

# StrategyLLM: Large Language Models as Strategy Generators, Executors, Optimizers, and Evaluators for Problem Solving

Chang Gao<sup>1</sup>, Haiyun Jiang<sup>2</sup>, Deng Cai<sup>3</sup>, Shuming Shi<sup>3</sup>, Wai Lam<sup>1</sup>

<sup>1</sup>The Chinese University of Hong Kong

<sup>2</sup>Shanghai Jiao Tong University

<sup>3</sup>Tencent AI Lab



香港中文大學  
The Chinese University of Hong Kong



# Chain-of-Thought (CoT) Prompting

## Standard Prompting

### Model Input

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how many apples do they have?

### Model Output

A: The answer is 27. ❌

## Chain-of-Thought Prompting

### Model Input

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

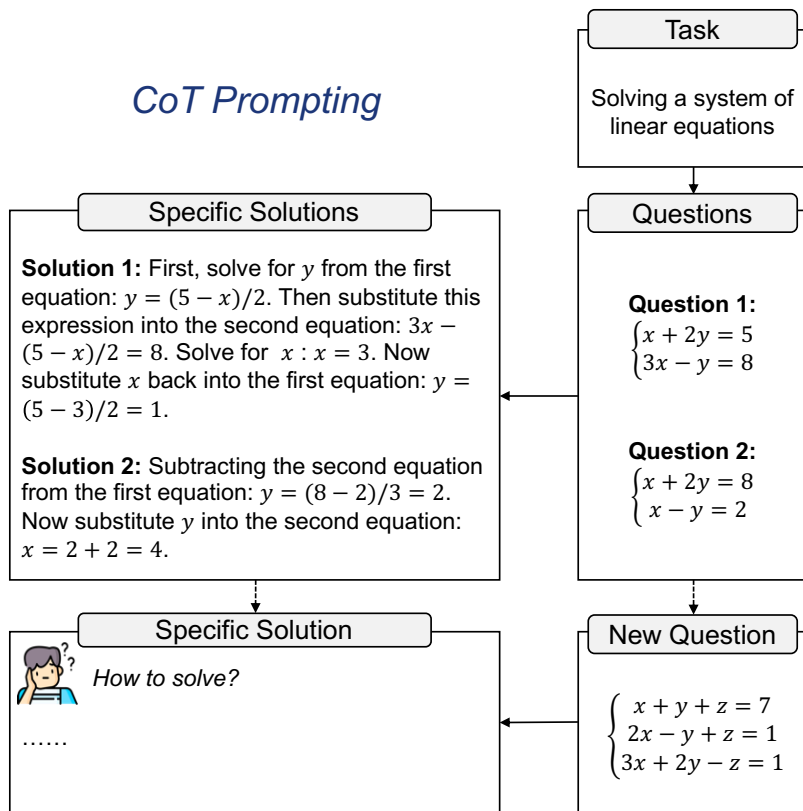
A: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls.  $5 + 6 = 11$ . The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how many apples do they have?

### Model Output

A: The cafeteria had 23 apples originally. They used 20 to make lunch. So they had  $23 - 20 = 3$ . They bought 6 more apples, so they have  $3 + 6 = 9$ . The answer is 9. ✅

# Limited Generalizability of CoT Prompting

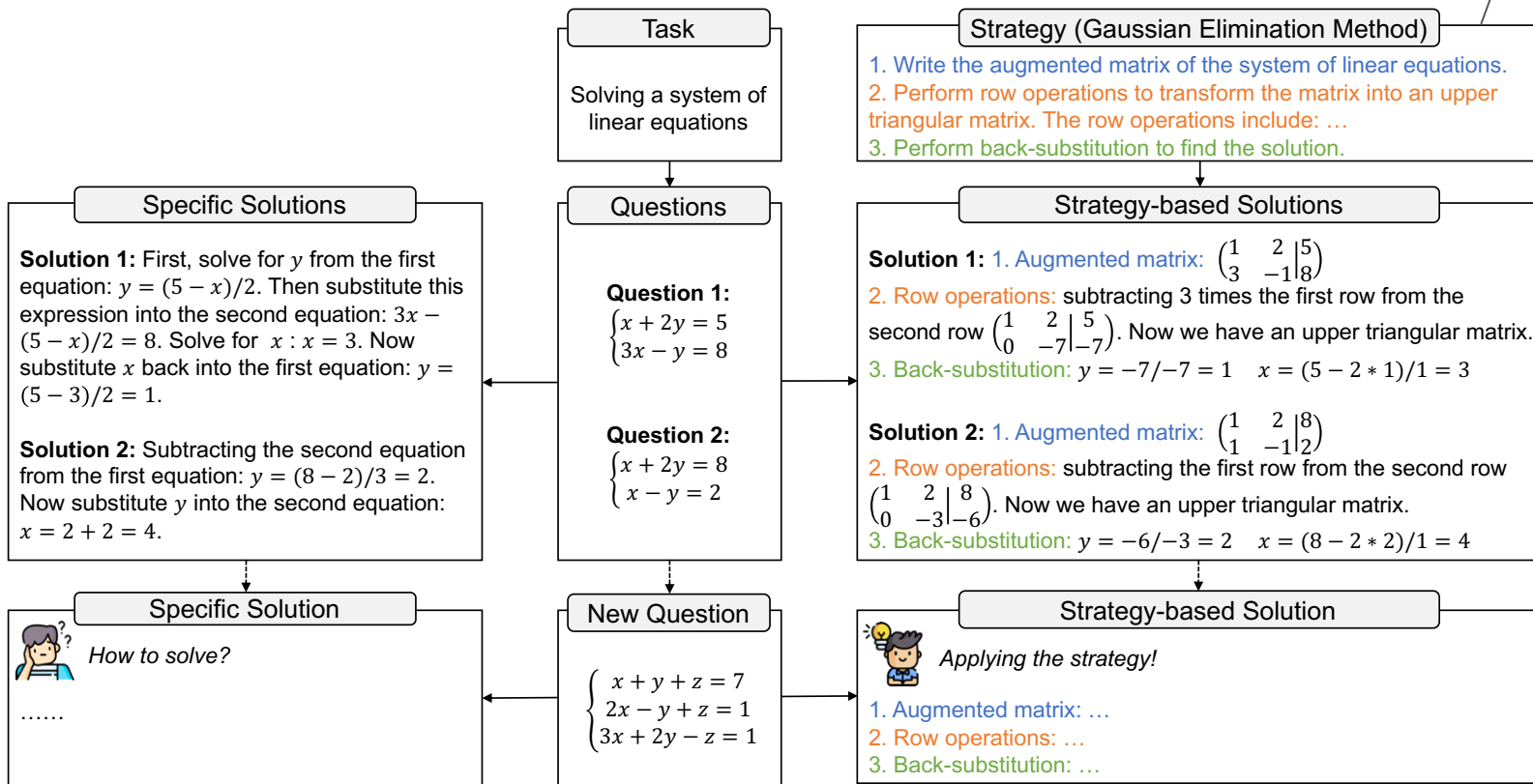


# Limited Generalizability of CoT Prompting

Task-Level Applicability  
Structured Organization  
Task Knowledge Encoding

CoT Prompting

Strategy-based Prompting

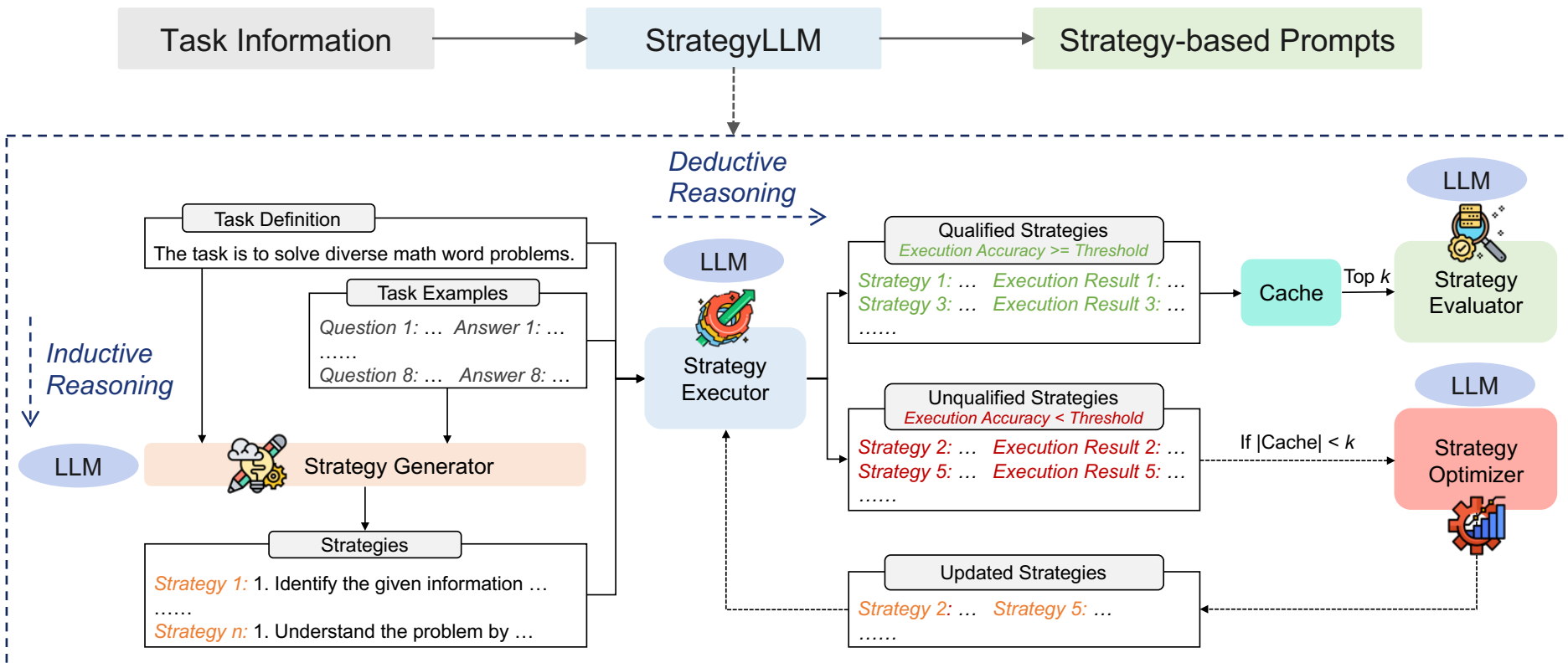


# Research Question

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*Can LLMs construct generalizable strategy-based few-shot prompts for various tasks automatically?*

# StrategyLLM: A Multi-agent Collaboration Framework



# Strategy-based Prompt

## MATH-Counting and Probability

### Strategy:

1. Identify the type of the counting or probability problem presented.
2. If the problem is regarding permutations or combinations, identify all possible outcomes and arrange or select the required number of objects accordingly.
3. If the problem is about probability, assess the sample space, the number of possible outcomes and the number of favorable outcomes.
4. For probability questions, use the formula of probability, which is the number of favorable outcomes divided by the number of total outcomes.
5. If the problem involves conditions or restrictions, ensure to factor in these when calculating probability or counting the possible arrangements or selections.
6. If the problem involves complex calculations, break them down into smaller manageable calculations.
7. Apply the suitable rule or formula according to the type of counting or probability problem, such as multiplication rule, addition rule, arrangement rules, rules of permutations and combinations, or probability rules.
8. Compute the result using arithmetic calculation or mathematical formulas, and write down the answer.
9. Review the process and the answer to ensure accuracy.

### Executing the strategy on a task example:

**Question:** Given that  $\binom{17}{7} = 19448$ ,  $\binom{17}{8} = 24310$  and  $\binom{17}{9} = 24310$ , calculate  $\binom{19}{9}$ .

### Solution:

1. The problem is about combinations, which is a type of counting problem.
2. The problem is asking for the number of ways to choose 9 items from a set of 19, which is represented by  $\binom{19}{9}$ .
3. We are given the values of  $\binom{17}{7}$ ,  $\binom{17}{8}$ , and  $\binom{17}{9}$ , which can be used to calculate  $\binom{19}{9}$ .
4. The formula for combinations is  $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ , where  $n$  is the total number of items,  $r$  is the number of items to choose, and  $!$  denotes factorial.
5. However, we can use the property of combinations that  $\binom{n}{r} = \binom{n}{n-r}$ , and the relationship between  $\binom{n+1}{r}$  and  $\binom{n}{r-1}$  and  $\binom{n}{r}$ , which is  $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$ .
6. Using these properties, we can express  $\binom{19}{9}$  as  $\binom{18}{8} + \binom{18}{9}$ .
7. Similarly,  $\binom{18}{8}$  can be expressed as  $\binom{17}{7} + \binom{17}{8}$ , and  $\binom{18}{9}$  can be expressed as  $\binom{17}{8} + \binom{17}{9}$ .
8. Substituting the given values, we get  $\binom{19}{9} = (19448 + 24310) + (24310 + 24310)$ .
9. Performing the arithmetic calculation, we get  $\binom{19}{9} = 43758 + 48620 = 92378$ .

**Answer:** The answer is 92378.

Generalization ↑

Interpretability ↑

Controllability ↑

# Inference

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## Candidate Strategies


*Strategy 1:* 1. Identify the given information ... -> *Prompt 1*

*Strategy 2:* 1. Identify the relevant numbers ... -> *Prompt 2*

.....

*Strategy k:* 1. Understand the problem by ... -> *Prompt k*

*Validation Accuracy*





# Inference

## Candidate Strategies

*Strategy 1:* 1. Identify the given information ... -> *Prompt 1*

*Strategy 2:* 1. Identify the relevant numbers ... -> *Prompt 2*

.....

*Strategy k:* 1. Understand the problem by ... -> *Prompt k*

*Validation Accuracy*



*Applying the best strategy!*

→ *Suitable for simple or specific tasks*

# Inference

## Candidate Strategies

*Strategy 1:* 1. Identify the given information ... -> *Prompt 1*

*Strategy 2:* 1. Identify the relevant numbers ... -> *Prompt 2*

.....

*Strategy k:* 1. Understand the problem by ... -> *Prompt k*

*Validation Accuracy*



*Applying the top m strategies!*

*StrategyLLM-SC*

*Taking a majority vote on all answers!*

*StrategyLLM-ZS*

*Let LLM determine the final answer!*

→ *Suitable for complex or diverse tasks*

# Evaluation Tasks

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## Math Reasoning

Seven datasets of different subjects, namely, Algebra (AL), Prealgebra (PA), Intermediate Algebra (IA), Counting and Probability (CP), Number Theory (NT), Geometry (GE), and Precalculus (PC) from the MATH benchmark

## Commonsense Reasoning

StrategyQA and Date Understanding (DU) from Big-Bench Hard

## Algorithmic Reasoning

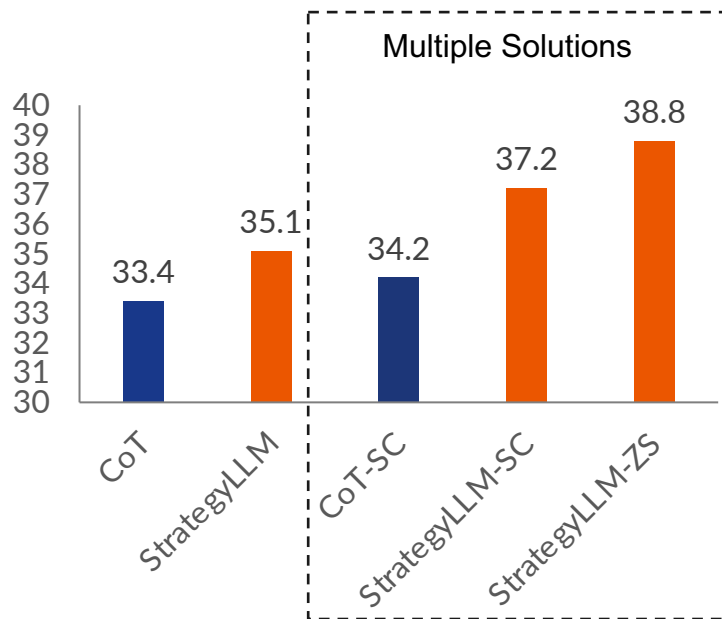
Word Sorting (WS) and Multi-step Arithmetic (MA) from Big-Bench Hard

## Symbolic Reasoning

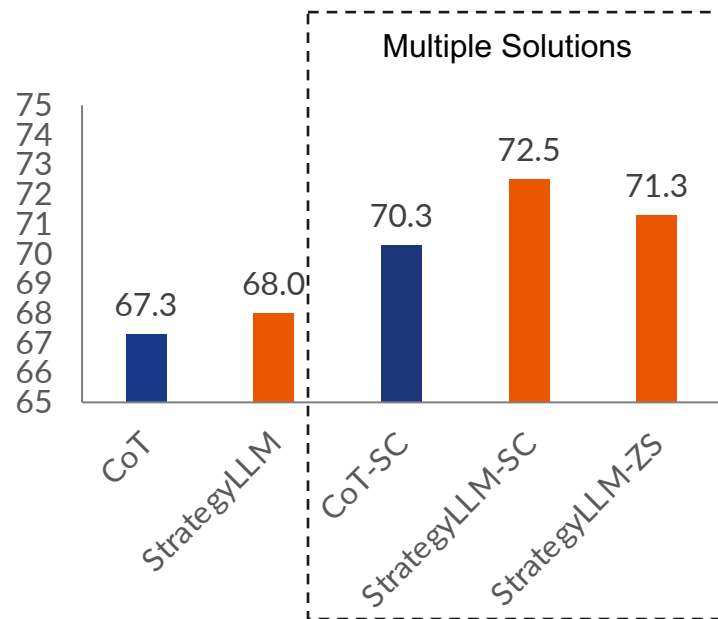
Last Letter Concatenation (LLC)  
**OOD setting**  
Prompt: 2 words  
Test: 4, 8, 16 words

# Results

*Adopting multiple strategies brings obvious benefits on complex or diverse tasks.*



Math Reasoning

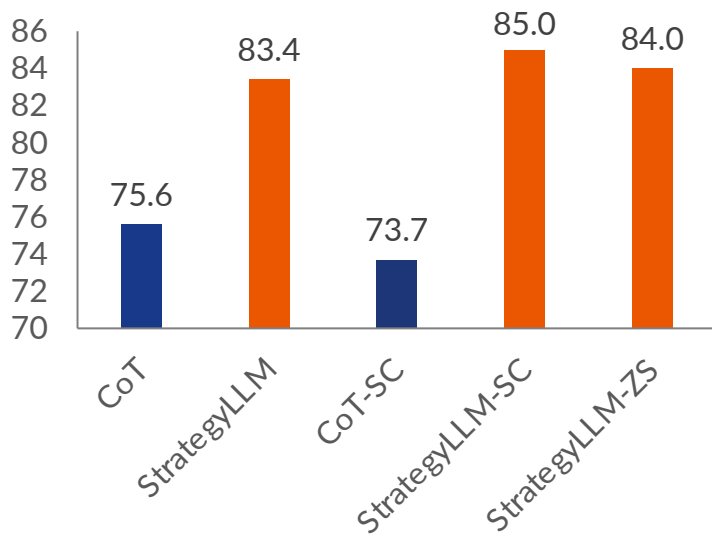


Commonsense Reasoning

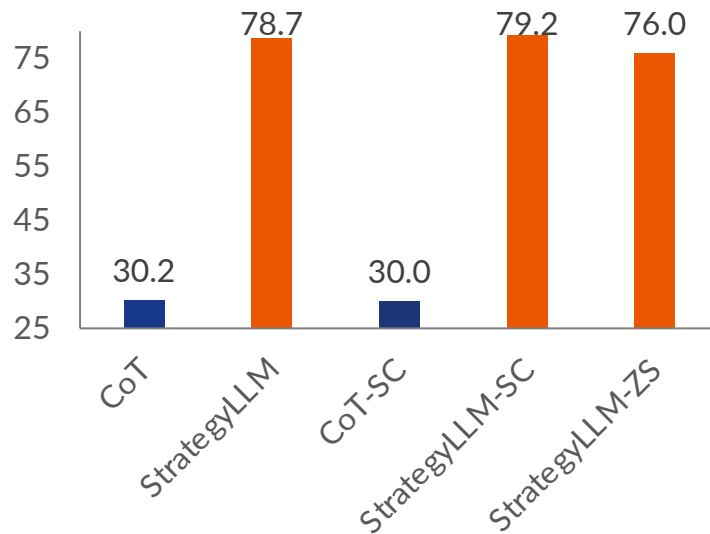
*StrategyLLM outperforms competitive baselines without human involvement!*

# Results

*Explicitly incorporating effective strategies significantly enhance the out-of-distribution (OOD) generalization abilities of LLMs.*

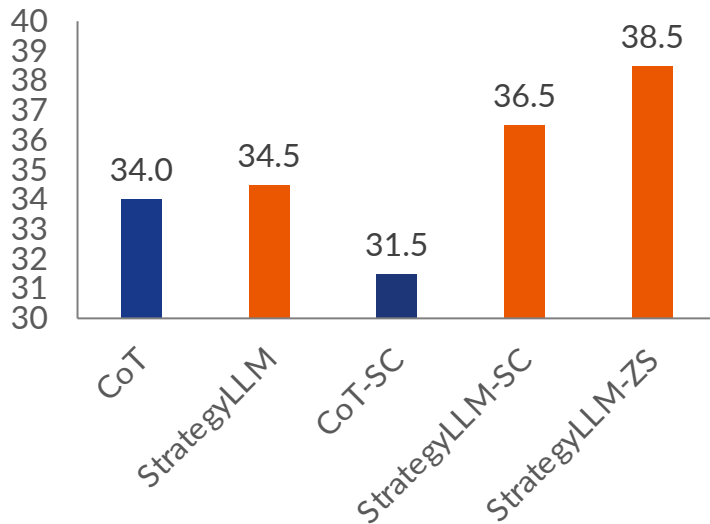


Algorithmic Reasoning

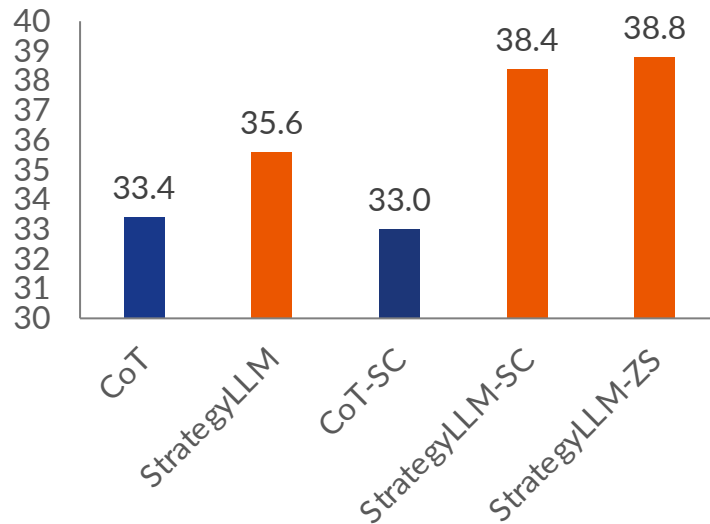


Symbolic Reasoning

# Robustness Results



Counting & Probability - Dev

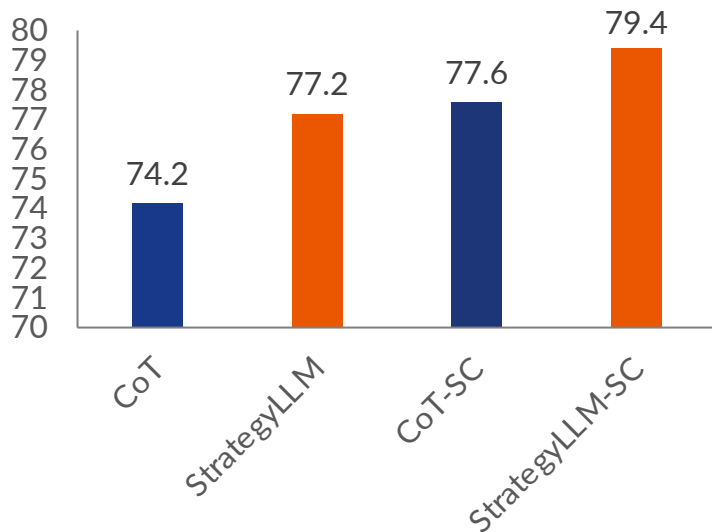


Counting & Probability – Random (5)

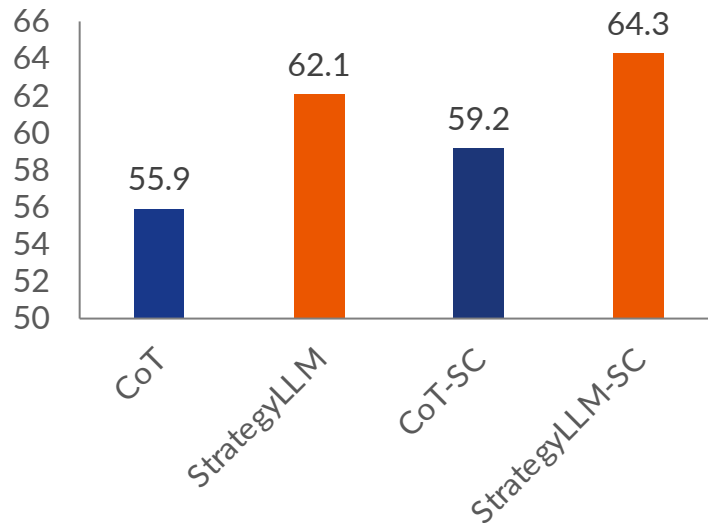
*StrategyLLM consistently delivers satisfactory performance across all groups of examples!*

# The Universality of StrategyLLM – Closed-Source LLMs

Average results on the CP, StrategyQA, and MA datasets, which represent three distinct reasoning tasks.



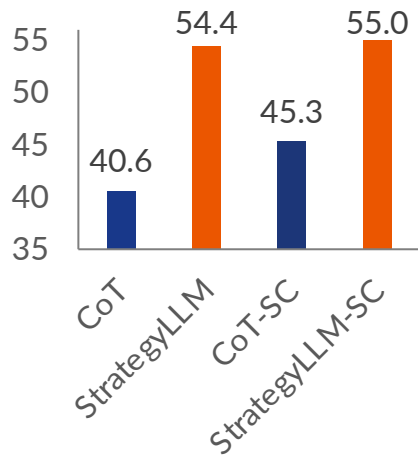
GPT-4



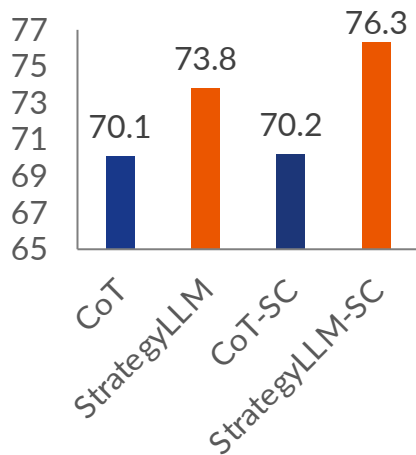
Claude-3-Sonnet

# The Universality of StrategyLLM – Open-Source LLMs

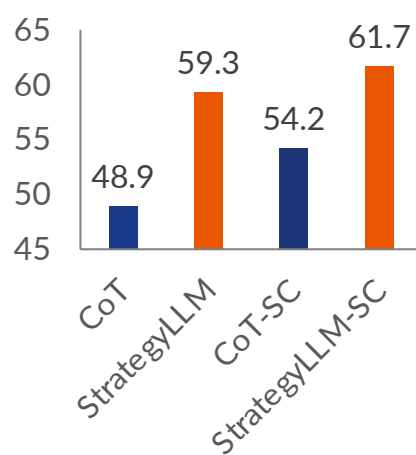
Average results on the CP, StrategyQA, and MA datasets, which represent three distinct reasoning tasks.



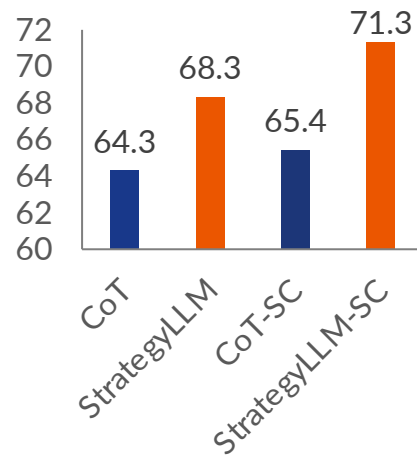
Llama-3-8B-Instruct



Llama-3-70B-Instruct



Mixtral-8x7B-Instruct

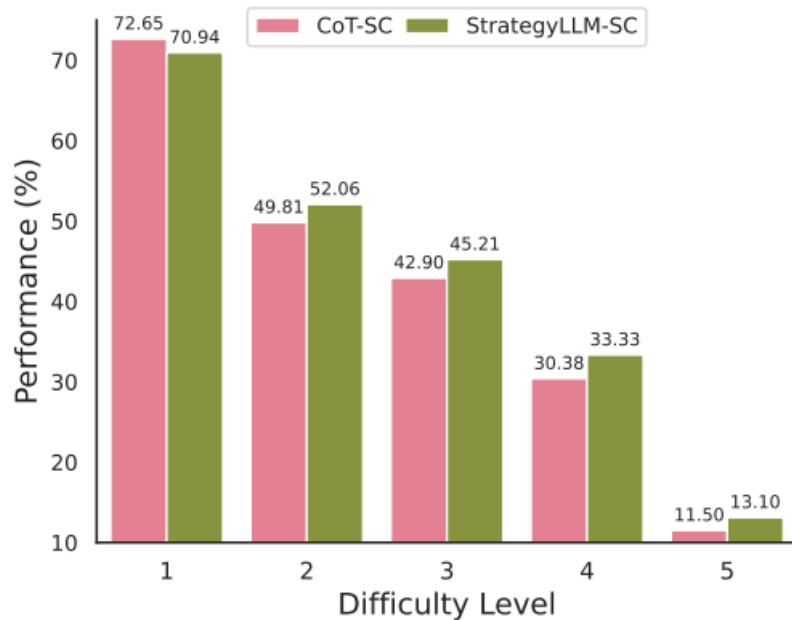


Mixtral-8x22B-Instruct



# Results across Different Difficulty Levels

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*Explicitly incorporating effective strategies significantly enhance complex reasoning!*

# Cost Analysis

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Methods	Math	Commonsense	Algorithmic	Symbolic
# Input Tokens	287.83K	228.67K	107.27K	70.94K
# Output Tokens	63.14K	33.15K	32.95K	28.48K
gpt-3.5-turbo-16k-0613	\$1.12	\$0.82	\$0.45	\$0.33
gpt-3.5-turbo-0125	\$0.24	\$0.16	\$0.10	\$0.08

*Our StrategyLLM framework is highly cost-efficient!*

# Insights

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*By effectively utilizing the extensive knowledge embedded in LLMs, StrategyLLM enables LLMs to address various tasks without human intervention!*

*Strategy is an excellent intermediate representation to improve reasoning paths in terms of generalization, robustness, and diversity!*

*LLMs can self-improve strategies through self-feedback!*