Theoretical Investigations and Practical Enhancements on Tail Task Risk Minimization in Meta Learning

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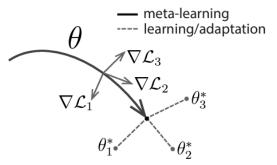
# Introduction

## □ Meta Learning

Leverages previous experience as priors to quickly adapt to unseen tasks <sup>(2)</sup>.

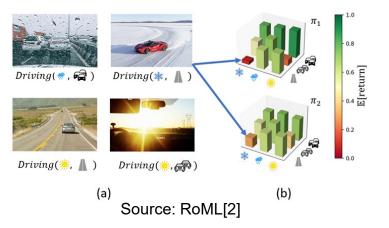


Worst fast adaptation can be catastrophic in risksensitive scenarios, e.g., autonomous driving ⊗.



*Figure 1.* Diagram of our model-agnostic meta-learning algorithm (MAML), which optimizes for a representation  $\theta$  that can quickly adapt to new tasks.

Source: MAML[1]

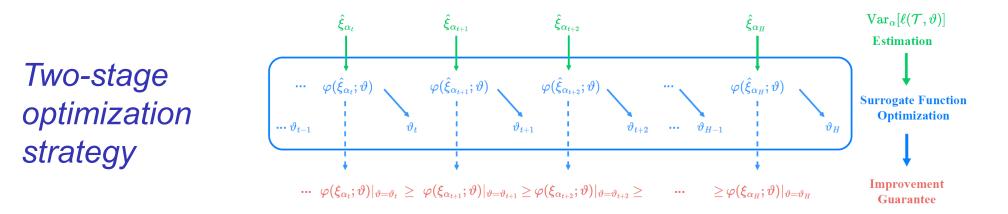


It is desirable to watch adaptation differences across tasks when deploying meta learning models.

## Introduction

## **Previous Works**

DR-MAML [3] increases task distributional robustness via employing the tail risk minimization principle for meta learning.



(i) Estimate the risk quantile  $VaR_{\alpha}$  with the crude Monte Carlo method in the task space.

(ii) Update the meta learning model parameters from the screened subset of tasks.

## Existing Limitations

## • Theoretically

(i) There constitutes no notion of solutions.

(ii) Lacks an algorithmic understanding of the two-stage optimization strategy.(iii) The analysis on generalization capability is ignored in the tail risk of tasks.

## Empirically

The use of the crude Monte Carlo might be less efficient in quantile estimates and suffers from a higher approximation error of the  $VaR_{\alpha}$ , degrading the adaptation robustness.

We propose translating the **two-stage optimization strategy** for distributionally robust meta learning into **a max-min optimization problem** <sup>(2)</sup>.



#### Notations

Task distribution  $p(\tau)$  defined in task space  $\Omega_{\tau}$ ; the set of all tasks  $\mathcal{T}$ ; Meta dataset  $\mathfrak{D}_{\tau}$ ,  $e.g., \mathfrak{D}_{\tau} = \{(x_i, y_i)\}_{i=1}^m = \mathfrak{D}_{\tau}^S \cup \mathfrak{D}_{\tau}^Q$  in few-shot regression problems; Parameter space  $\Theta$ 

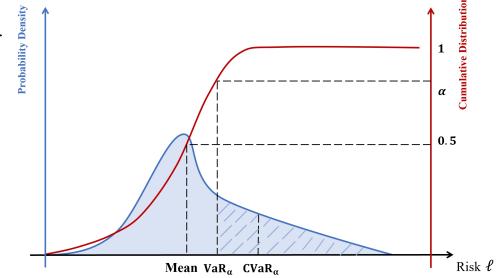
Meta risk function  $\ell: \mathfrak{D}_{\tau} \times \Theta \mapsto \mathbb{R}^+$  evaluating fast adaptation performance;

Cumulative distribution of the meta risk function

$$F_{\ell}(l;\theta) := \mathbb{P}\left(\left\{\ell\left(\mathfrak{D}_{\tau}^{Q},\mathfrak{D}_{\tau}^{S};\theta\right) \leq l; \tau \in \mathcal{T}, l \in \mathbb{R}^{+}\right\}\right)$$

## Notations

Value-at-risk (VaR<sub> $\alpha$ </sub>) VaR<sub> $\alpha$ </sub>[ $\ell(\mathcal{T}, \theta)$ ] = inf<sub> $l \in \mathbb{R}^+$ </sub>{ $l|F_{\ell}(l; \theta) \ge \alpha, \tau \in \mathcal{T}$ } Conditional value-at-risk (CVaR<sub> $\alpha$ </sub>) CVaR<sub> $\alpha$ </sub> =  $\mathbb{E}_{p(\tau)}[\ell|\ell \ge VaR_{\alpha}]$ Normalized cumulative distribution  $F_{\ell}^{\alpha}(l; \theta)$ ; Tail risk task subspace  $\Omega_{\alpha,\tau}$ ; Density function  $p_{\alpha}(\tau; \theta)$ 



Expected Risk Minimization. It minimizes meta risk based on the sampling chance of tasks from the original task distribution:

$$\min_{\theta \in \Theta} \mathcal{E}(\theta) := \mathbb{E}_{p(\tau)} \big[ \ell \big( \mathfrak{D}^Q_{\tau}, \mathfrak{D}^S_{\tau}; \theta \big) \big].$$

Worst-case Risk Minimization. Noticing that the worst fast adaptation can be disastrous in some risk sensitive scenarios, [4] proposes to conduct the worstcase optimization in meta learning:

$$\min_{\theta \in \Theta} \max_{\tau \in \mathcal{T}} \mathcal{E}_{\mathsf{W}}(\theta) := \ell \big( \mathfrak{D}_{\tau}^{Q}, \mathfrak{D}_{\tau}^{S}; \theta \big).$$

**Expected Tail Risk Minimization (** $CVaR_{\alpha}$ **).** To balance the average performance and the worst-case performance, [3] minimizes the expected tail risk, or equivalently  $CVaR_{\alpha}$  risk measure:

$$\min_{\theta \in \Theta, \xi \in \mathbb{R}} \mathcal{E}_{\alpha}(\theta, \xi) := \frac{1}{1 - \alpha} \int_{\alpha}^{1} v_{\beta} \, d\beta = \xi + \frac{1}{1 - \alpha} \mathbb{E}_{p(\tau)} \left[ \left[ \ell \left( \mathfrak{D}_{\tau}^{Q}, \mathfrak{D}_{\tau}^{S}; \theta \right) - \xi \right]^{+} \right],$$

 $v_{\beta} := F_{\ell}^{-1}(\beta)$  denotes the quantile statistics

$$\left[\ell\left(\mathfrak{D}^Q_{\tau},\mathfrak{D}^S_{\tau};\theta\right)-\xi\right]^+:=\max\{\ell\left(\mathfrak{D}^Q_{\tau},\mathfrak{D}^S_{\tau};\theta\right)-\xi,0\}$$
 is the hinge risk.

#### Example 1 (DR-MAML).

Given  $p(\tau)$  and vanilla MAML [1], the distributionally robust MAML within CVaR<sub> $\alpha$ </sub> can be written as a bi-level optimization problem:

$$\min_{\substack{\theta \in \Theta \\ \xi \in \mathbb{R}}} \xi + \frac{1}{1 - \alpha} \mathbb{E}_{p(\tau)} \left[ \left[ \ell \left( \mathfrak{D}^{Q}_{\tau}; \theta - \lambda \nabla_{\theta} \ell(\mathfrak{D}^{S}_{\tau}; \theta) \right) - \xi \right]^{+} \right],$$

where the gradient update w.r.t. the support set  $\nabla_{\theta} \ell(\mathfrak{D}^{S}_{\tau}; \theta)$  indicates the inner loop with a learning rate  $\lambda$ . The outer loop executes the gradient updates and seeks the robust meta initialization in the parameter space.

## **Two-Stage Optimization Strategies**

The pipelines of DR-MAML (**Example 1**):

**Stage-I** includes the fast adaptation *w.r.t.* individual task in Eq. (1) and the quantile estimate in Eq. (2)

**Stage-II** applies the sub-gradient updates to the model parameters in Eq. (3)/(4).

$$\theta_t^{\tau_i} = \theta_t^{meta} - \lambda_1 \nabla_\theta \ell(\mathfrak{D}_{\tau_i}^S; \theta), \ i = 1, \dots, \mathcal{B}$$
(1)

$$\hat{\xi} = \hat{F}_{MC-\mathcal{B}}^{-1}(\alpha), \tag{2}$$

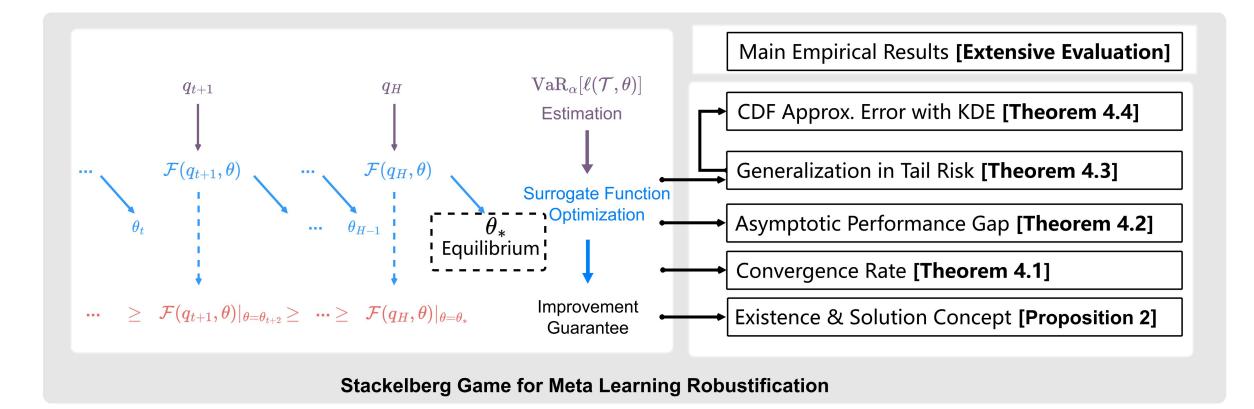
$$\delta(\tau_i) = \mathbb{1}\big[\ell\big(\mathfrak{D}^Q_{\tau_i}; \theta_t^{\tau_i}\big) \ge \hat{\xi}\big], \ i = 1, \dots, \mathcal{B}$$
(3)

$$\theta_{t+1}^{meta} \leftarrow \theta_t^{meta} - \lambda_2 \left[ \sum_{i=1}^{\mathcal{B}} \nabla_{\theta} \left[ \delta(\tau_i) \cdot \ell(\mathfrak{D}_{\tau_i}^Q; \theta_t^{\tau_i}) \right] \right].$$
(4)

These two stages repeat until reaching the convergence required iterations.

# **Theoretical Investigations**

# The Sketch of our Study



On the left side is the two-stage distributionally robust strategy.

The contributed theoretical understanding is right-down, with the right-up the empirical improvement.

In the two-stage optimization strategy, the default is the minimization of the risk measure *w.r.t.* the parameter space after the maximization of the risk measure *w.r.t.* the task subspace.

Max-Min Optimization. With the pre-assigned decision-making orders, the studied problem can be characterized as:

$$\max_{q(\tau)\in\mathcal{Q}_{\alpha}}\min_{\theta\in\Theta}\mathcal{F}(q,\theta):=\mathbb{E}_{q(\tau)}[\ell(\mathfrak{D}_{\tau}^{Q},\mathfrak{D}_{\tau}^{S};\theta)],$$

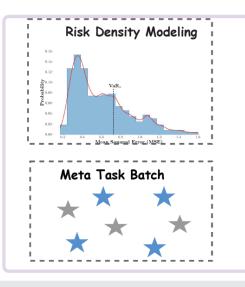
where  $Q_{\alpha} := \{q(\tau) | \mathcal{T}_q \subseteq \mathcal{T}, \int_{\tau \in \mathcal{T}_q} p(\tau) d\tau = 1 - \alpha\}$  constitutes a collection of uncertainty sets over task subspace  $\mathcal{T}_q$ , and  $q(\tau)$  is the normalized probability density over the task subspace.

Stackelberg Game: the example pipelines can be understood as approximately solving a stochastic two-player zero-sum Stackelberg game

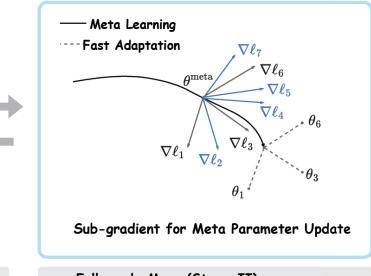
 $\mathcal{SG} := \langle \mathcal{P}_L, \mathcal{P}_F; \{q \in Q_\alpha\}, \{\theta \in \Theta\}; \mathcal{F}(q, \theta) \rangle.$ 

## Best Responses:

$$\begin{split} \mathcal{SG}: \ q_t &= \arg \max_{q \in \mathcal{Q}_{\alpha}} \mathbb{E}_q \big[ \ell \big( \mathfrak{D}^Q_{\tau}, \mathfrak{D}^S_{\tau}; \theta_t \big) \big], \\ & Leader Player \\ \theta_{t+1} &= \arg \min_{\theta \in \Theta} \mathbb{E}_{q_t} \big[ \ell \big( \mathfrak{D}^Q_{\tau}, \mathfrak{D}^S_{\tau}; \theta \big) \big], \\ & Follower Player \end{split}$$



Leader's Move (Stage-I): (1) Risk Distribution Modeling with KDEs (2) Optimal Subset Selection in the Task Batch



$$\begin{array}{l} \textbf{Follower's Move (Stage-II):} \\ \theta_{t+1}^{\text{meta}} \leftarrow \theta_t^{\text{meta}} - \lambda_2 \Big[ \sum_{i=1}^{\mathcal{B}} \nabla_{\theta} [\delta(\tau_i) \cdot \ell(\mathfrak{D}_{\tau_i}^Q; \theta_t^{\tau_i})] \Big] \end{array}$$

## **Definition** (Local Stackelberg Equilibrium).

Let  $(q_*, \theta_*) \in Q_{\alpha} \times \Theta$  be the solution. With the leader  $q_* \in Q_{\alpha}$  and the follower  $\theta_* \in \Theta$ ,  $(q_*, \theta_*)$  is called a *local Stackelberg equilibrium* for the leader if the following inequalities hold,

$$\begin{split} & \inf_{\theta \in \mathcal{S}_{\Theta'}(q_*)} \mathcal{F}(q_*, \theta) \geq \inf_{\theta \in \mathcal{S}_{\Theta'}(q)} \mathcal{F}(q, \theta), \\ & \text{where } \mathcal{S}_{\Theta'}(q) := \{ \bar{\theta} \in \Theta' \, | \mathcal{F}(q, \bar{\theta}) \leq \mathcal{F}(q, \theta), \forall \theta \in \Theta' \}. \end{split}$$

#### **\Box** Interpretation of the Obtained Equilibrium ( $q_*$ , $\theta_*$ ).

Given the follower's decision  $\theta_*$  and the induced task risk distribution  $F_{\ell}(l; \theta_*)$ , the leader cannot further raise a proposal of a task subset with a probability  $1 - \alpha$  to degrade the tailed expected performance.

## **Convergence** Rate

#### **Assumption 1**

- 1. The meta risk function  $\ell(\mathfrak{D}^Q_{\tau}, \mathfrak{D}^S_{\tau}; \theta)$  is  $\beta_{\tau}$ -Lipschitz continuous *w.r.t.*  $\theta$ ;
- 2. The cumulative distribution  $F_{\ell}(l; \theta)$  is  $\beta_{\ell}$ -Lipschitz continuous *w.r.t.* l, and the normalized density function  $p_{\alpha}(\tau; \theta)$  is  $\beta_{\theta}$ -Lipschitz continuous *w.r.t.*  $\theta$ ;
- 3. For arbitrary valid  $\theta \in \Theta$  and corresponding  $p_{\alpha}(\tau; \theta)$ ,  $\ell(\mathfrak{D}_{\tau}^{Q}, \mathfrak{D}_{\tau}^{S}; \theta)$  is bounded:

$$\sup_{\tau\in\Omega_{\alpha,\tau}}\ell(\mathfrak{D}^Q_{\tau_i},\mathfrak{D}^S_{\tau_i};\theta)\leq\mathcal{L}_{\max}.$$

## **Assumption 2**

The implicit function  $h(\cdot)$  is  $\beta_h$ -Lipschitz continuous w.r.t.  $\theta \in \Theta$ , and  $\nabla_{\theta} \mathcal{F}(q, \theta)$  is  $\beta_q$ -Lipschitz continuous w.r.t.  $q \in Q_{\alpha}$ .

#### **Theorem 1** (Convergence Rate for the Second Player).

Let the iteration sequence in optimization be:  $\dots \mapsto \{q_{t-1}, \theta_t\} \mapsto \{q_t, \theta_{t+1}\} \mapsto \dots \mapsto \{q_*, \theta_*\}$ , with the converged equilibirum  $(q_*, \theta_*)$ . Under the **Assumption** 2 and suppose that  $||I - \lambda \nabla^2_{\theta\theta} \mathcal{F}(q_*, \theta_*)||_2 < 1 - \lambda \beta_q \beta_h$ , we can have  $\lim_{t\to\infty} \frac{\|\theta_{t+1} - \theta_*\|_2}{\|\theta_t - \theta_*\|_2} \leq 1$ , and the iteration converges with the rate  $(||I - \lambda \nabla^2_{\theta\theta} \mathcal{F}(q_*, \theta_*)||_2 + \lambda \beta_q \beta_h)$ .

#### **Theorem 2** (Asymptotic Performance Gap in Tail Task Risk).

Under the Assumption 1 and given a batch of tasks  $\{\tau_i\}_{i=1}^{\mathcal{B}}$ we can have  $CVaR_{\alpha}\left(\theta_T^{meta}\right) - CVaR_{\alpha}(\theta_*) \leq \beta_{\tau} \parallel \theta_T^{meta} - \theta_* \parallel$   $+ \frac{VaR_{\alpha}^*}{1 - \alpha} \left(\mathbb{P}(\mathcal{T}_1) - \mathbb{P}(\mathcal{T}_2)\right),$  $\mathcal{T} = \{\tau: \ell^* < \operatorname{VaR}_{\alpha}^*, \ell^{\operatorname{meta}} \geq \operatorname{VaR}_{\alpha}^{\operatorname{meta}}\}$ 

For sufficiently large T, the first term can be bounded by a small number due to the convergence, and the second term vanishes.

#### **Theorem 3** (Generalization Bound in the Tail Risk Cases).

Given a collection of task samples  $\{\tau_i\}_{i=1}^{\mathcal{B}}$  and corresponding meta datasets, we can derive the following generalization bound in the presence of tail risk:

$$R(\theta_*) \leq \widehat{R}(\theta_*) + \sqrt{\frac{2\left(\frac{\alpha}{1-\alpha}\mathcal{L}_{\max}^2 + \mathbb{V}_{\tau_i \sim p_\alpha(\tau)}\left[\ell\left(\mathfrak{D}_{\tau_i}^Q, \mathfrak{D}_{\tau_i}^S; \theta_*\right)\right]\right)\ln\left(\frac{1}{\epsilon}\right)}{\mathcal{B}}} + \frac{1}{3(1-\alpha)}\frac{\mathcal{L}_{\max}}{\mathcal{B}}\left(2\ln\left(\frac{1}{\epsilon}\right) + 3\alpha\mathcal{B}\right),$$

where the inequality holds with probability at least  $1 - \epsilon$  and  $\epsilon \in (0,1)$ ,  $\mathbb{V}[\cdot]$  denotes the variance operation, and  $\mathcal{L}_{max}$  is from the **Assumption** 1.

KDE can handle arbitrary complex distributions compared to crude Monte Carlo (MC) methods

$$F_{\ell} - \mathsf{KDE}(l;\theta) = \int_{-\infty}^{l} \frac{1}{\mathcal{B}h_{\ell}} \sum_{i=1}^{\mathcal{B}} K\left(\frac{t - \ell(\mathfrak{D}_{\tau_{i}}^{Q}, \mathfrak{D}_{\tau_{i}}^{S}; \theta)}{h_{\ell}}\right) dt,$$

□ Theorem 4. Let  $F_{\ell-\mathsf{KDE}}^{-1}(\alpha;\theta) = \mathsf{VaR}_{\alpha}^{\mathsf{KDE}}[\ell(\mathcal{T},\theta)]$  and  $F_{\ell}^{-1}(\alpha;\theta) = \mathsf{VaR}_{\alpha}[\ell(\mathcal{T},\theta)]$ . Suppose that K(x) is lower bounded by a constant,  $\forall x$ . For any  $\epsilon > 0$ , with probability at least  $1 - \epsilon$ , we can have the following bound:  $\sup_{\theta \in \Theta} \left(F_{\ell-\mathsf{KDE}}^{-1}(\alpha;\theta) - F_{\ell}^{-1}(\alpha;\theta)\right) \leq \mathcal{O}\left(\frac{h_{\ell}}{\sqrt{\mathcal{B}*\log \mathcal{B}}}\right)$ .

**Remark 1.** The crude Monte Carlo used in typically incurs an error of approximately  $\mathcal{O}\left(\frac{1}{\sqrt{\mathcal{B}}}\right)$  in estimating quantiles . In contrast, that of KDE is no more than  $\mathcal{O}\left(\frac{h_{\ell}}{\sqrt{\mathcal{B}*\log \mathcal{B}}}\right)$  from **Theorem 4**.



## **Benchmarks & Baselines**

Benchmarks.

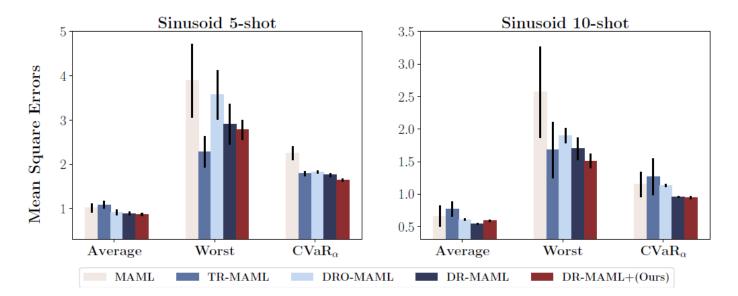


Baselines. MAML mainly works as the base meta learner, and we consider vanilla MAML, TR-MAML, DRO-MAML and DR-MAML.

Evaluations. Expected/empirical risk minimization (Average), worst-case risk minimization (Worst), and tail risk minimization (CVaR<sub>α</sub>).

# Sinusoid Regression

**Problem Setup.** The goal of the sinusoid regression is to quickly fit an underlying function  $f(x) = A\sin(x - B)$  from *K* randomly sampled data points, and tasks are specified by (A, B).

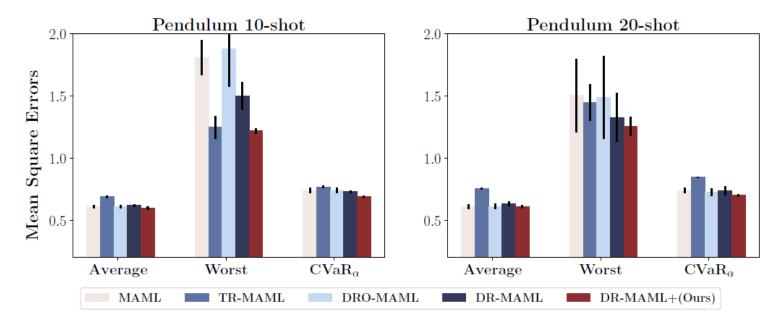


Result Analysis. (1) DR-MAML+ consistently outperforms all baselines across average and CVaR<sub>α</sub> indicators in the 5-shot case. DR-MAML+ exhibits more robustness in challenging task distributions, *e.g.*, 5-shot case.

(2) The standard error associated with our method is significantly smaller than others, underscoring the stability of DR-MAML+.

# **System Identification**

Problem Setup. The system identification corresponds to learning a dynamics model from a few collected transitions in physics systems.



- Result Analysis. (1) There is no significant difference between 10-shot and 20-shot cases. DR-MAML+ dominates the performance across all indicators in both cases.
  - (2) TR-MAML behaves well in the worst-case but sacrifices too much average performance.
  - (3) DR-MAML+ exhibits an advantage over DR-MAML regarding  $CVaR_{\alpha}$ .

## Few-Shot Image Classification

Problem Setup. The task is a 5-way 1-shot classification problem. And 64 classes are selected for constructing meta-training tasks, with the remaining 32 classes for meta-testing.

Table 1: Average 5-way 1-shot classification accuracies in *mini*-ImageNet with reported standard deviations (3 runs). With  $\alpha = 0.5$ , the best results are in **bold**.

	Eight Meta-Training Tasks		Four Meta-Testing Tasks			
Method	Average	Worst	$\text{CVaR}_{\alpha}$	Average	Worst	$\mathrm{CVaR}_{lpha}$
MAML [23]	$70.1 \pm 2.2$	$48.0 \pm 4.5$	$63.2 \pm 2.6$	$46.6 \pm 0.4$	$44.7 \pm 0.7$	44.6±0.7
TR-MAML [37]	$63.2 \pm 1.3$	$60.7 \pm 1.6$	$62.1 \pm 1.2$	$48.5 \pm 0.6$	$45.9 \pm 0.8$	$46.6 \pm 0.5$
DRO-MAML [60]	$67.0 \pm 0.2$	$56.6 \pm 0.4$	$61.6 \pm 0.2$	$49.1 \pm 0.2$	$46.6 \pm 0.1$	$47.2 \pm 0.2$
_ <u>DR-MAML [1]</u>	$70.2\pm0.2$	$63.4 \pm 0.2$	$67.2 \pm 0.1$	$49.4 \pm 0.1$	$47.1 \pm 0.1$	<u>47.5±0.1</u>
DR-MAML+(Ours)	$\overline{70.4{\pm}0.1}$	$\overline{63.8{\pm 0.2}}$	$67.5 \pm 0.1$	$49.9 \pm 0.1$	$47.2{\pm}0.1$	$48.1 \pm 0.1$

Result Analysis. (1) Methods within a two-stage distributionally robust strategy, namely DR-MAML and DR-MAML+, show superiority to others across all indicators in both training and testing scenarios.

(2) DR-MAML+ and DR-MAML are comparable in most scenarios, and we attribute this to the small batch size in training, which weakens KDE' quantile approximation advantage.

# Meta Reinforcement Learning

Problem Setup. Take 2-D point robot navigation as the benchmark. The goal is to reach the target destination with the help of a few exploration transitions for fast adaptation.

The chart reports average return and $\text{CVaR}_{\alpha}$ return with $\alpha = 0.5$ .				
Method	Average	$\mathrm{CVaR}_{lpha}$		
MAML [23]	$-21.1 \pm 0.69$	$-29.2 \pm 1.37$		
DRO-MAML [60]	$-20.9 \pm 0.41$	$-29.0 \pm 0.66$		
$\_$ DR-MAML [1] $\_$ $\_$	$-19.6 \pm 0.49$	$-28.9 \pm 1.20$		
DR-MAML+(Ours)	$-19.2\pm0.44$	$\textbf{-28.4} \pm \textbf{0.86}$		

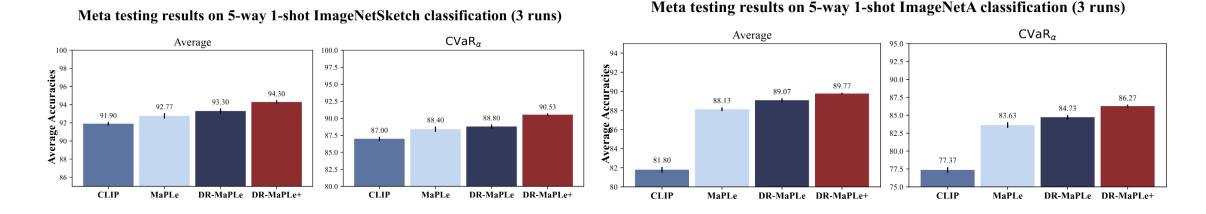
Table 2: Meta testing returns in point robot navigation (4 runs).

Result Analysis. (1) DR-MAML+ benefits from a more reliable quantile estimate and achieves superior performance.

(2) The application of distributional robustness to reinforcement learning yields improvements in returns.

# **Extensions on Large Models**

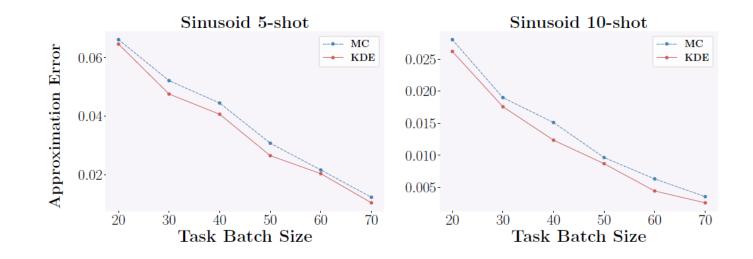
Benchmarks. Tiered-ImageNet, ImageNetA, ImageNetSketch
 Baselines. CLIP, MaPLe, DR-MaPLe



**Result Analysis.** (1) DR-MaPLe and DR-MaPLe+ consistently outperform baselines across both average and  $CVaR_{\alpha}$  indicators in **5-way 1-shot** cases, demonstrating the advantage of the two-stage strategy in enhancing the robustness of few-shot learning

(2) DR-MaPLe+ achieves better results as KDE quantiles are more accurate with large batch sizes. These results examine the scalability and compatibility of our method on large models..

## Assessment of Quantile Estimators



The VaR<sub> $\alpha$ </sub> approximation error decreases with more tasks.

The KDE produces more accurate estimates with a sharper decreasing trend.

The above well verifies the conclusion in **Theorem** 3.

## **Other Investigations**

#### **Evaluation with Other Robust Meta Learners**

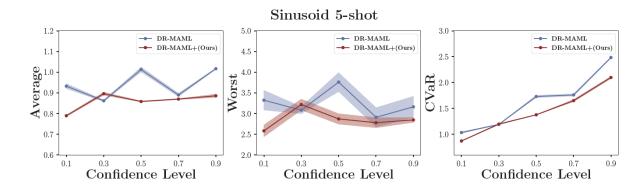
Table 4: MSEs for Sinusoid 5-shot with reported standard deviations (5 runs). With  $\alpha = 0.7$ the best results are in bold.

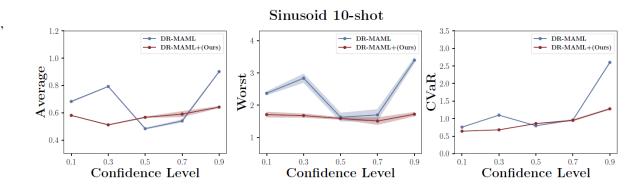
Method	Average	Worst	$CVaR_{\alpha}$
CNP [15]	0.09±0.00	$2.71 \pm 0.54$	$0.24 \pm 0.01$
TR-CNP [37]	$0.10 \pm 0.01$	$1.51 \pm 0.30$	$0.22 \pm 0.03$
DRO-CNP [60]	$0.09 \pm 0.02$	$2.54 \pm 1.81$	$0.21 \pm 0.05$
DR-CNP [1]	$0.09 \pm 0.01$	1. <u>62+0.4</u> 5	0.20+0.02
DR-CNP+(Ours)	0.08±0.01	<b>1.47</b> ±0.90	$0.17{\scriptstyle\pm0.02}$

Table 5: MSEs for Pendulum 10-shot with reported standard deviations (5 runs). With  $\alpha = 0.5$ , the best results are in bold.

Method	Average	Worst	$\mathrm{CVaR}_{\alpha}$
CNP [15]	0.75±0.01	$1.51 \pm 0.23$	$0.87 \pm 0.02$
TR-CNP [37]	$0.76 \pm 0.00$	$1.24 \pm 0.02$	$0.85 \pm 0.01$
DRO-CNP [60]	$0.73 \pm 0.01$	$1.51 \pm 0.16$	$0.85 \pm 0.01$
$\_$ <u>DR-CNP[1]</u>	0.75 <u>+0.01</u>	<u>1.40+0.16</u>	$0.86 \pm 0.01$
DR-CNP+(Ours)	0.72±0.01	$1.36 \pm 0.07$	$0.82{\pm}0.00$

#### Sensitivity Analysis to Confidence Level







Principle	Meta Learner	Generalization	Convergence	Robustness Type
MAML	$\min_{\theta \in \Theta} \mathbb{E}_{p(\tau)} \Big[ \ell(\mathfrak{D}^Q_{\tau}, \mathfrak{D}^S_{\tau}; \theta) \Big]$	1	1	
DRO-MAML	$\Big  \max_{q(\tau) \in \mathcal{Q}} \min_{\theta \in \Theta} \mathbb{E}_{q(\tau)} \Big[ \ell(\mathfrak{D}^Q_{\tau}, \mathfrak{D}^S_{\tau}; \theta) \Big]$	×	×	Uncertainty Set (Not tail risk)
TR-MAML	$\min_{\theta \in \Theta} \max_{\tau \in \mathcal{T}} \ell(\mathfrak{D}^Q_{\tau}, \mathfrak{D}^S_{\tau}; \theta)$	<ul> <li>✓</li> </ul>	✓	Worst-Case Task
DR-MAML	$\min_{\theta \in \Theta} \mathbb{E}_{p_{\alpha}(\tau;\theta)} \Big[ \ell(\mathfrak{D}_{\tau}^{Q}, \mathfrak{D}_{\tau}^{S}; \theta) \Big]$	×	×	Tail Task Risk
DR-MAML+(Ours)	$\left \max_{q(\tau)\in\mathcal{Q}_{\alpha}}\min_{\theta\in\Theta}\mathbb{E}_{q(\tau)}\left[\ell(\mathfrak{D}_{\tau}^{Q},\mathfrak{D}_{\tau}^{S};\theta)\right]\right.$	1	✓	Tail Task Risk

- DRO-MAML [5] includes the uncertainty set Q for robust fast adaptation, there exists no theoretical analysis.
- TR-MAML [4] only focuses on the worst-case, which considers a bit extreme and rarely occurred cases.
- DR-MAML [3] *lacks generalization capability and convergence rate analysis w.r.t.* the meta learner.
- DR-MAML+ is a more specific instantiation of that in DR-MAML.

- Proposes to understand the two-stage distributionally robust strategy from optimization processes.
- Defines the convergence solution, and derives the generalization bound in the presence of tail task risk.
- Extensive evaluations demonstrate the significance of our proposal and its scalability to multimodal large models in boosting robustness.

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[1] Finn, C., Abbeel, P., and Levine, S. Model-agnostic meta-learning for fast adaptation of deep networks. In International conference on machine learning, pages 1126–1135. PMLR, 2017.

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# Thanks for your attention .

