P²C²Net: Pde-Preserved Coarse Correction Network for Efficient Prediction of Spatiotemporal Dynamics

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Ø Background&Motivation

l The limitations of Numerical methods

- Numerical method for solving PDEs, which typically relies on fine grids, is highly time-consuming and resource-intensive.

l Limited capacity of Machine Learning models

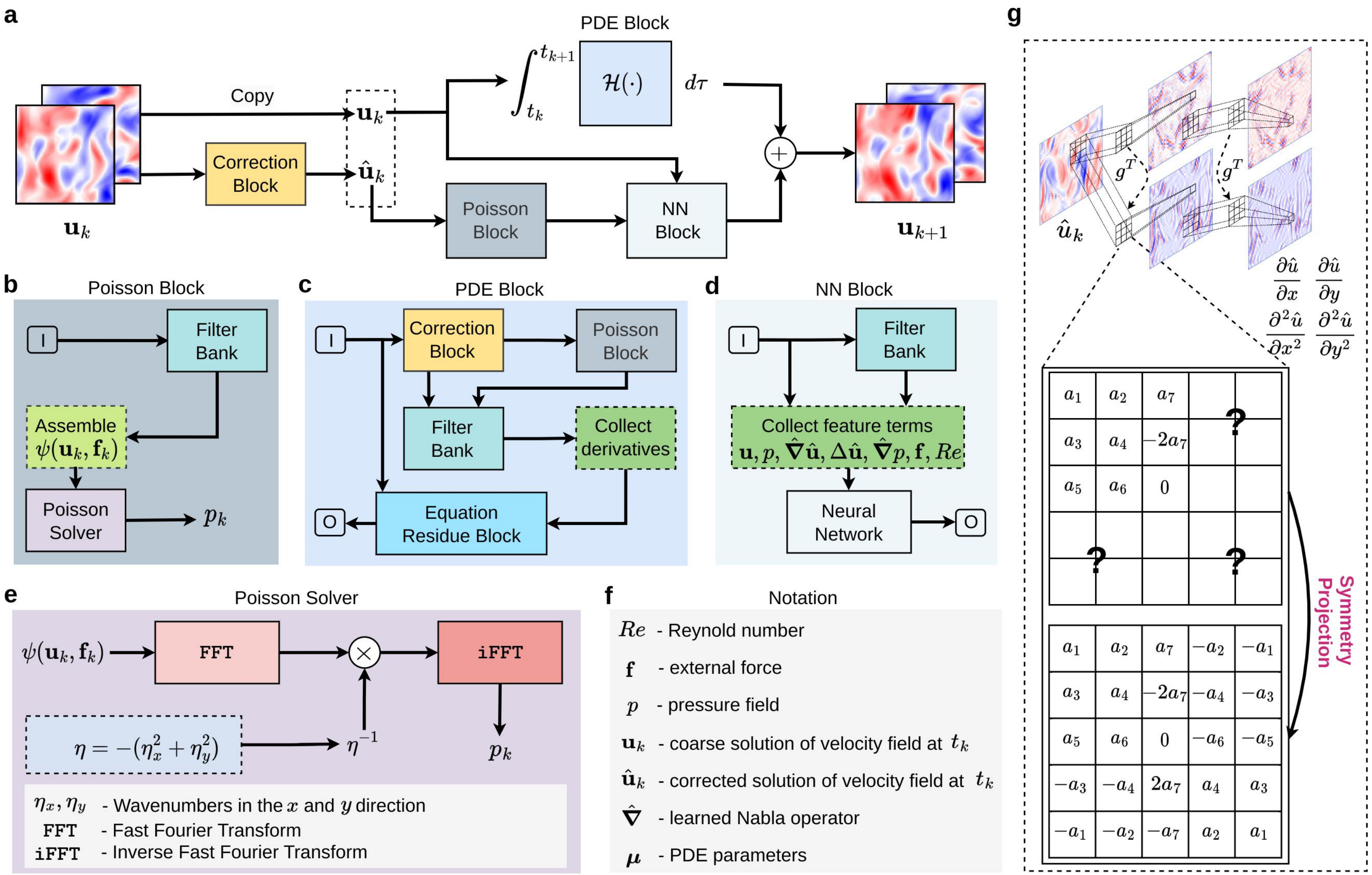
- Traditional machine learning methods for solving PDEs require large training data, produce models with limited generalization ability, and often lack interpretability.

l Major concerns of Encoding Physics into the Network

- Embedding physical knowledge enables the model to focus on fitting the underlying equation, accelerating convergence and reducing the dependency on large training datasets.

- This approach enhances the model's generalization capacity, allowing it to adapt to flow fields under various conditions.

Ø Method



Problem formulation :

$$\frac{\partial \mathbf{u}}{\partial t} - \mathcal{F}(\mathbf{u}, \mathbf{u}^2, \cdots, \nabla \mathbf{u}, \nabla^2 \mathbf{u}, \cdots; \boldsymbol{\mu}) = \mathbf{f}$$

RK4 integration scheme :

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \int_{t_k}^{t_{k+1}} \left[\mathcal{H}\left(\mathbf{u}(\tilde{\mathbf{x}}, \tau), \mathbf{u}^2(\tilde{\mathbf{x}}, \tau), \cdots, \nabla \mathbf{u}(\tilde{\mathbf{x}}, \tau), \nabla^2 \mathbf{u}(\tilde{\mathbf{x}}, \tau), \cdots; \boldsymbol{\mu}\right) + \mathbf{f}(\tau)\right] d\tau$$

Learnable PDE Block :

$$\mathcal{H}\left(\mathbf{u}_k, \mathbf{u}_k^2, \cdots, \nabla \mathbf{u}_k, \nabla^2 \mathbf{u}_k, \cdots; \boldsymbol{\mu}\right) \leftarrow \mathcal{F}\left(\mathbf{u}_k, \mathbf{u}_k^2, \cdots, \hat{\nabla} \hat{\mathbf{u}}_k, \hat{\nabla}^2 \hat{\mathbf{u}}_k, \cdots; \boldsymbol{\mu}\right)$$

Loss function:

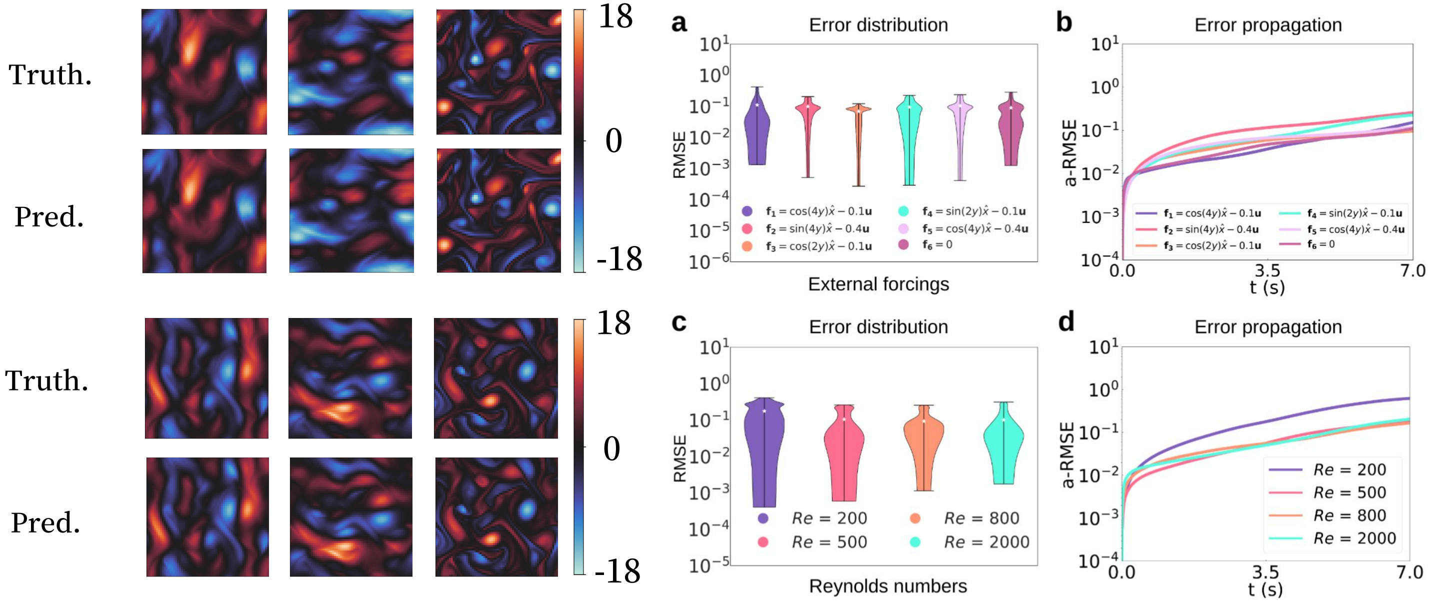
$$\mathcal{J}(\boldsymbol{\theta}) = \frac{1}{BN} \sum_{i=1}^{B} \sum_{j=1}^{N} MSE\left(\breve{\mathcal{S}}_{ij}, \mathcal{S}_{ij}\right)$$

Ø Experiments

l Quantitative results

Case	Model	RMSE	MAE	MNAD	HCT (s)
Burgers	FNO	0.0980	0.0762	0.062	0.3000
	UNet	0.3316	0.2942	0.2556	0.0990
	DeepONet	0.2522	0.2106	0.1692	0.0020
	PeRCNN	0.0967	0.1828	0.1875	0.4492
	P^2C^2Net (Ours)	**0.0064**	**0.0046**	**0.0037**	**1.4000**
	Promotion (↑)	93.4%	94.0%	94.0%	211.7%
GS	FNO	NaN	NaN	NaN	354
	UNet	NaN	NaN	NaN	4
	DeepONet	0.3921	0.2670	0.2670	852
	PeRCNN	0.1586	0.0977	0.0976	954
	P^2C^2Net (Ours)	**0.0135**	**0.0062**	**0.0062**	**2000.0**
	Promotion (↑)	91.5%	93.7%	93.6%	109.6%
FN	FNO	0.8935	0.5447	0.2593	3.5000
	UNet	0.1730	0.0988	0.0470	6.5000
	DeepONet	0.5474	0.3737	0.1779	0.5128
	PeRCNN	0.5703	0.2258	0.1075	5.3750
	P^2C^2Net (Ours)	**0.0390**	**0.0149**	**0.0071**	**10.000**
	Promotion (↑)	77.5%	84.9%	84.9%	53.8%
NS	FNO	1.0100	0.7319	0.0887	2.5749
	UNet	0.8224	0.5209	0.0627	3.9627
	LI	NaN	NaN	NaN	3.5000
	PeRCNN	1.2654	0.9787	0.1192	0.6030
	P^2C^2Net (Ours)	**0.3533**	**0.1993**	**0.0235**	**7.1969**
	Promotion (↑)	57.0%	61.7%	62.5%	81.6%

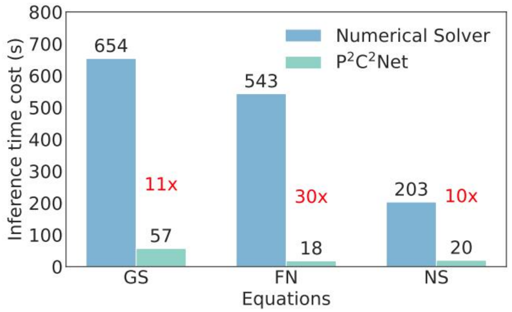
l Generalization test



u Our model is trained with $\mathbf{f} = \sin(4y)\mathbf{n}_x - 0.1u$ and $Re = 1000$, where $\mathbf{n}_x = [1, 0]^T$

u P^2C^2Net is able to generalize to different external forces f and Reynolds numbers *Re*.

l Computational time for comparison



u remarkable speedup

l Impact of noise on P^2C^2Net performance

Training	RMSE	MAE	MNAD	HCT (s)
+ 1% noise	0.0092	0.0088	0.0062	1.4
+ 0.5% noise	0.0078	0.0057	0.0047	1.4
w/o Noise noise	0.0064	0.0046	0.0037	1.4

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Thank You!

For further details, feel free to get in touch with us.

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