## **BMRS: Bayesian Model Reduction** for Structured Pruning

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**BACKGROUND:** We want to balance accuracy and compression in a principled way. We do so with variational inference and Bayesian Model Reduction (BMR). **BMRS** in a nutshell

 Apply multiplicative noise to network structures (Neklyudov et al. 2017)

$$h_{i} = \theta_{i} \cdot (\boldsymbol{w}_{i} \boldsymbol{h}_{i-1})$$
  

$$q_{\phi}(\theta_{i}) = \operatorname{LogN}_{[a,b]}(\theta_{i} | \mu_{i}, \sigma_{i}^{2})$$
  

$$p(\theta_{i}) = \operatorname{LogU}_{[a,b]}(\theta_{i})$$

 Use Bayesian Model Reduction (BMR) to find what to prune

$$\Delta F \triangleq \log \frac{\tilde{p}(D)}{p(D)} = \log \mathbb{E}_{\tilde{p}}\left[\frac{q_{\phi}(\theta)}{p(\theta)}\right]$$

## RESULTS

- BMRS finds near-optimal point w/ post-training pruning
- Good compression + maintains accuracy w/ continuous pruning



	MP	NIST	Fash-	MNIST	CIFAR10						
Pruning Method	Comp. (%)	Acc.	Comp. (%)	Acc.	Comp. (%)	Acc.					
	MLP										
None	$0.00 \pm 0.00$	$97.43 \pm 0.14$	$0.00\pm0.00$	$88.17 \pm 0.20$	$0.00\pm0.00$	$44.94\pm0.40$					
L2	$43.11 \pm 2.06$	$10.39\pm0.32$	$87.86 \pm 2.27$	$18.23 \pm 10.22$	$42.89 \pm 2.64$	$10.00\pm0.00$					
$E[\theta]$	$52.08 \pm 1.71$	$96.88 \pm 0.15$	$91.76 \pm 0.81$	$85.59 \pm 0.26$	$77.99 \pm 1.54$	$43.39\pm0.46$					
SNR	$58.57 \pm 2.01$	$96.92 \pm 0.08$	$99.83 \pm 0.00$	$10.00\pm0.00$	$75.93 \pm 1.26$	$43.97\pm0.46$					
$BMRS_N$	$48.86 \pm 1.32$	$96.95 \pm 0.19$	$93.20\pm0.66$	$84.99 \pm 0.35$	$76.36 \pm 1.08$	$43.59 \pm 0.29$					
BMRS <sub>U</sub> -8	$48.73 \pm 1.90$	$96.93 \pm 0.16$	$93.02\pm0.81$	$85.01 \pm 0.32$	$77.17 \pm 0.98$	$43.45\pm0.42$					
$BMRS_{\mathcal{U}}-4$	$54.47 \pm 1.74$	$96.99 \pm 0.13$	$91.57\pm0.71$	$85.79 \pm 0.34$	$76.63 \pm 0.94$	$44.06 \pm 0.40$					
	Lenet5										
None	$0.00 \pm 0.00$	$99.07 \pm 0.09$	$0.00\pm0.00$	$89.16 \pm 0.27$	$0.00\pm0.00$	$67.62 \pm 0.77$					
L2	$83.42 \pm 1.92$	$11.35\pm0.00$	$83.62 \pm 1.69$	$10.00\pm0.00$	$52.29 \pm 2.18$	$10.00\pm0.00$					
$E[\theta]$	$88.29 \pm 1.00$	$51.30 \pm 41.12$	$89.71 \pm 0.56$	$50.93 \pm 33.45$	$66.19 \pm 1.36$	$65.83 \pm 0.90$					
SNR	$92.66 \pm 5.77$	$62.70 \pm 41.93$	$98.47 \pm 3.45$	$17.01 \pm 21.03$	$70.29 \pm 2.02$	$67.68 \pm 0.52$					
$BMRS_N$	$86.90 \pm 1.15$	$95.59 \pm 0.94$	$88.02 \pm 1.00$	$77.90 \pm 2.44$	$62.87 \pm 1.64$	$66.14 \pm 0.70$					
$BMRS_{\mathcal{U}}-8$	$86.11 \pm 1.37$	$95.27 \pm 1.02$	$87.61 \pm 0.72$	$77.23 \pm 3.49$	$62.54 \pm 1.49$	$66.28 \pm 1.07$					
$BMRS_{U}-4$	$87.58 \pm 1.01$	$96.66 \pm 0.59$	$88.72\pm0.73$	$81.10 \pm 1.50$	$68.07 \pm 1.95$	$67.66 \pm 0.59$					



## **Bayesian structured pruning** without threshold tuning



Structures are pruned using an efficient calculation based on the statistics of the variational distribution

Also read our critical perspective of Al efficiency





Two variants

• BMRS  $_{\mathcal{N}} \Delta F$ :

$$\frac{1}{2}\log\frac{\tilde{\sigma}_q^2}{2\pi\tilde{\sigma}_p^2\sigma_q^2} + \log\frac{Z_{\tilde{q}}(\log b - \log a)}{Z_{\tilde{p}}Z_q}$$
$$-\frac{1}{2}(\frac{\mu_q^2}{\sigma_q^2} + \frac{\tilde{\mu}_p^2}{\tilde{\sigma}_p^2} - \frac{\tilde{\mu}_q^2}{\tilde{\sigma}_q^2})$$

• BMRS<sub>u</sub> 
$$\Delta F$$
:  

$$\frac{\log b' - \log a'}{\log b - \log a} \le q_{\phi}(a' \le \theta \le b')$$
• BMRS<sub>N</sub> is thresholdless  
• BMRS<sub>u</sub> can be tuned...  
CNN performance on CIFAR10  

$$\int_{0}^{0} \int_{0}^{0} \int_{0$$

...and has a connection to **floating** ulletpoint precision

Mantissas:  $m \sim (m \log B)^{-1}$ ,  $\frac{1}{B} \leq m \leq 1$ 

$$\tilde{p}(\theta): \begin{cases} \left(\theta \log \frac{2^{p_2}}{2^{p_1}}\right)^{-1}, \frac{1}{2^{p_2}} \le \theta \le \frac{1}{2^{p_1}}\\ 0, & \text{otherwise} \end{cases}$$

## Different pruning functions are learned

Spearman rank correlation of pruning methods

SNR	1	1	0.86	0.95	0.47	0.27					
Etheta	1	1	0.86	0.95	0.47	0.27					
L2	0.86	0.86	1	0.81	0.33	0.21					
BMRS_N	0.95	0.95	0.81	1	0.49	0.32					
BMRS_U-8	0.47	0.47	0.33	0.49	1	0.8					
BMRS_U-4	0.27	0.27	0.21	0.32	0.8	1					
	SNR	Etheta		BMRS_N	BMRS_U-8	BMRS_U-4					



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