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GACL: Exemplar-Free Generalized Analytic Continual Learning

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Continual learning, Incremental learning, Lifelong learning

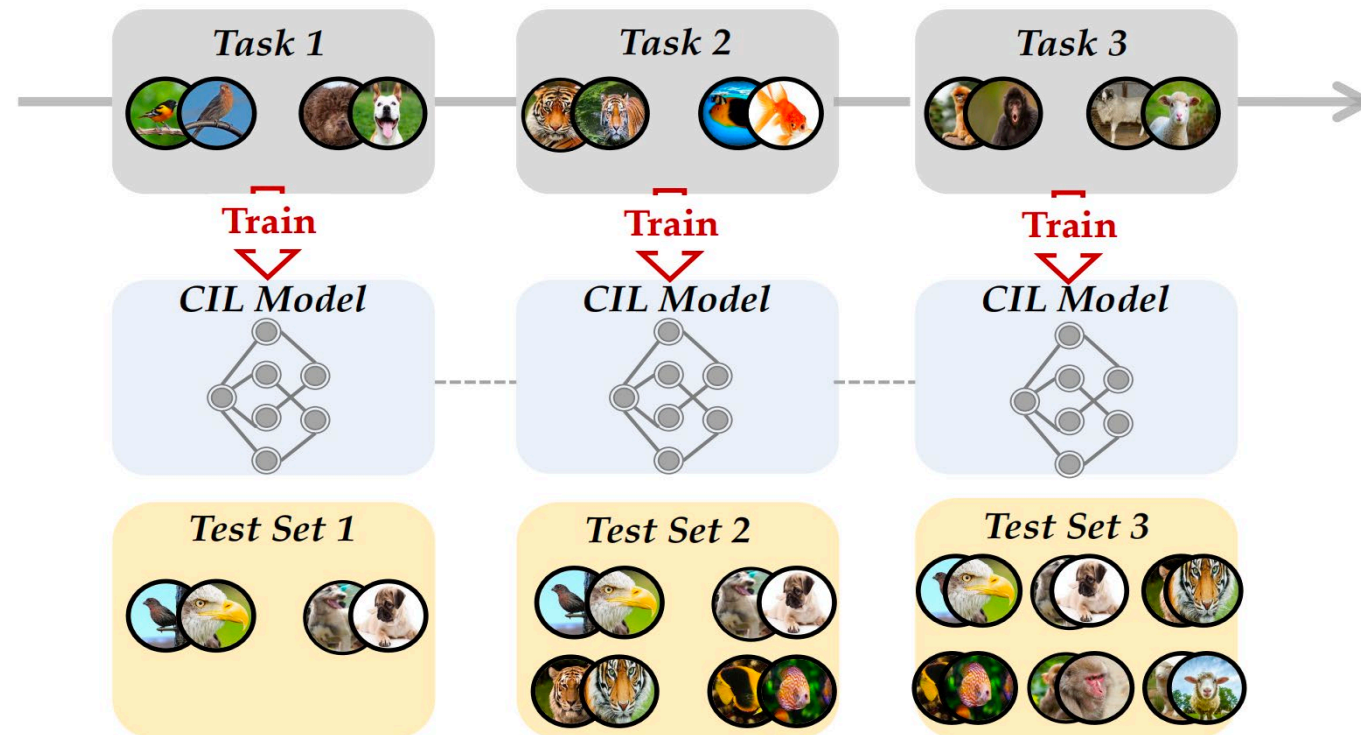
Enables **continuous knowledge acquisition**, mimicking human behavior.



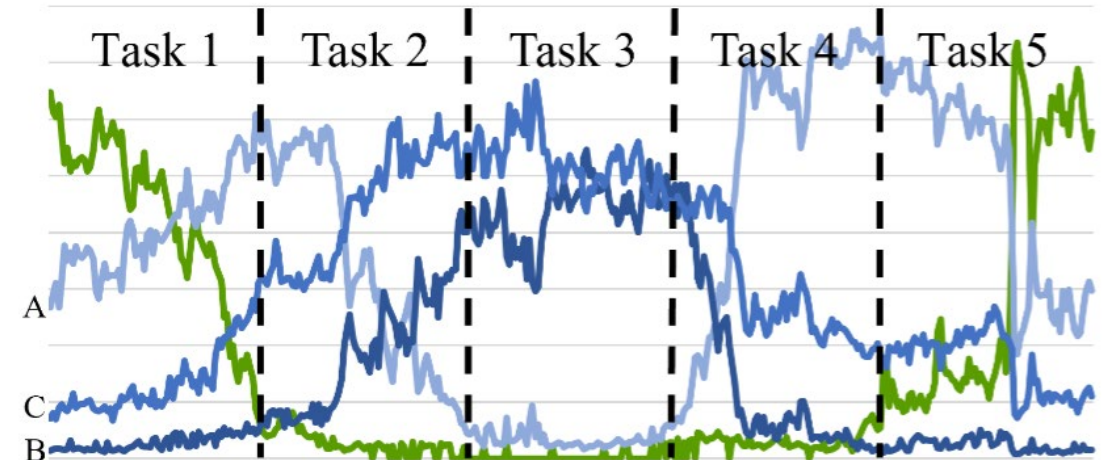
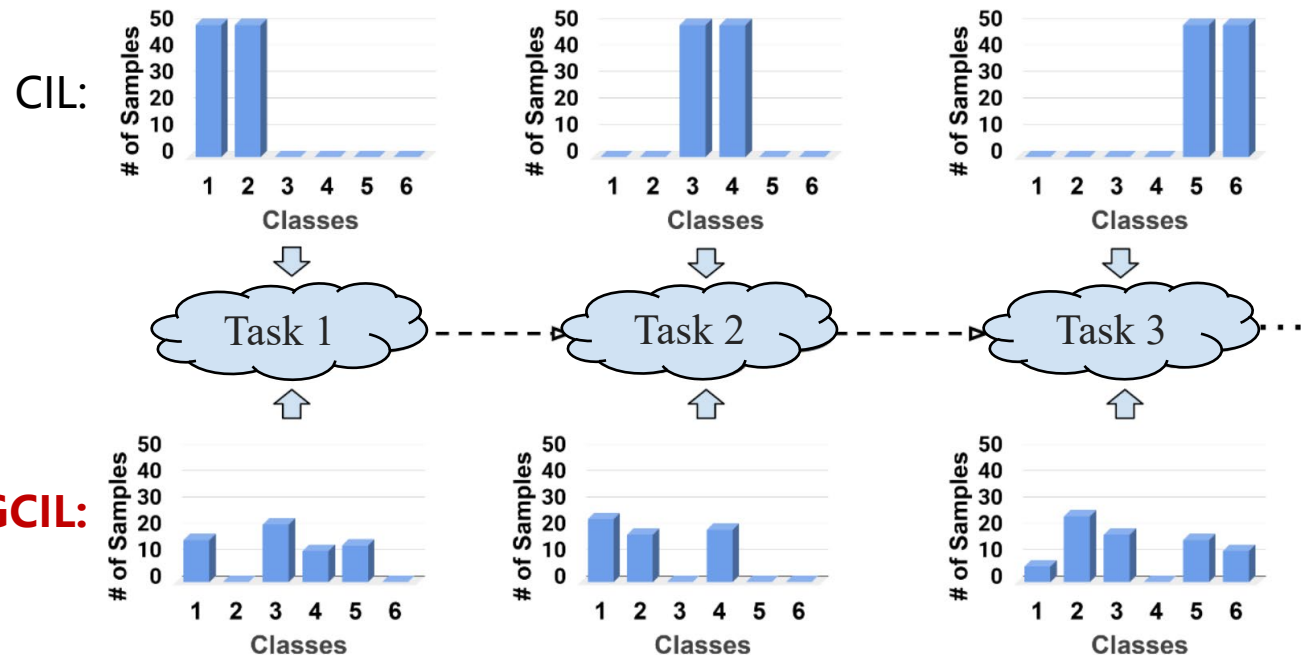
Practical Significance:

- No need for retraining
- Adapt used models to new tasks

Class Incremental Learning (CIL) aims to continually build a holistic classifier among **all seen classes** with **non-overlapping classes** arriving sequentially.

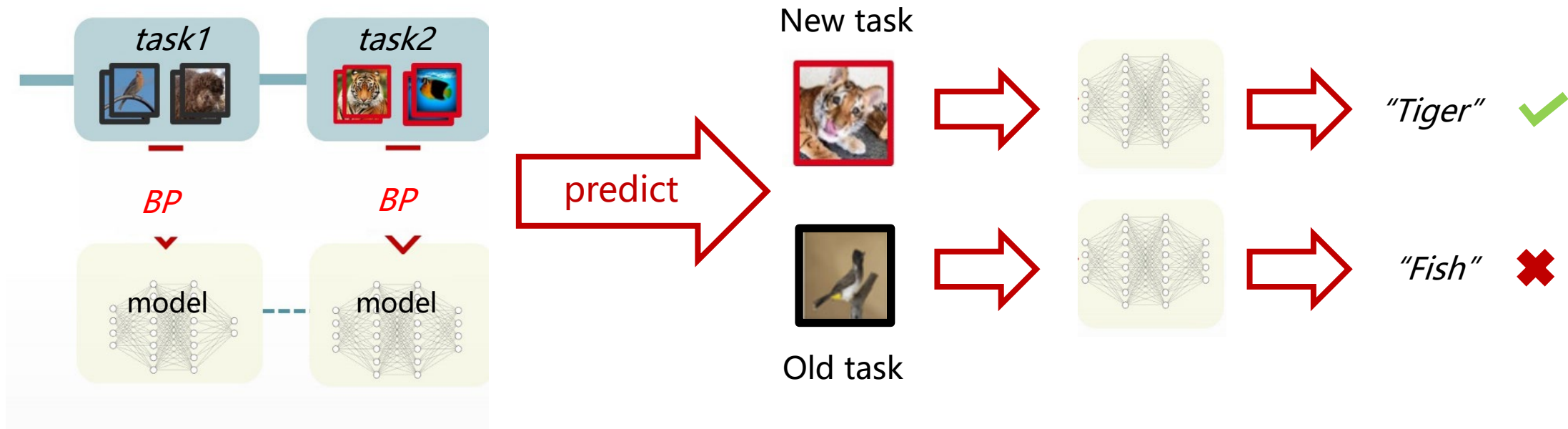


Generalized Class Incremental Learning (GCIL) simulates **real-world** incremental learning, as **distributions of data category and sample size could be uneven** between tasks.

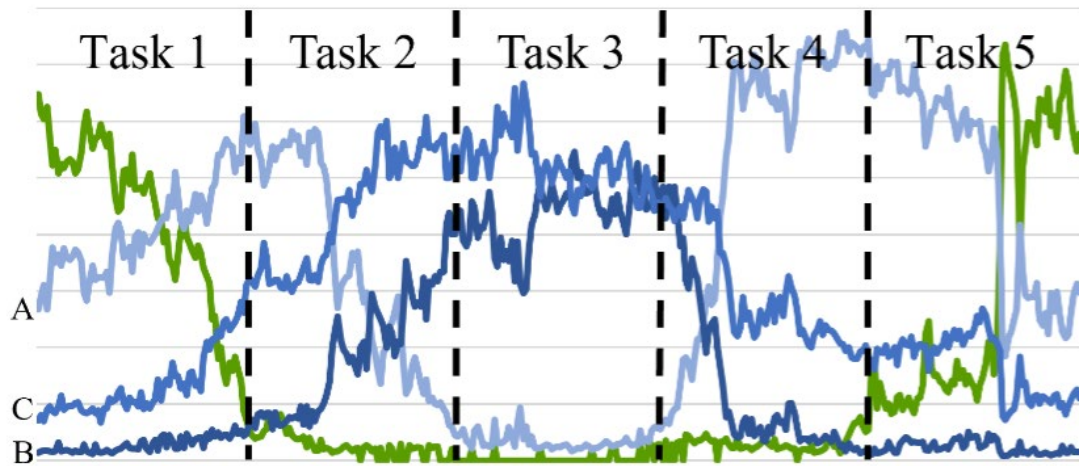


Real-World Data Example: Normalized Search History from Dec 28, 2021, to Dec 28, 2022 (NAVER Search Trend API)

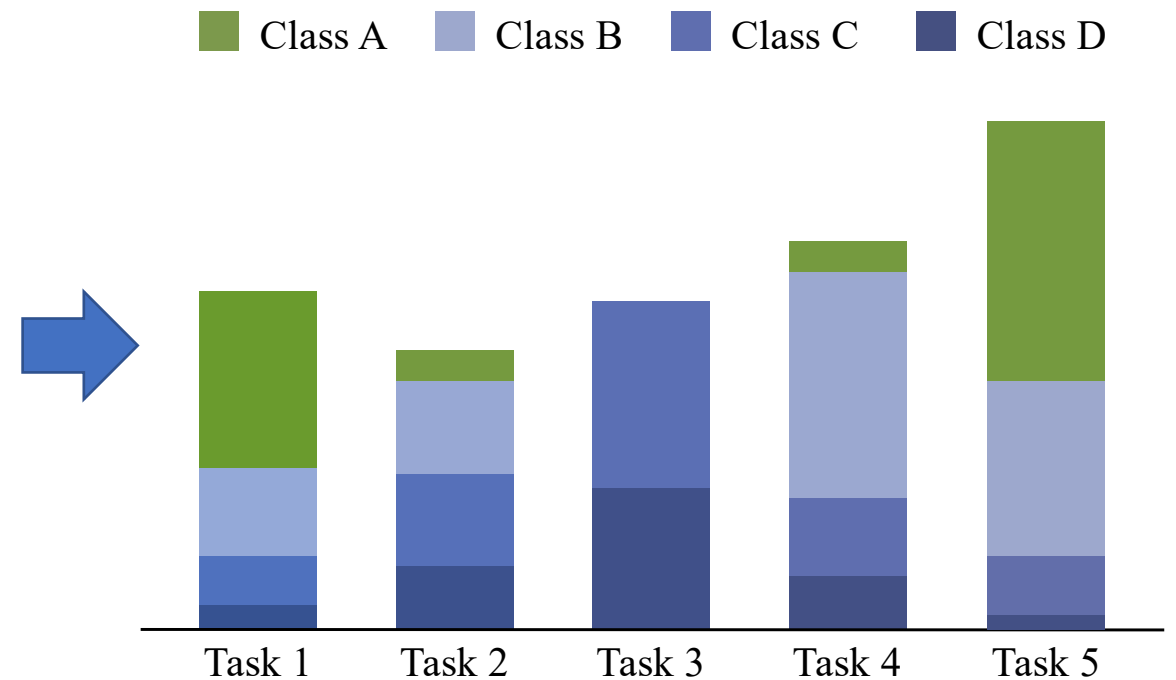
- Model Learns in **multiple stages** and different tasks.
- New model does well in **new tasks**.
- Performance **decreases for the previous** tasks.



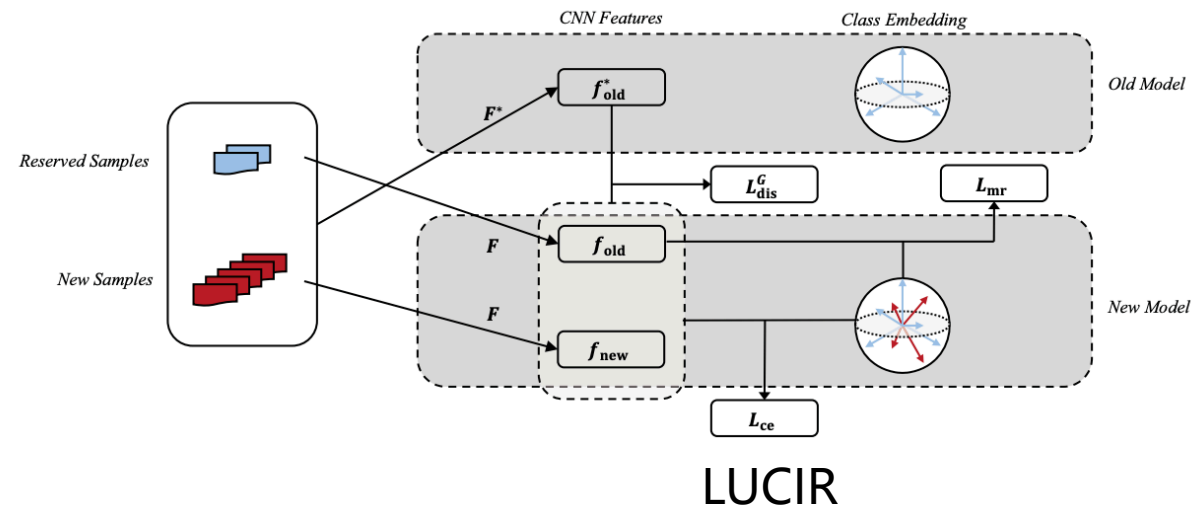
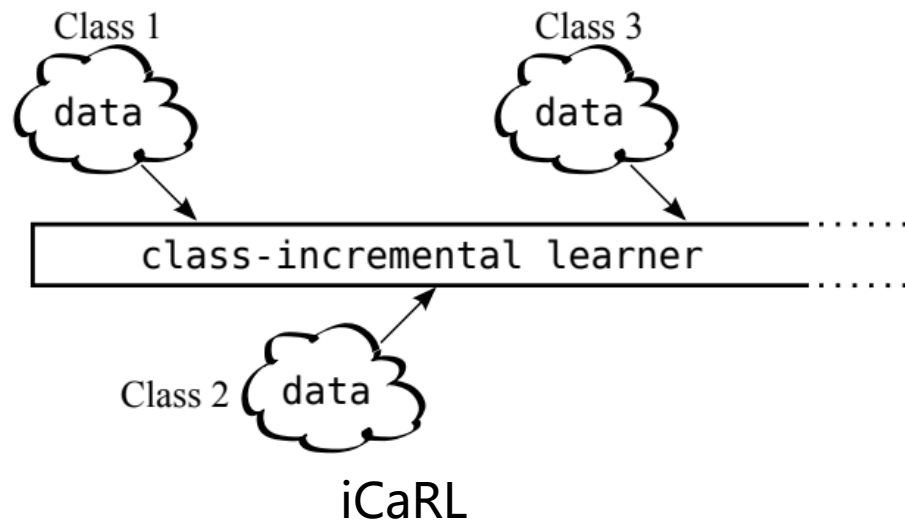
- Uneven distribution of data category and sample → **Class imbalance**
- Neglect of minority samples → **Undermining performance**



Real-World Data Example: Normalized Search History from Dec 28, 2021, to Dec 28, 2022 (NAVER Search Trend API)



- Replay-based CIL uses historical exemplars to replay.
- Invade the data privacy
- Hard to cross domain



➤ Exemplar-free CIL (EFCIL) learns without storing any historical exemplars.

- | | | |
|----------------------|---|---|
| Regularization-based | ➤ Prevent large drift of important weights | } Hard to resist forgetting
Hard to cross domain |
| Prototype-based | ➤ A prototype is selected for each category | |

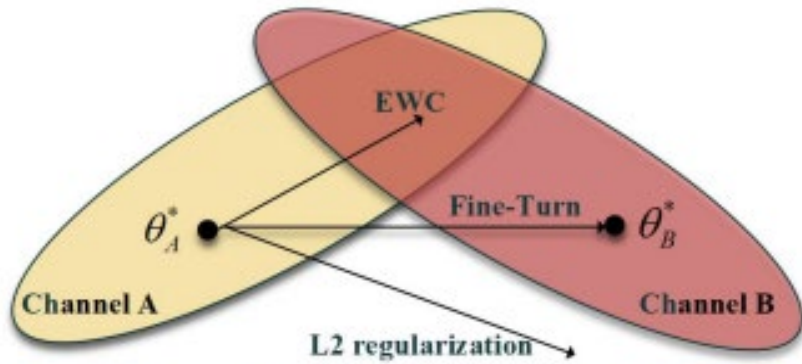
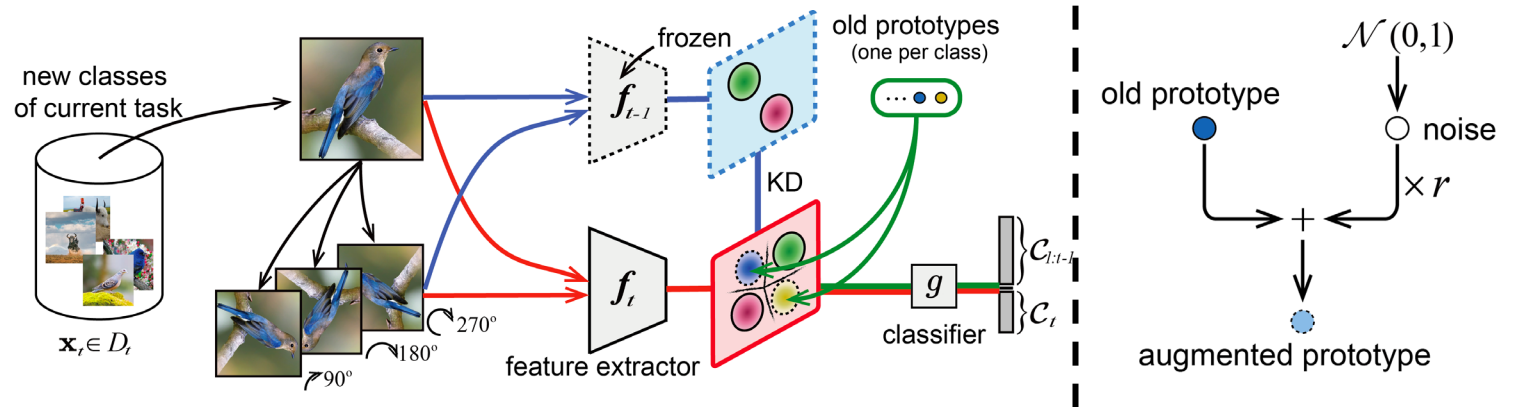


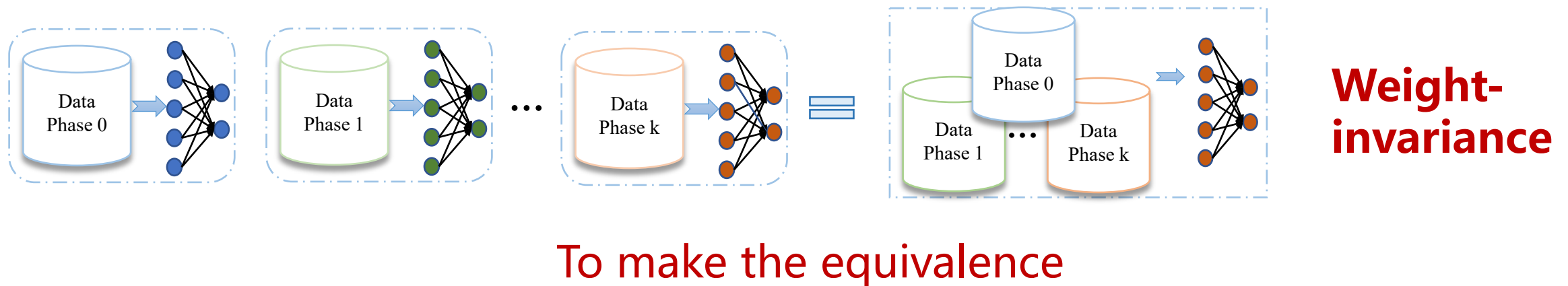
Fig. 2. The trajectory of parameter changes in parameter space

Elastic Weight Consolidation



Prototype Augmentation and Self-Supervision for Incremental Learning

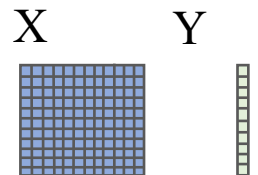
Analytic Continual Learning (ACL) makes the **equivalence** between CIL and joint learning in **linear classifiers**.



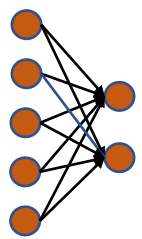
However, existing ACL are designed for traditional CIL scenarios.

Analytic learning (AL): Use **Least Square** (LS) solutions to train the network
Block-wise recursive Moore-Penrose inverse (BRMP): convert LS to **block-wise calculation**

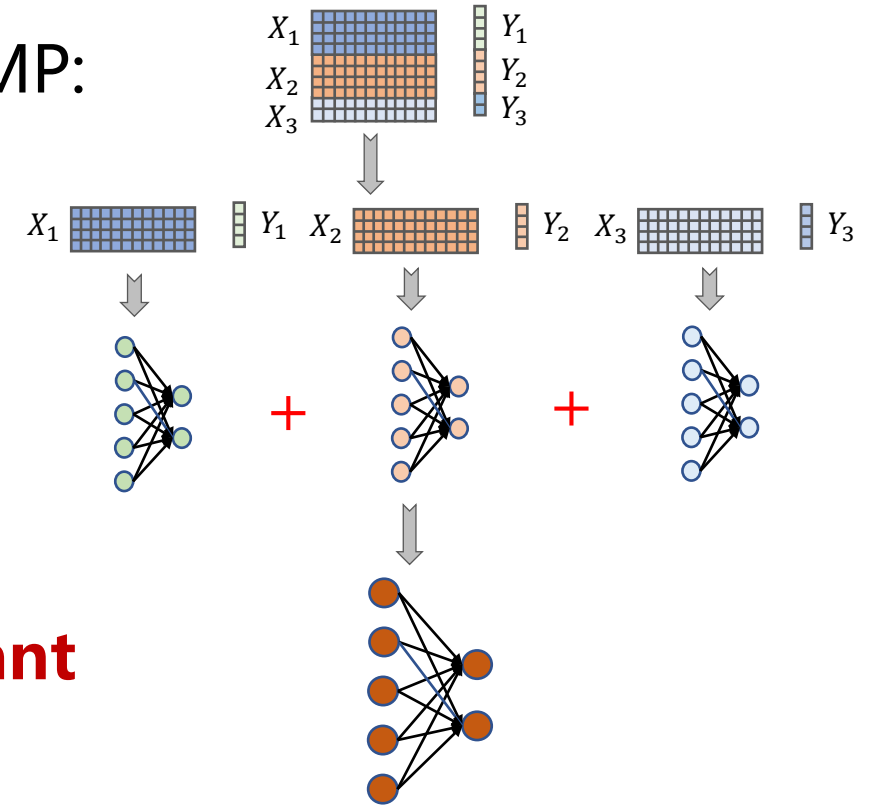
Train with LS:



Calculate **optimal weight** via LS

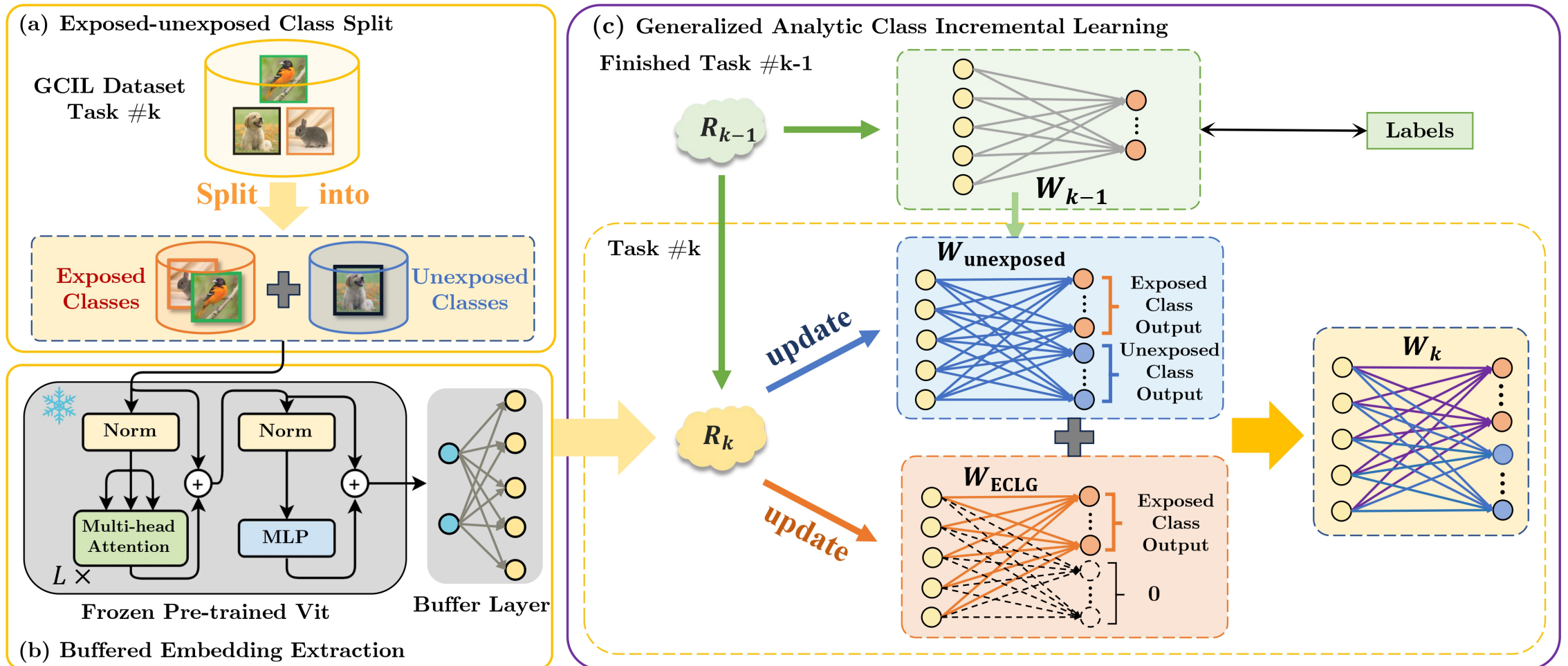


BRMP:

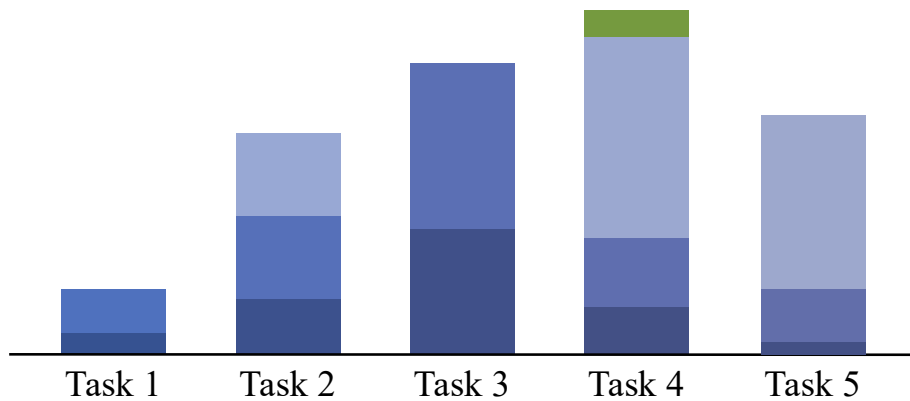


Weight-invariant

- Address the uneven data distributions of GCIL
- Extend ACL to GCIL tasks

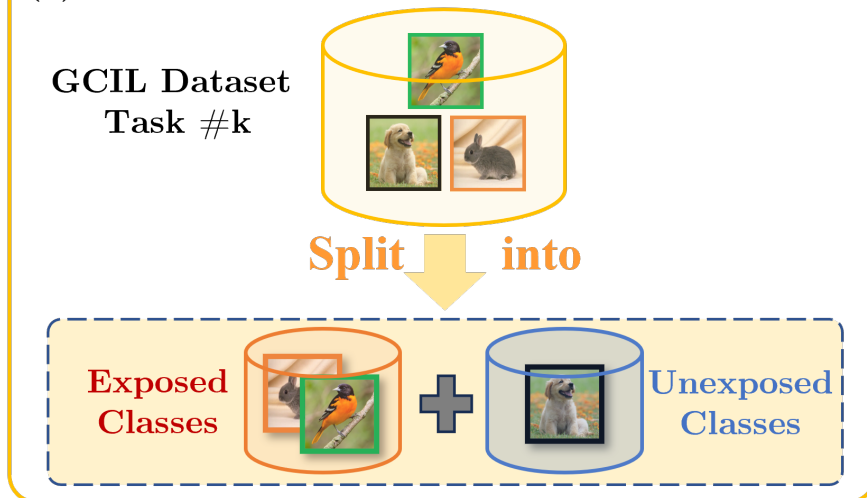


■ Class A
 ■ Class B
 ■ Class C
 ■ Class D



- **Exposed classes** in task k : classes that **have appeared** in previous tasks 1 to $k - 1$
 - **Unexposed classes** in task k : classes that **make their initial appearance**
- e.g., in task 2, exposed classes: C & D, unexposed class: B

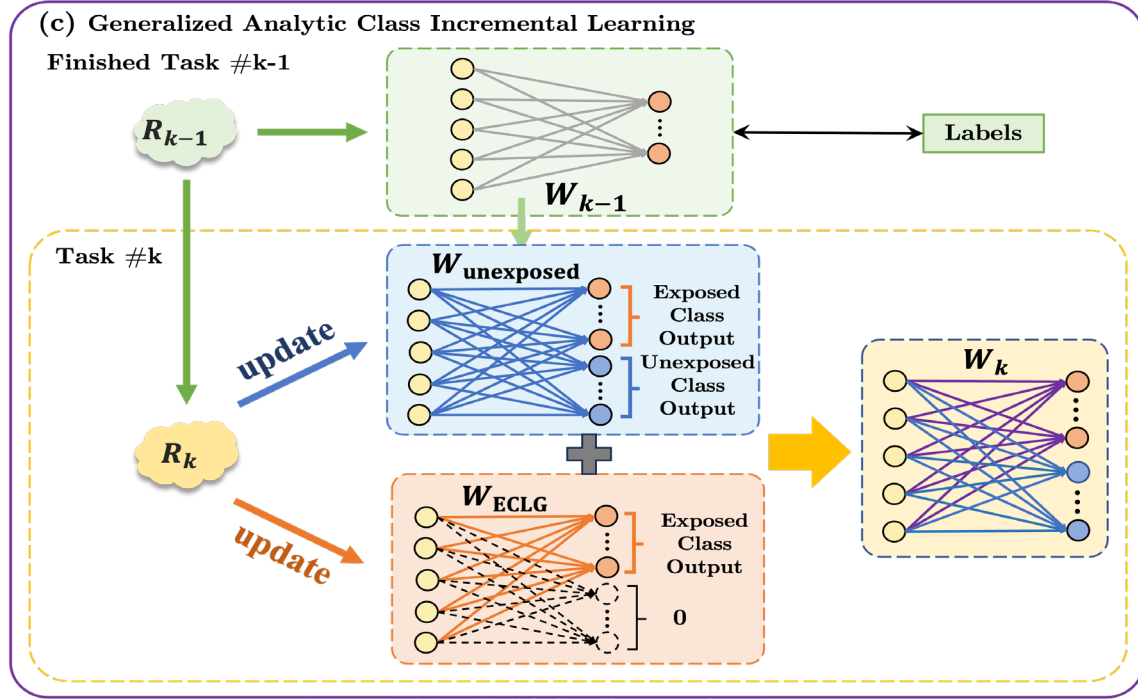
(a) Exposed-unexposed Class Split



Learning problem in task k

$$\mathbf{Y}_k^{\text{train}} = \left[\begin{array}{c} \bar{\mathbf{Y}}_k^{\text{train}} \\ \tilde{\mathbf{Y}}_k^{\text{train}} \end{array} \right]$$

Unexposed Class Label Exposed Class Label



Learning problem in task k

$$\mathbf{X}_{1:k}^{\text{total}} = \begin{bmatrix} \mathbf{X}_{1:k-1}^{\text{total}} \\ \mathbf{X}_k^{(B)} \end{bmatrix} \quad \mathbf{Y}_{1:k}^{\text{total}} = \begin{bmatrix} \mathbf{Y}_{1:k-1}^{\text{total}} & \mathbf{0} \\ \tilde{\mathbf{Y}}_k^{\text{train}} & \tilde{\mathbf{Y}}_k^{\text{train}} \end{bmatrix}$$

$$\underset{\mathbf{W}_{\text{FCN}}^{(k)}}{\text{argmin}} \left\| \begin{bmatrix} \mathbf{Y}_{1:k-1}^{\text{total}} & \mathbf{0} \\ \tilde{\mathbf{Y}}_k^{\text{train}} & \tilde{\mathbf{Y}}_k^{\text{train}} \end{bmatrix} - \begin{bmatrix} \mathbf{X}_{1:k-1}^{\text{total}} \\ \mathbf{X}_k^{(B)} \end{bmatrix} \mathbf{W}_{\text{FCN}}^{(k)} \right\|_{\text{F}}^2 + \gamma \left\| \mathbf{W}_{\text{FCN}}^{(k)} \right\|_{\text{F}}^2$$

Unexposed Class Label Exposed Class Label

➤ Solution of Least Square

=

➤ Solution of Recursive Least Square

$$\hat{\mathbf{W}}_{\text{FCN}}^{(k)} = (\mathbf{X}_{1:k}^{\text{total}\top} \mathbf{X}_{1:k}^{\text{total}} + \gamma \mathbf{I})^{-1} \mathbf{X}_{1:k}^{\text{total}\top} \mathbf{Y}_{1:k}^{\text{total}}$$

$$\mathbf{R}_k = (\mathbf{X}_{1:k}^{\text{total}\top} \mathbf{X}_{1:k}^{\text{total}} + \gamma \mathbf{I})^{-1}$$

$$\hat{\mathbf{W}}_{\text{FCN}}^{(k)} = \left[\hat{\mathbf{W}}_{\text{FCN}}^{(k-1)} - \mathbf{R}_k \mathbf{X}_k^{(B)\top} \mathbf{X}_k^{(B)} \hat{\mathbf{W}}_{\text{FCN}}^{(k-1)} + \mathbf{R}_k \mathbf{X}_k^{(B)\top} \tilde{\mathbf{Y}}_k^{\text{train}} \quad \mathbf{R}_k \mathbf{X}_k^{(B)\top} \tilde{\mathbf{Y}}_k^{\text{train}} \right]$$

$$= \hat{\mathbf{W}}_{\text{unexposed}}^{(k)} + \hat{\mathbf{W}}_{\text{ECLG}}^{(k)}$$

$$\hat{\mathbf{W}}_{\text{unexposed}}^{(k)} = \left[\hat{\mathbf{W}}_{\text{FCN}}^{(k-1)} - \mathbf{R}_k \mathbf{X}_k^{(B)\top} \mathbf{X}_k^{(B)} \hat{\mathbf{W}}_{\text{FCN}}^{(k-1)} \quad \mathbf{R}_k \mathbf{X}_k^{(B)\top} \tilde{\mathbf{Y}}_k^{\text{train}} \right]$$

$$\hat{\mathbf{W}}_{\text{ECLG}}^{(k)} = \left[\mathbf{R}_k \mathbf{X}_k^{(B)\top} \tilde{\mathbf{Y}}_k^{\text{train}} \quad \mathbf{0} \right] \quad \text{ECLG Module}$$

$$\mathbf{R}_k = \mathbf{R}_{k-1} - \mathbf{R}_{k-1} \mathbf{X}_k^{(B)\top} (\mathbf{I} + \mathbf{X}_k^{(B)} \mathbf{R}_{k-1} \mathbf{X}_k^{(B)\top})^{-1} \mathbf{X}_k^{(B)} \mathbf{R}_{k-1}$$

- **Outperform existing** EFCIL methods by a considerable margin
- **Better than** most SOTA replay-based methods

Mem Size	Method	EFCIL	CIFAR-100 (%)			ImageNet-R (%)			Tiny-ImageNet (%)		
			\mathcal{A}_{AUC}	\mathcal{A}_{Avg}	\mathcal{A}_{Last}	\mathcal{A}_{AUC}	\mathcal{A}_{Avg}	\mathcal{A}_{Last}	\mathcal{A}_{AUC}	\mathcal{A}_{Avg}	\mathcal{A}_{Last}
2000	EWC++ [16]	×	53.31 \pm 1.70	50.95 \pm 1.50	52.55 \pm 0.71	36.31 \pm 0.72	39.87 \pm 1.35	29.52 \pm 0.43	52.43 \pm 0.52	54.61 \pm 1.54	37.67 \pm 0.77
	ER [35]	×	56.17 \pm 1.84	53.80 \pm 1.46	55.60 \pm 0.69	39.31 \pm 0.70	43.03 \pm 1.19	32.09 \pm 0.44	55.69 \pm 0.47	57.87 \pm 1.42	41.10 \pm 0.57
	RM [32]	×	53.22 \pm 1.82	52.99 \pm 1.69	55.25 \pm 0.61	32.34 \pm 1.88	36.46 \pm 2.23	25.26 \pm 1.08	49.28 \pm 0.43	57.74 \pm 1.57	41.79 \pm 0.34
	MVP-R [30]	×	60.62 \pm 1.03	57.58 \pm 0.56	64.30 \pm 0.29	47.16 \pm 1.00	50.36 \pm 0.90	42.05 \pm 0.15	61.15 \pm 0.86	62.41 \pm 0.50	51.12 \pm 0.67
500	EWC++ [16]	×	48.31 \pm 1.81	44.56 \pm 0.96	40.52 \pm 0.83	32.81 \pm 0.76	35.54 \pm 1.69	23.43 \pm 0.61	45.30 \pm 0.61	46.34 \pm 2.05	27.05 \pm 1.35
	ER [35]	×	51.59 \pm 1.94	48.03 \pm 0.80	44.09 \pm 0.80	35.96 \pm 0.72	39.01 \pm 1.54	26.14 \pm 0.44	48.95 \pm 0.58	50.44 \pm 1.71	29.97 \pm 0.75
	RM [32]	×	41.07 \pm 1.30	38.10 \pm 0.59	32.66 \pm 0.34	22.45 \pm 0.62	22.08 \pm 1.78	9.61 \pm 0.13	36.66 \pm 0.40	38.83 \pm 2.33	18.23 \pm 0.22
	MVP-R [30]	×	56.20 \pm 1.47	53.61 \pm 0.04	55.35 \pm 0.43	43.28 \pm 1.41	45.74 \pm 0.97	35.60 \pm 1.18	55.28 \pm 1.42	55.45 \pm 1.02	40.12 \pm 0.40
0	LwF [14]	✓	40.71 \pm 2.13	38.49 \pm 0.56	27.03 \pm 2.92	29.41 \pm 0.83	31.95 \pm 1.86	19.67 \pm 1.27	39.88 \pm 0.90	41.35 \pm 2.59	24.93 \pm 2.01
	L2P [36]	✓	42.68 \pm 2.70	39.89 \pm 0.45	28.59 \pm 3.34	30.21 \pm 0.91	32.21 \pm 1.73	18.01 \pm 3.07	41.67 \pm 1.17	42.53 \pm 2.52	24.78 \pm 2.31
	DualPrompt [33]	✓	41.34 \pm 2.59	38.59 \pm 0.68	22.74 \pm 3.40	30.44 \pm 0.88	32.54 \pm 1.84	16.07 \pm 3.20	39.16 \pm 1.13	39.81 \pm 3.03	20.42 \pm 3.37
	MVP [30]	✓	45.07 \pm 2.43	44.93 \pm 0.54	39.94 \pm 0.47	35.77 \pm 2.55	35.58 \pm 1.20	22.06 \pm 5.01	46.43 \pm 3.07	45.41 \pm 1.09	28.21 \pm 2.89
	SLDA [37]	✓	53.00 \pm 3.85	50.09 \pm 2.77	61.79 \pm 3.81	33.11 \pm 3.17	33.78 \pm 1.76	39.02 \pm 1.30	46.43 \pm 3.07	45.41 \pm 4.43	53.13 \pm 2.29
	GACL (ours)	✓	57.99\pm2.46	56.24\pm3.12	70.31\pm0.06	41.68\pm0.78	47.30\pm0.84	42.22\pm0.10	63.14\pm0.66	69.32\pm0.87	62.68\pm0.08

- The ECLG module captures knowledge from exposed-class labels
- Demonstrate competence with ECLG module

r_B	Dataset	With ECLG			Without ECLG		
		$\mathcal{A}_{AUC}(\%)$	$\mathcal{A}_{Avg}(\%)$	$\mathcal{A}_{Last}(\%)$	$\mathcal{A}_{AUC}(\%)$	$\mathcal{A}_{Avg}(\%)$	$\mathcal{A}_{Last}(\%)$
10%	CIFAR-100	57.99 ± 2.46	56.24 ± 3.12	70.31 ± 0.06	45.68 ± 7.74	42.04 ± 4.52	47.30 ± 2.61
	ImageNet-R	41.68 ± 0.78	47.30 ± 0.84	42.22 ± 0.10	40.29 ± 2.23	46.95 ± 1.15	41.67 ± 0.36
	Tiny-ImageNet	63.14 ± 0.66	69.32 ± 0.87	62.68 ± 0.08	60.21 ± 1.86	65.80 ± 1.20	60.13 ± 0.37
30%	CIFAR-100	57.33 ± 1.03	58.74 ± 1.59	69.90 ± 0.01	42.53 ± 1.97	42.26 ± 1.75	45.49 ± 1.17
	ImageNet-R	42.19 ± 0.44	47.82 ± 1.11	42.90 ± 0.08	42.01 ± 0.26	46.95 ± 1.15	41.67 ± 0.56
	Tiny-ImageNet	60.73 ± 1.15	67.31 ± 1.14	59.73 ± 2.55	60.63 ± 1.86	57.03 ± 1.98	60.13 ± 0.55
50%	CIFAR-100	56.74 ± 1.14	58.29 ± 1.95	70.02 ± 0.05	40.91 ± 3.57	47.25 ± 2.64	58.61 ± 2.62
	ImageNet-R	41.33 ± 1.46	46.42 ± 2.30	42.92 ± 0.17	40.44 ± 3.14	42.50 ± 3.43	39.05 ± 1.65
	Tiny-ImageNet	60.96 ± 1.83	66.28 ± 2.69	62.24 ± 0.10	60.32 ± 4.20	60.70 ± 4.30	56.97 ± 1.89

- Investigate with various disjoint class ratio and the blurry sample ratio
- Demonstrate the robustness of the GACL

r_D	Method	$\mathcal{A}_{\text{AUC}}(\%)$	$\mathcal{A}_{\text{Avg}}(\%)$	$\mathcal{A}_{\text{Last}}(\%)$
0%	SLDA [37]	<u>55.51\pm1.93</u>	<u>53.94\pm0.92</u>	67.45 \pm 0.26
	MVP-R [30]	53.49 \pm 1.40	50.73 \pm 0.37	60.54 \pm 2.03
	GACL (ours)	49.96 \pm 0.61	50.56 \pm 0.49	<u>69.94\pm0.09</u>
50%	SLDA [37]	53.00 \pm 3.85	50.09 \pm 2.77	61.79 \pm 3.81
	MVP-R [30]	56.20 \pm 1.47	53.61 \pm 0.04	55.35 \pm 0.43
	GACL (ours)	<u>57.99\pm2.46</u>	<u>56.24\pm3.12</u>	<u>70.31\pm0.06</u>
100%	SLDA [37]	65.46 \pm 4.79	67.29 \pm 5.28	63.56 \pm 2.68
	MVP-R [30]	68.43 \pm 0.28	68.04 \pm 1.48	53.14 \pm 0.72
	GACL (ours)	<u>70.72\pm0.32</u>	<u>77.57\pm1.02</u>	<u>69.97\pm0.03</u>

The performance at different disjoint class ratio with blurry sample ratio = 10% on CIFAR-100.

r_B	Method	$\mathcal{A}_{\text{AUC}}(\%)$	$\mathcal{A}_{\text{Avg}}(\%)$	$\mathcal{A}_{\text{Last}}(\%)$
10%	SLDA [37]	53.00 \pm 3.85	50.09 \pm 2.77	61.79 \pm 3.81
	MVP-R [30]	56.20 \pm 1.47	53.61 \pm 0.04	55.35 \pm 0.43
	GACL (ours)	<u>57.99\pm2.46</u>	<u>56.24\pm3.12</u>	<u>70.31\pm0.06</u>
30%	SLDA [37]	54.55 \pm 4.66	54.06 \pm 2.41	63.04 \pm 2.56
	MVP-R [30]	<u>59.65\pm2.04</u>	58.31 \pm 1.52	58.16 \pm 1.38
	GACL (ours)	<u>57.33\pm1.03</u>	<u>58.74\pm1.59</u>	<u>69.90\pm0.01</u>
50%	SLDA [37]	53.81 \pm 3.43	52.93 \pm 2.36	63.45 \pm 2.72
	MVP-R [30]	<u>59.10\pm1.98</u>	57.34 \pm 1.96	54.81 \pm 0.21
	GACL (ours)	<u>56.74\pm1.14</u>	<u>58.29\pm1.95</u>	<u>70.02\pm0.05</u>

The performance at different blurry sample ratio with disjoint class ratio = 50% on CIFAR-100.

Thanks for Watching!

- Our paper is available at:

<https://arxiv.org/abs/2403.15706>



- Our codes are available at:

<https://github.com/CHEN-YIZHU/GACL>

