Reparameterized Multi-Resolution Convolutions for Long Sequence Modelling

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Global convolution sequence models:

✔ **Effective general-purpose sequence models**

- *FlexConv, S4, S4D, SGConv, Hyena*

✔ **Efficient computation via FFTs**

- *FlashFFTConv*

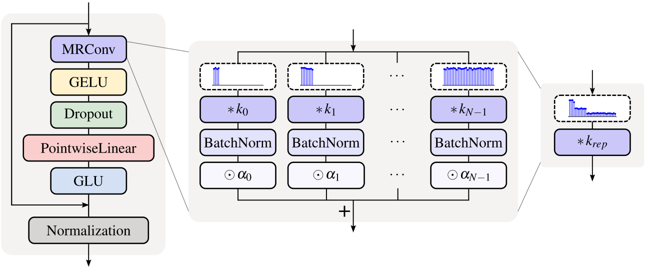
✗ **Difficult to train**

- *Explicitly parameterized kernels are prone to overfitting*
- *Implicit kernel parameterization, regularization, composition of sub kernels*

✗ **Hand-crafted inductive biases**

- *Fixed kernel decay*

MRConv: Multi-Resolution Convolutions



(a) Block (b) Training (c) Inference

1. **Multi-Resolution Convolutions**
   - *Introduces learnable kernel decay*
2. **Causal Structural Reparameterization**
   - *Improves training by introducing training-time non-linearity*
3. **Low-Rank Kernel Parameterizations**
   - *Explicitly parameterized kernels are prone to overfitting*

Multi-Resolution Convolutions

We define **multi-resolution convolutions** as the weighted sum of normalized convolutions of different length

$$y = \alpha_0 \mathrm{BN}_0(k_0 * u) + \alpha_1 \mathrm{BN}_1(k_1 * u) + \cdots + \alpha_{N-1} \mathrm{BN}_{N-1}(k_{N-1} * u) \tag{1}$$

- At each resolution the kernel k_i is of length $2^i l_0$
- Weighted sum of multi-resolution implicitly learns kernel decay
- BatchNorm required for learning weighted sum due to impact of kernel size on output statistics

Causal Structural Reparameterization

We can merge multiple causal convolutions into one as,

$$y = \underbrace{\sum_{n=0}^{N-1}(u * k_n)}_{\text{Sum of convolutions}} = \underbrace{\left(u * \left(\sum_{n=0}^{N-1} k_n\right)\right)}_{\text{Convolution of sum}} = (u * k_{rep}), \tag{2}$$

But what about BatchNorm?

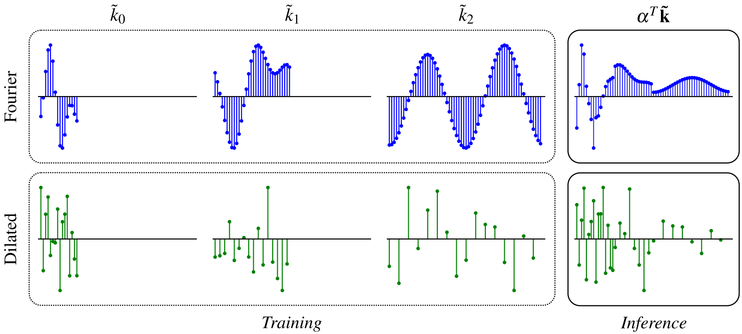
✗ Non-linear during training → Cannot Merge

$$y = \underbrace{\alpha_0 \mathrm{BN}_0(k_0 * u)}_{\text{Sum of convolutions}} + \underbrace{\alpha_1 \mathrm{BN}_1(k_1 * u)}_{\text{Sum of convolutions}} + \cdots + \underbrace{\alpha_{N-1} \mathrm{BN}_{N-1}(k_{N-1} * u)}_{\text{Sum of convolutions}} \tag{3}$$

✔ Linear during inference → Merge

$$y = u * \underbrace{(\alpha_0 \mathrm{BN}_0(k_0) + \alpha_1 \mathrm{BN}_1(k_1) + \cdots + \alpha_{N-1} \mathrm{BN}_{N-1}(k_{N-1}))}_{\text{Convolution of sum}} \tag{4}$$

Low-Rank Kernel Parameterizations



1. **Dilated Kernels** $y[t] = (u * k_{dilated})[t] = \sum_{\tau=0}^{l-1} k[\tau]u[t - p\tau]$

2. **Fourier Kernels** $k_{fourier}[t] = \text{IFFT}[\text{ZeroPad}(\hat{k}, L - m)])[t]$

3. **Sparse Kernels** $k_{sparse}[t] = \delta_{t \in \mathcal{T}} \cdot k_t$

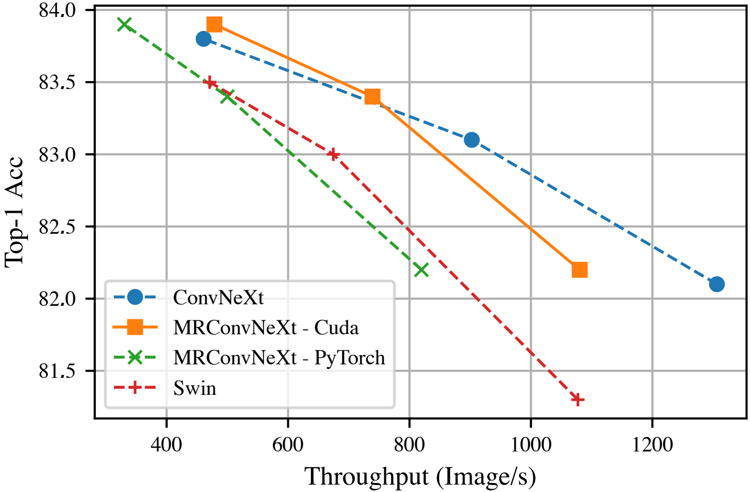
Experiments: Long Range Arena

MRConv is competitive with other sub-quadratic complexity models, including SSMs and linear-time transformers.

Model (Input length)	ListOps (2,048)	Text (4,096)	Retrieval (4,000)	Image (1,024)	Pathfinder (1,024)	Path-X (16,384)	Avg.
Transformer	36.37	64.27	57.46	42.44	71.40		53.66
Linear-Time Transformers:							
MEGA-Chunk	58.76	**90.19**	90.97	85.80	94.41	93.81	85.66
State Space Models:							
S4D-LegS	60.47	86.18	89.46	88.19	93.06	91.95	84.89
S4-LegS	59.60	86.82	90.90	88.65	94.20	96.35	86.09
Liquid-S4	**62.75**	89.02	91.20	89.50	94.8	96.66	87.32
S5	62.15	89.31	91.40	88.00	95.33	**98.58**	**87.46**
Convolutional Models:							
CCNN	43.60	84.08	-	88.90	91.51		-
Long Conv	62.2	89.6	91.3	87.0	93.2	96.0	86.6
SGConv	61.45	89.20	91.11	87.97	95.46	97.83	87.17
MRConv	62.40	89.26	**91.44**	**90.37**	**95.55**	97.82	**87.81**

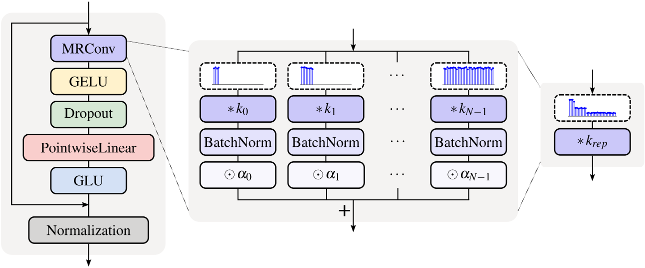
Experiments: ImageNet Classification

Using optimized CUDA kernels for 1D FFT convolutions, we close the gap between theoretical and empirical throughput.



Summary

Thank you for listening!



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More in our paper:

- More experiments
- More ablations
- More implementation details