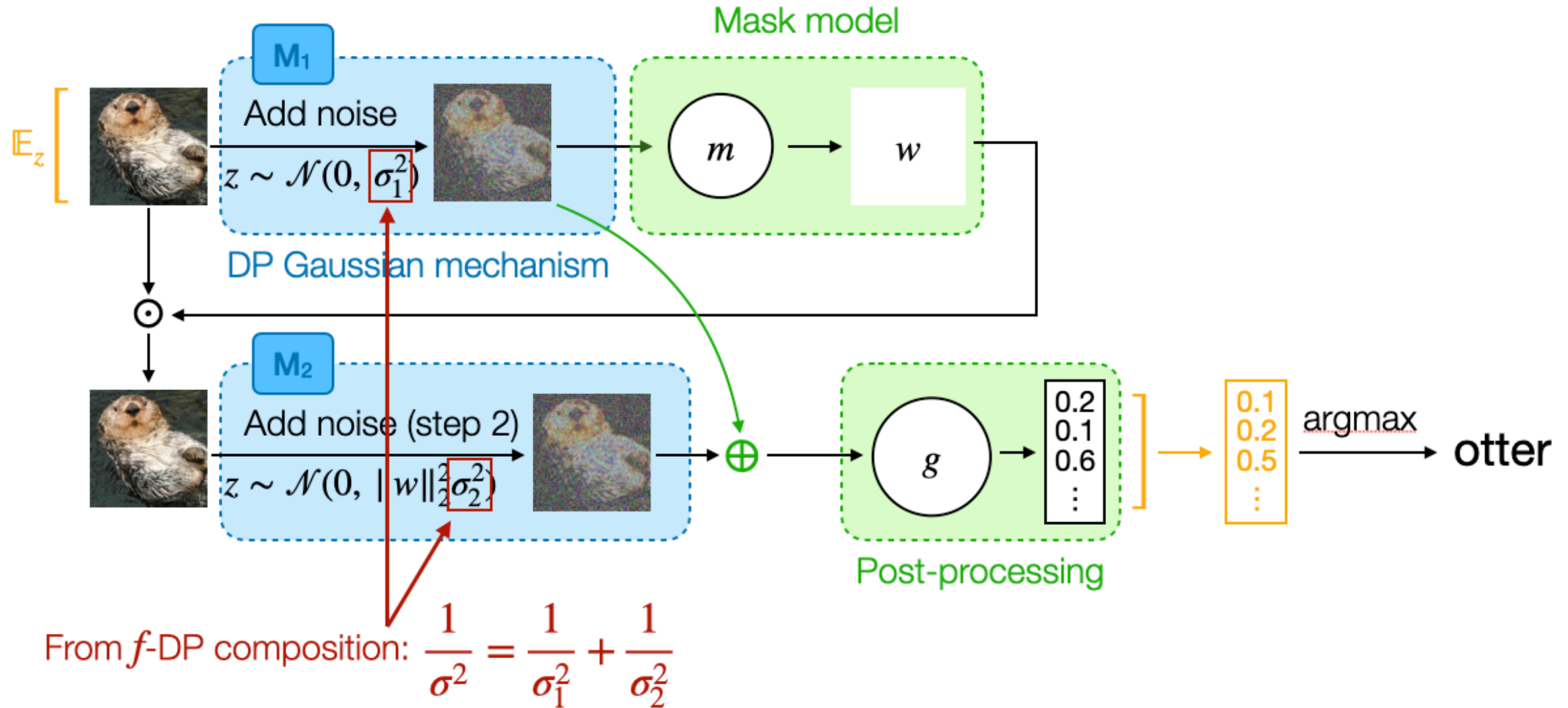


# Adaptive Randomized Smoothing for $\ell_\infty$

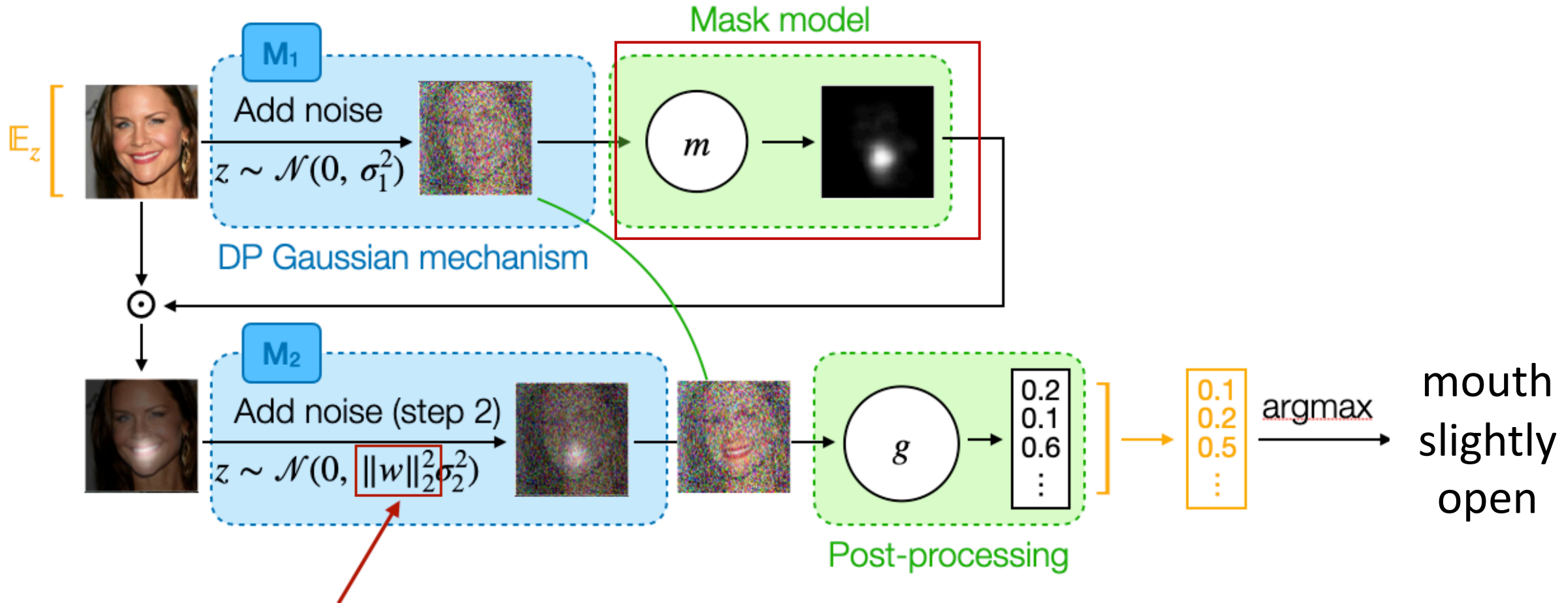




# Adaptive Randomized Smoothing for $\ell_\infty$



# Why does ARS help?



Noise reduction from masking based dimension reduction!

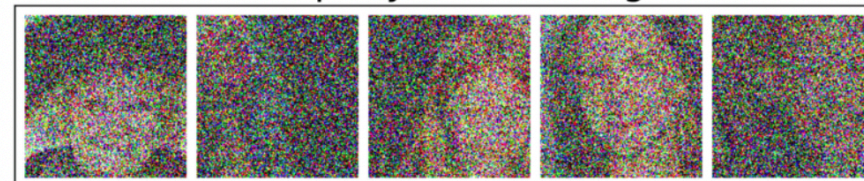
# Evaluations

- CelebA

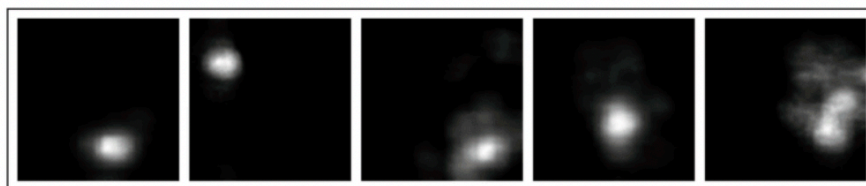
Input images



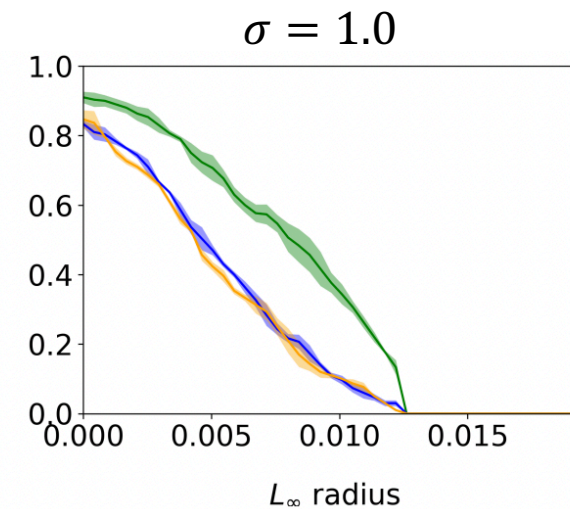
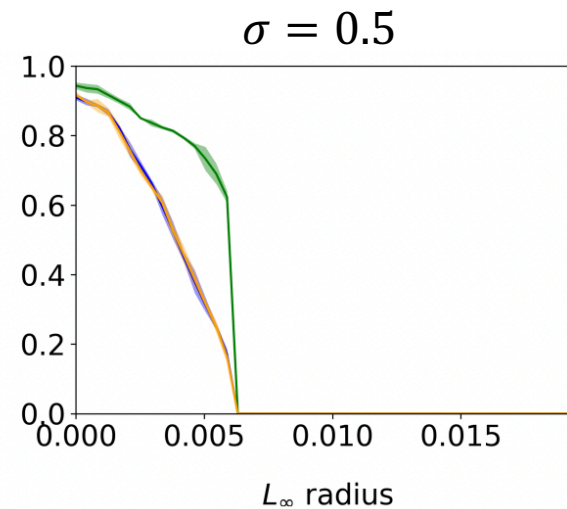
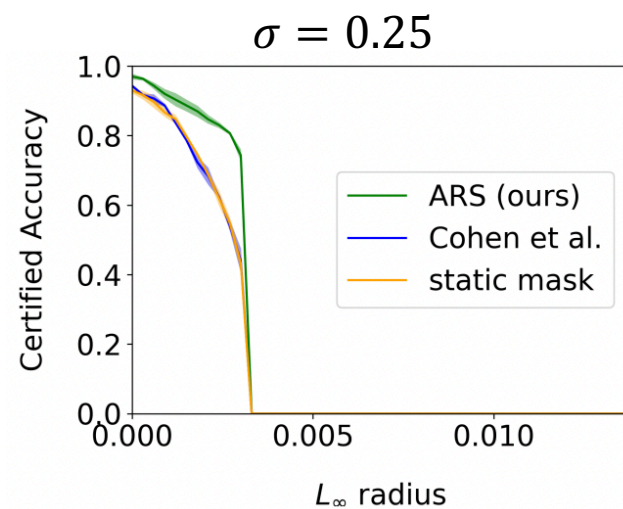
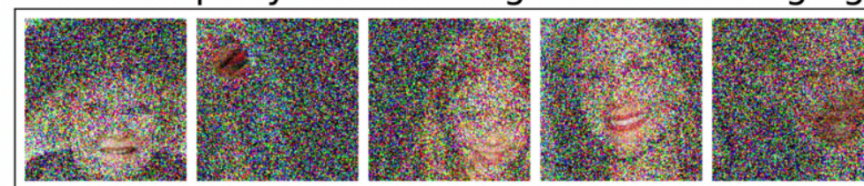
First query noised images



ARS Masks



Second query noised images after averaging



# Conclusion

- Adaptive Randomized Smoothing (ARS) uses DP composition post-processing properties to certify **adaptive multi-step models**.
- ARS learns to **adjust the scale of noise** based on the test input.
- ARS provides **higher accuracy** at a given level of provable robustness.

**Link to our code**



