Alias-Free Mamba Neural Operator

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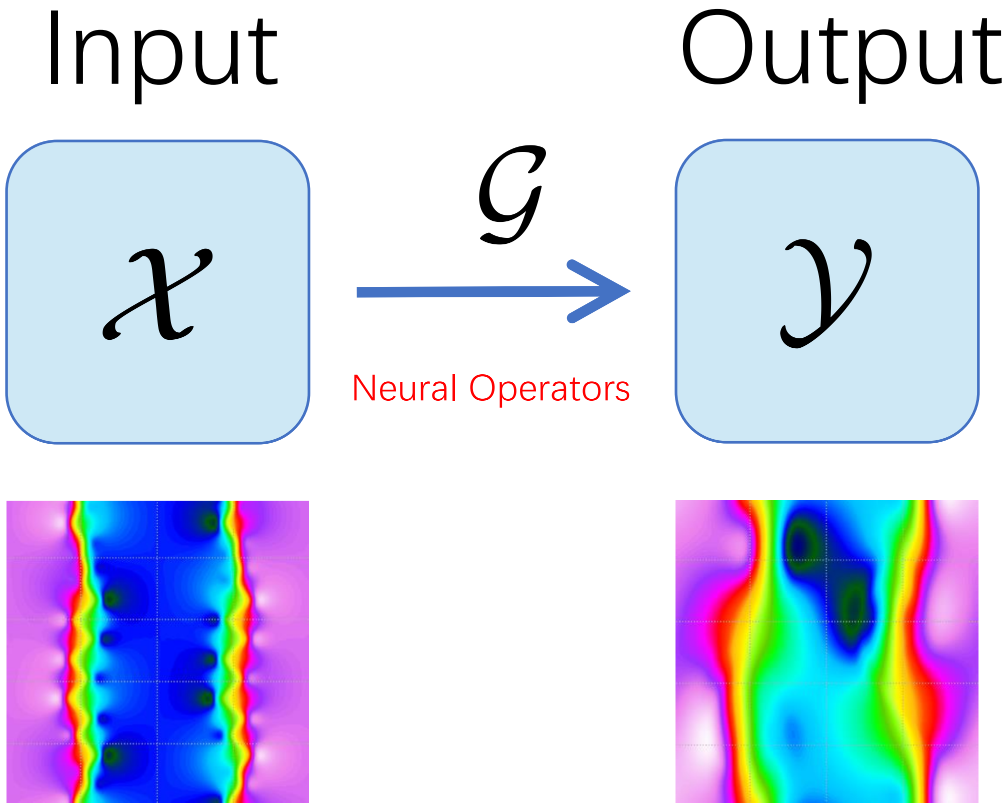
Partial Differential Equation

Numerous scientific and engineering problems entail recurrently resolving intricate Partial Differential Equation (PDE) for various parameter values, including **fluid flows**, **heat transfer analysis**, and **structural deformation studies**.



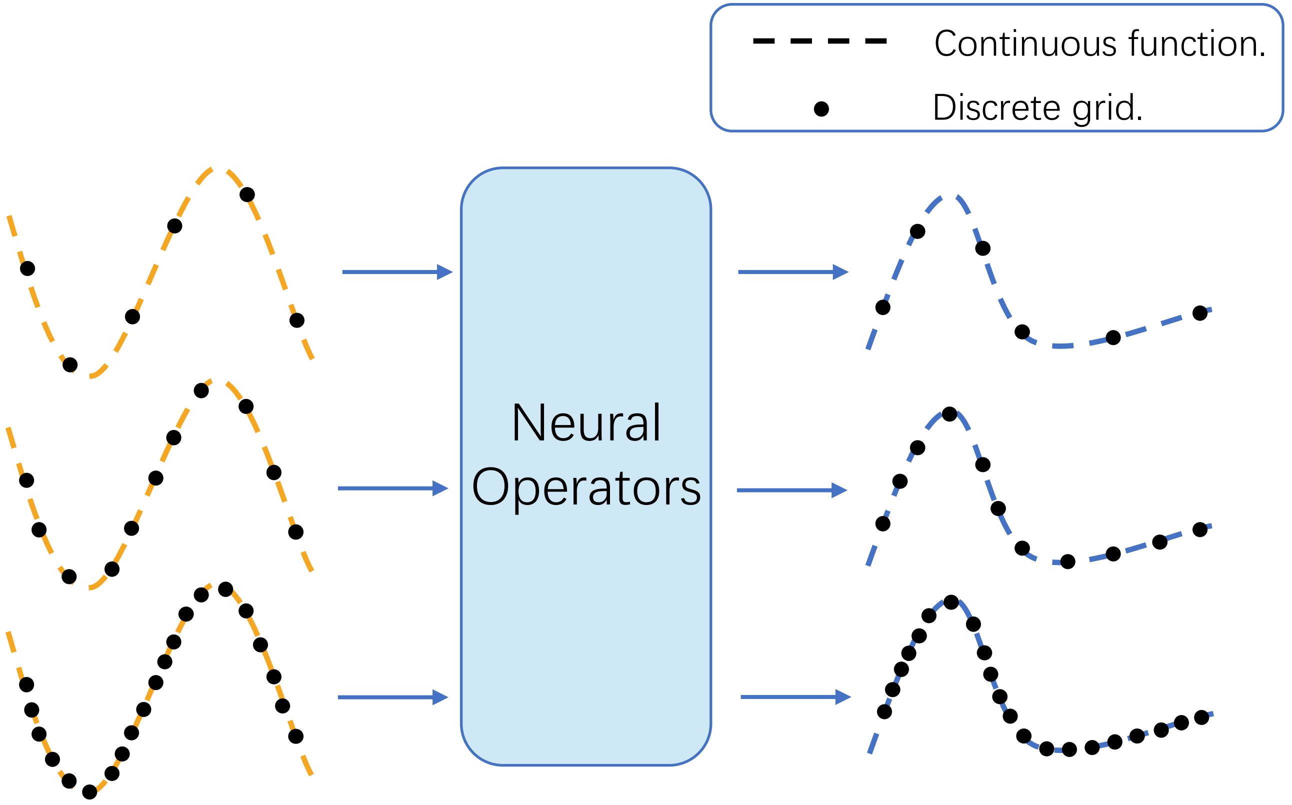
- Input: initial and boundary conditions, coefficients, source terms. etc.
- Output: solution, e.g. at some given time.

Advantages: Neural operators are faster, more accurate, and more flexible than traditional methods.

Neural Operators

Characteristic: Learning mappings between infinite-dimensional function spaces, ensuring discretization invariance.

Can adapt to different levels of discretization!



Different discrete representations correspond to the same underlying continuous function.

Related Work and Our Contributions

Existing neural operators are mainly categorized into FNO type based on frequency domain transformation, Transformer and CNN type based on spatial integration.

Contributions.

- A novel Mamba scanning integration is proposed as kernel integration.
- Propose MambaNO, combining global Mamba and local convolution integration.
- Prove that the proposed MambaNO is a representation-equivalent neural operator.
- Demonstrated MambaNO's outstanding performance in solving PDEs.

Methodologies

$$\int_D K_t(x, y, v(x), v(y))v(y)\mathrm{d}y$$

Kernel integration formula

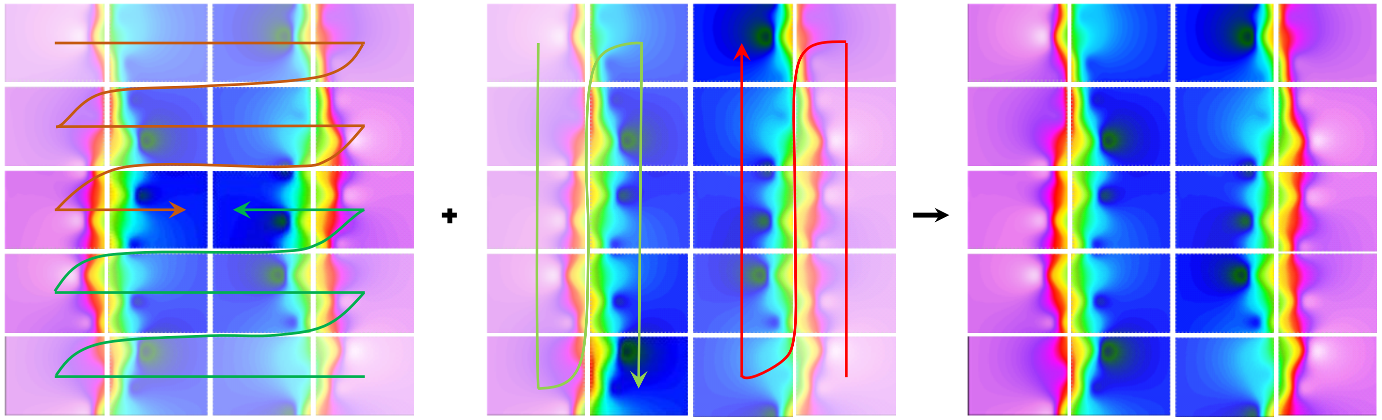
This Paper

$$h'(x) = Ah(x) + Bv(x),$$
$$u(x) = Ch(x).$$

State Space Model

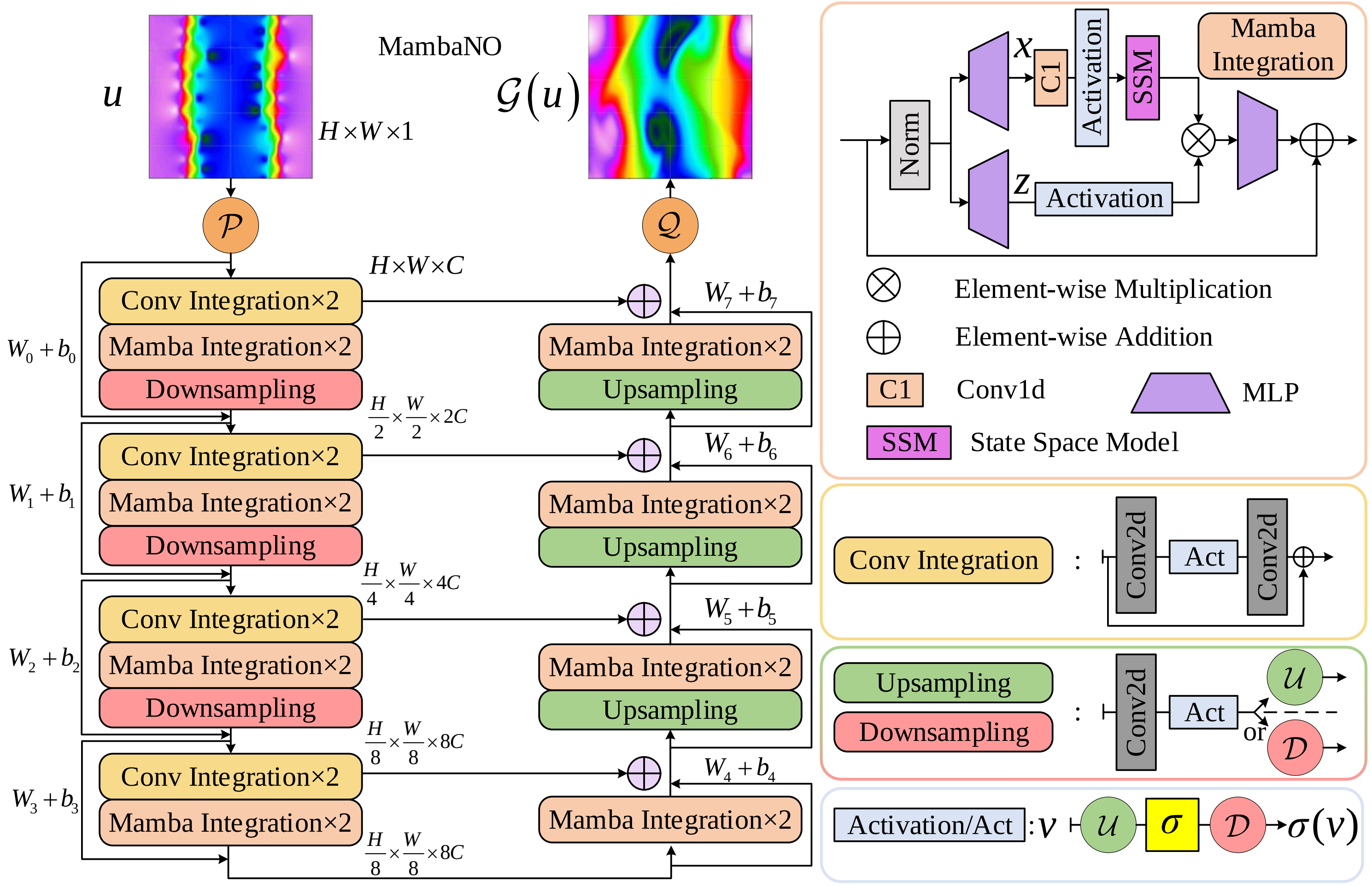
Our paper establishes a connection between the kernel integration formula and the state-space Model through rigorous mathematical derivation, enabling the global integration process to be implemented with linear complexity scanning!

Cross-Scan
$O(N)$ complexity



Cross-scanning operation in Mamba integration with a computational complexity of O(N).

MambaNO

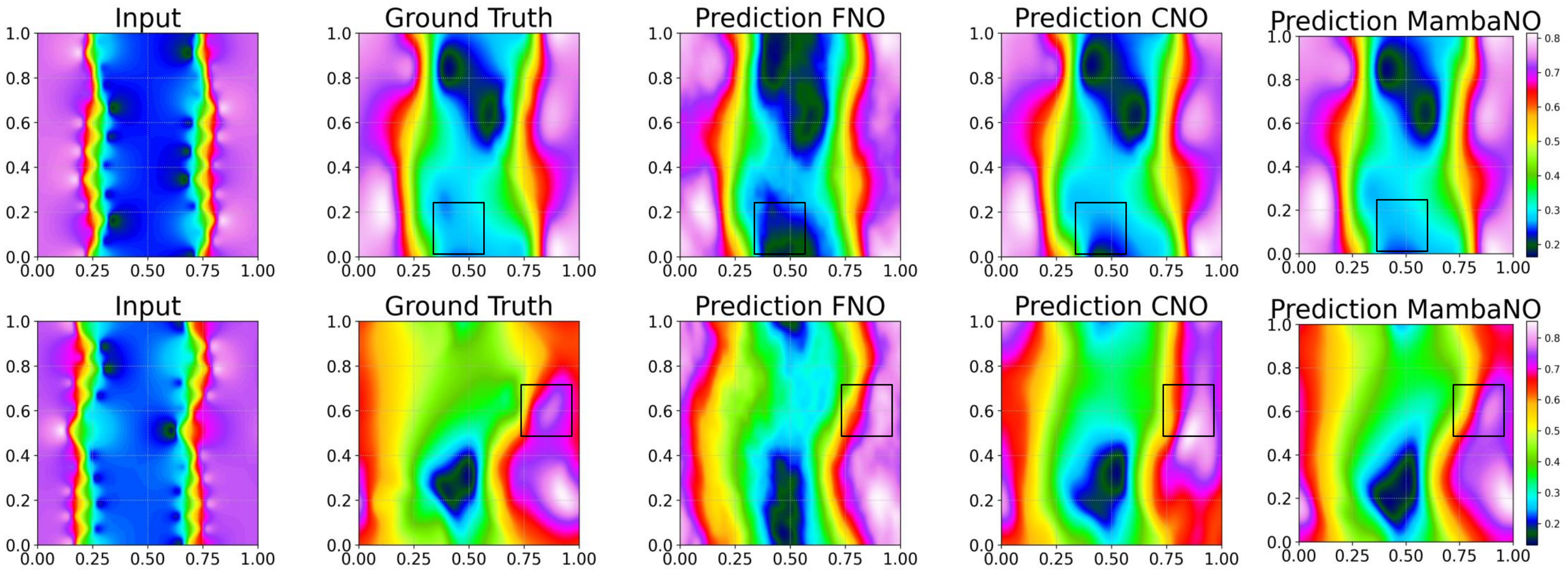


Mamba integration, conv integration, up/downsampling, and activation function layers are all constrained within a band-limited function space to prevent aliasing.

Experiments

- We ran fair grid search for each baseline and each benchmark.

Visualization results on the 2D Navier-Stokes equations.



Experiments

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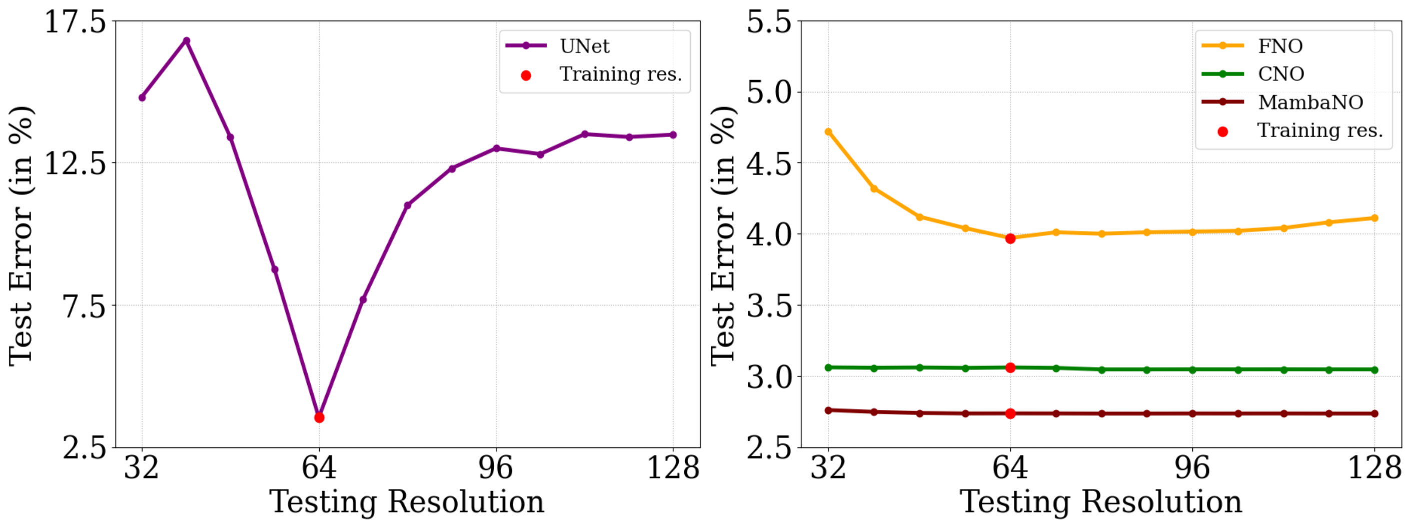
Table 1: Relative median L^1 test errors for various benchmarks and models.

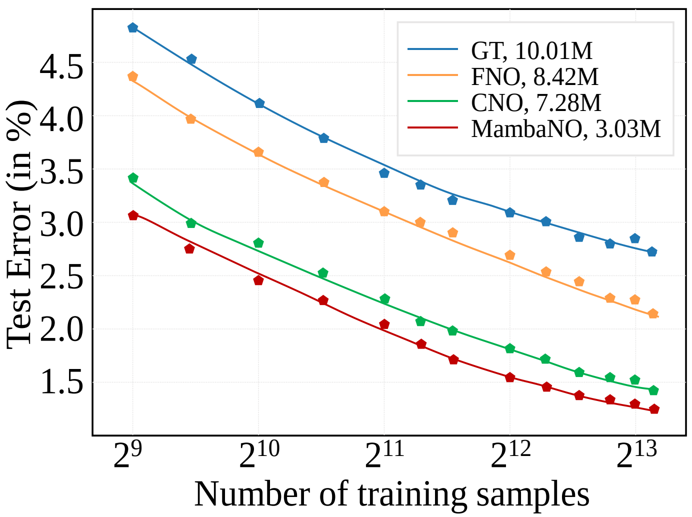
	In/Out	GT	Unet	ResNet	DON	FNO	CNO	MambaNO
Poisson Equation	In	4.09%	1.05%	0.63%	19.07%	7.35%	0.31%	0.17%
	Out	3.47%	1.55%	1.34%	11.18%	8.62%	0.33%	0.21%
Wave Equation	In	0.91%	0.96%	0.70%	1.43%	0.65%	0.40%	0.38%
	Out	1.97%	2.24%	2.50%	3.12%	1.95%	1.29%	1.22%
Smooth Transport	In	1.18%	0.59%	0.47%	1.38%	0.34%	0.29%	0.26%
	Out	666.07%	2.97%	2.73%	119.61%	1.97%	0.35%	0.34%
Discontinuous Transport	In	1.70%	1.44%	1.41%	6.35%	1.26%	1.11%	1.08%
	Out	27270.96%	1.62%	1.54%	140.73%	3.47%	1.31%	1.21%
Allen-Cahn Equation	In	1.30%	1.38%	2.36%	22.97%	0.87%	0.91%	0.72%
	Out	3.03%	3.28%	3.91%	20.75%	2.18%	2.33%	2.11%
Navier-Stokes Equation	In	4.61%	4.94%	4.10%	12.95%	3.97%	3.07%	2.74%
	Out	17.23%	16.98%	15.04%	23.39%	14.89%	10.94%	5.95%
Darcy Flow	In	0.86%	0.54%	0.42%	1.13%	0.80%	0.38%	0.33%
	Out	1.17%	0.64%	0.60%	1.61%	1.11%	0.50%	0.44%
Compressible Euler	In	2.33%	0.72%	1.89%	2.15%	0.49%	0.39%	0.34%
	Out	3.14%	0.91%	2.20%	3.08%	0.74%	0.63%	0.61%

Experiments

- Variation in test errors across various resolutions and scaling laws.



Varying the input resolution after training.



Test error vs. Number of training samples.

Thanks for listening