Soft Tensor Product Representations for Fully Continuous, Compositional Visual Representations

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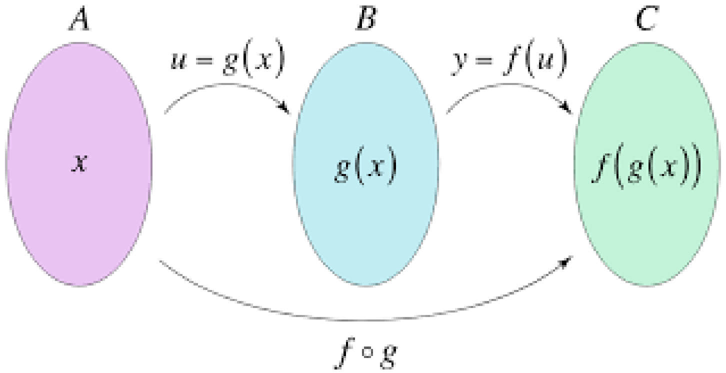
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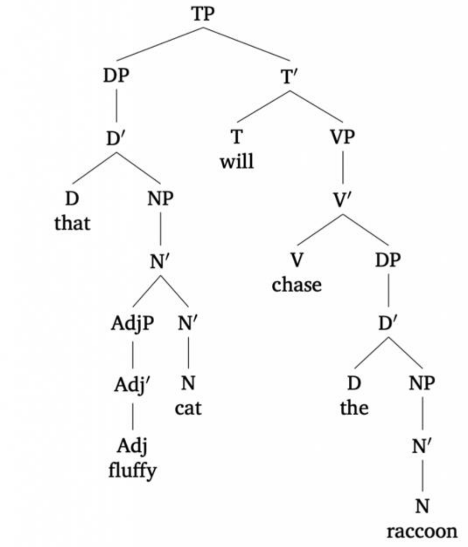
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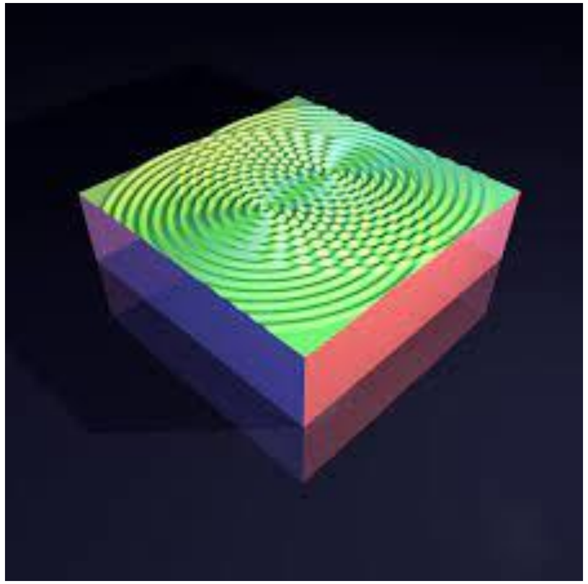
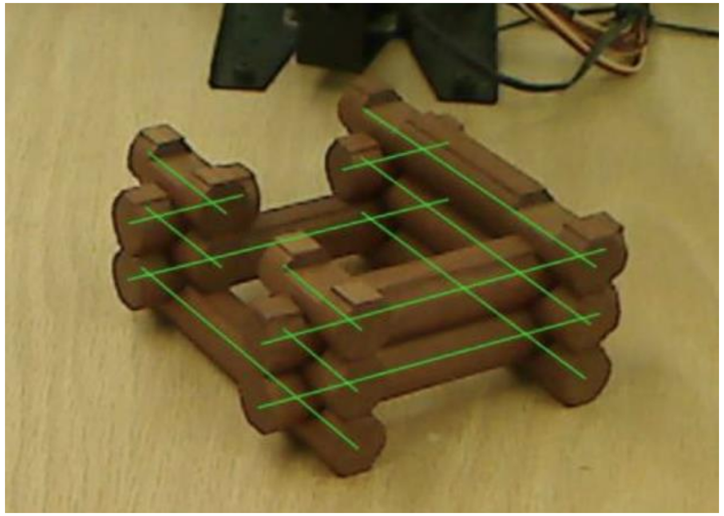
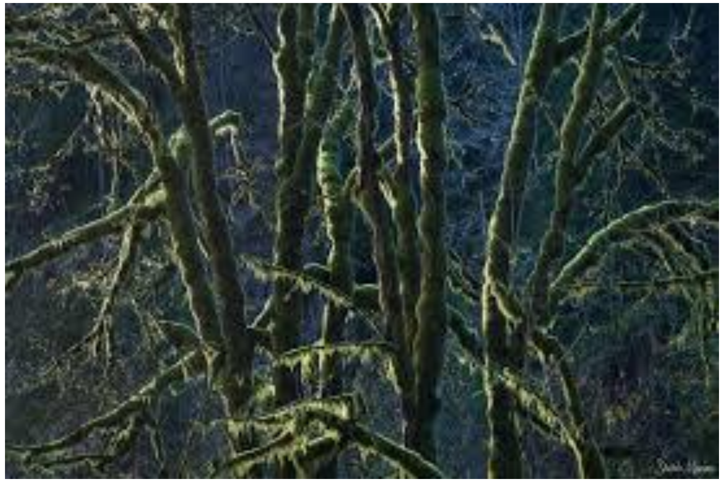
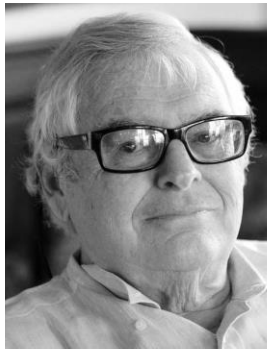




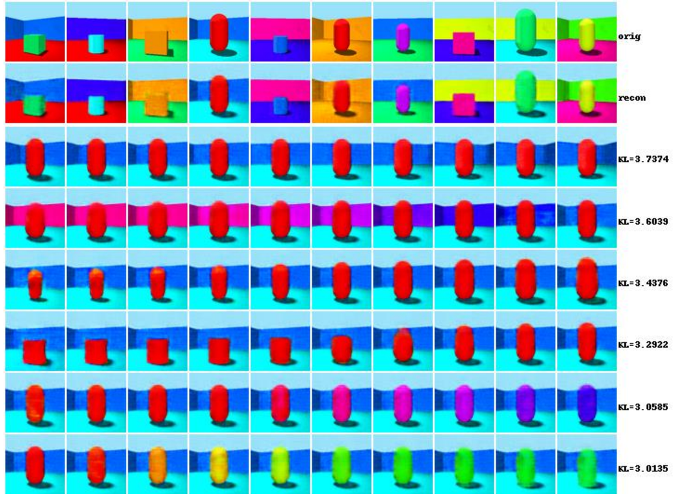


Motivation

- **Theoretical**: long philosophical tradition (Fodor, Chomsky) of inductively arguing from key properties of human cognition that cognition itself *must* be underpinned by a compositional system [1, 2].

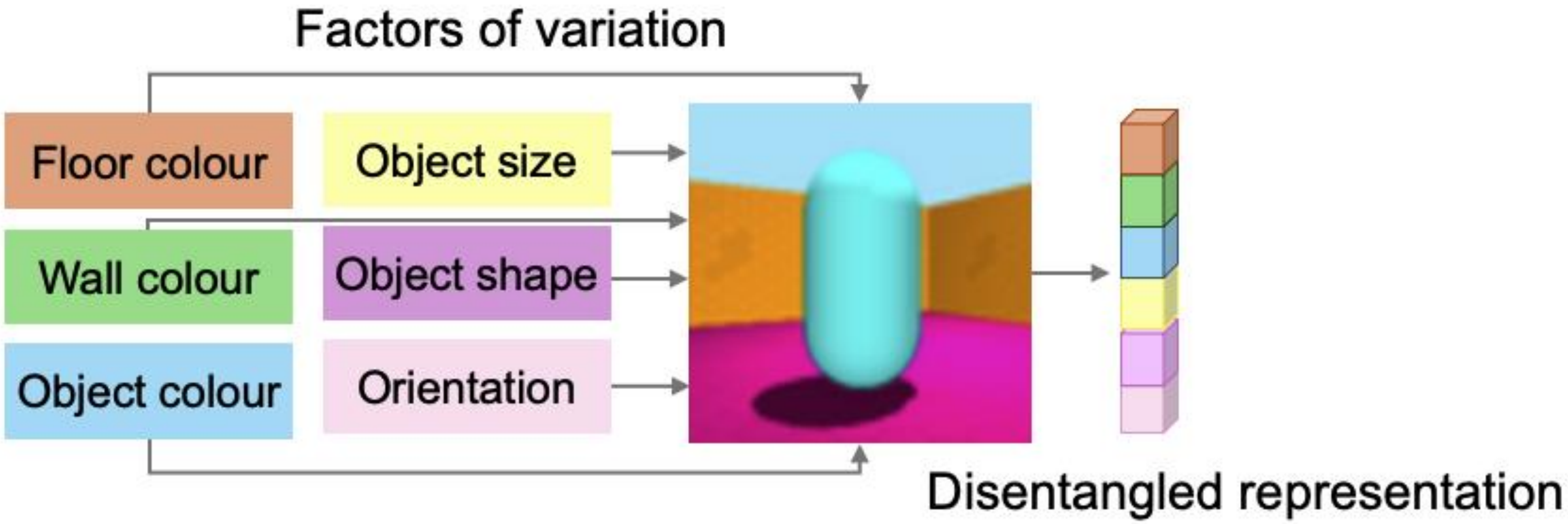


- **Empirical**: compositional representations enhance interpretability [4, 5], sample efficiency [6, 7], fairness [8, 9, 10], robustness to OOD settings [7, 11, 12].



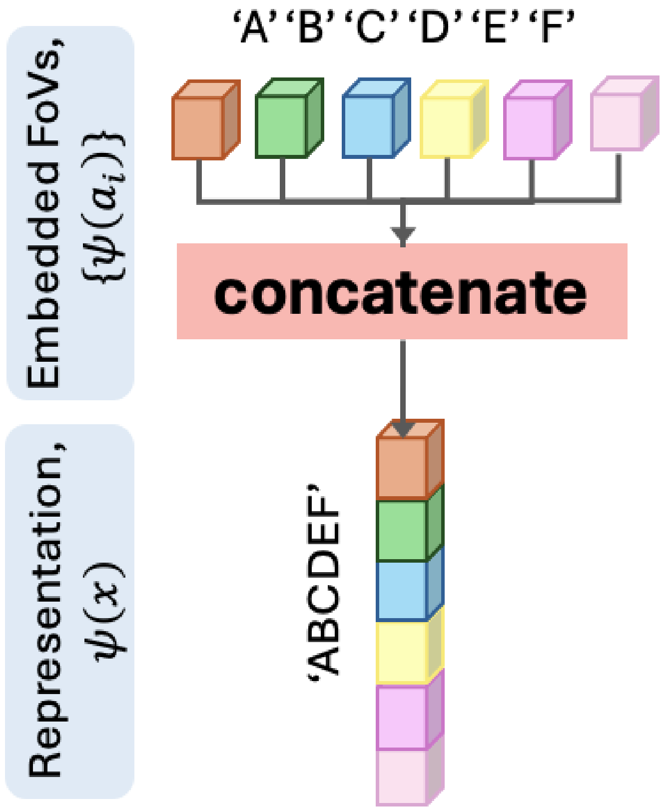
Existing Work

- **Disentanglement** is a key approach for compositional representation learning.
- Aims to isolate underlying factors of variation (FoVs) into distinct parts of the representation.
  - i.e., FoVs should be 1-1 mapped to representational parts – the Jacobian requirement of [13].



Disentanglement and Symbolic Compositionality

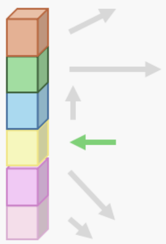
- Disentanglement enforces a fundamentally **symbolic** treatment of compositional structure.
  - This is because disentanglement essentially allocates FoVs to distinct representational **slots**.
  - The overall representation is thus analogous to a **string** formed by the **concatenation** of FoV slots (***tokens***).



Our Key Hypothesis

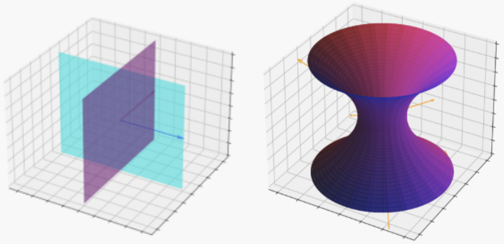
- Symbolic compositional representations are **fundamentally incompatible** with the **continuous** vector spaces of deep learning:

Gradient Fragmentation



- Updating a *single* FoV (shown in yellow) restricts gradient propagation to dimensions associated with that slot (i.e., no gradient flows across other dimensions, indicated by gray arrows).
- This inhibits the *smooth flow* of gradient across the **entire** vector space, i.e., $\mathbb{R}^6$.
- Transitions between such updates can be *abrupt* and *discontinuous*, potentially **complicating learning**.

Incompatible Representational Structure

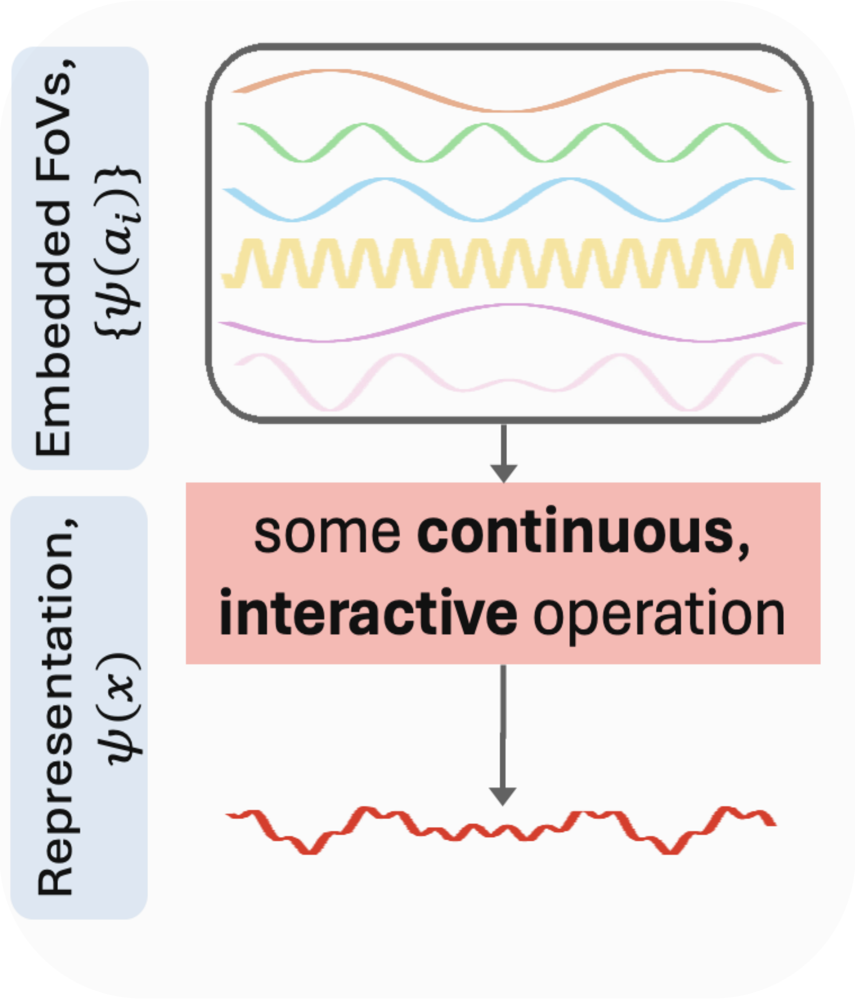


- Consider a disentangled representation in $\mathbb{R}^3$, with 3 1D FoVs.
- By encoding FoVs in separate, independent slots, this approach prevents FoVs from being encoded as *flexible combinations* of basis vectors spanning the **entire space** (i.e., $\mathbb{R}^3$), limiting representational expressivity.
- We illustrate this concept using the hyperboloid (right), combining all 3 basis vectors in $\mathbb{R}^3$, versus a pair of orthogonal planes (left).

- The symbolic/continuous mismatch manifests in broadly **suboptimal** deep learning **model behaviour.**

A New Way of Treating Compositional Structure?

- Can we align compositional structure with continuous vector spaces, by formulating a fundamentally **continuous** compositional representation?
  - Such an approach smoothly blends FoVs into the representation – like the continuous *superimposition* of multiple waves into an aggregate wave (in red on the RHS)



Soft TPR Framework

- To do this, we propose a **new** compositional representation learning framework, the *Soft Tensor Product Representation (TPR) framework*, which comprises:

1. *Soft TPR*: a new, inherently continuous compositional **representational form**.
2. *Soft TPR Autoencoder*: a theoretically-principled **method** for learning Soft TPRs.

Soft TPR

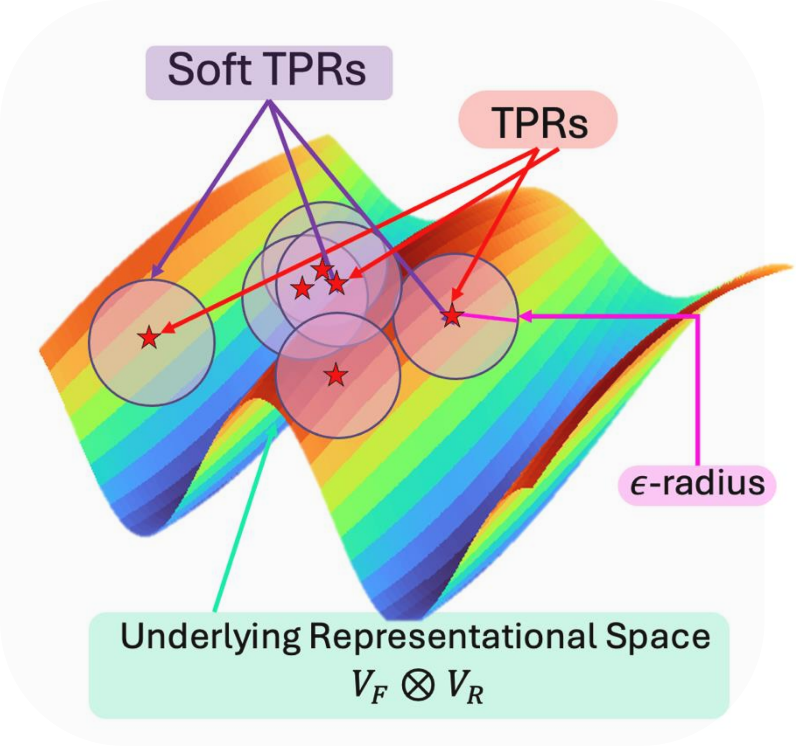
- Our Soft TPR form is a new mathematical specification that represents compositional structure **continuously**:

Soft TPR Form:

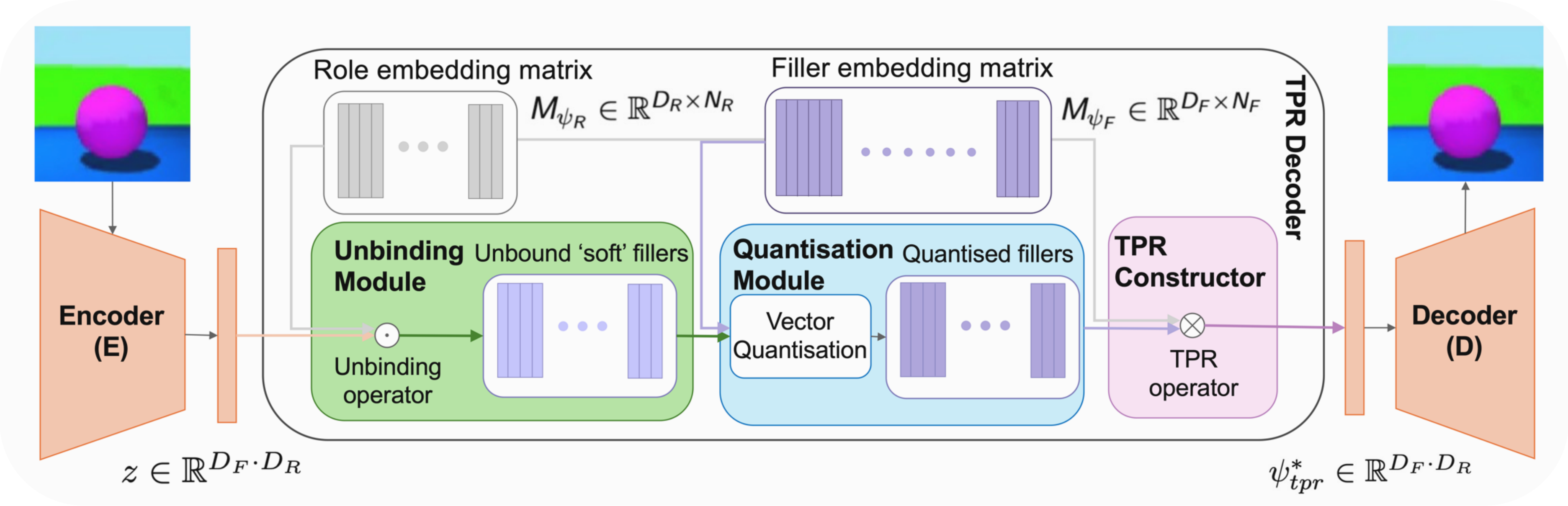
$$z \in \{x \in V_F \otimes V_R \mid \|x - \psi_{tpr}\|_F \leq \epsilon\}$$

where $\|A\|_F$ denotes Frobenius norm of A,
ϵ some small, +ve scalar-valued constant,
ψ_{tpr} a (traditional) TPR produced by TPR function mapping from data to TPRs in $V_F \otimes V_R$

- It extends upon Smolensky's Tensor Product Representation [3].
- Soft TPR preserves the traditional TPR's useful mathematical & structural properties (see paper for proofs & further details).
- Soft TPRs have the added benefits of being **easier** to learn and more representationally **flexible** than TPRs.
  - ➞ This allows Soft TPRs to be applied in **broader settings** compared to traditional TPRs [14, 15, 16, 17, 18, 19, 20] e.g., the non-formal domain of *vision* with a more realistic *weak supervision* requirement.



Soft TPR Autoencoder



- A novel framework introduced to learn Soft TPRs. 3 main components (please see paper for more details):
- **Encoder**: Produces a **candidate** Soft TPR, z.
- **TPR Decoder**: Leverages the mathematical properties of the Soft TPR/TPR framework to encourage z to have the correct **mathematical form** of a Soft TPR (unsupervised loss).
- **Weak Supervision**: Apply a weakly supervised loss inspired by prior disentanglement work [21, 22, 23, 24, 25] to encourage z to contain the correct **semantic content**.

Results

- Our results empirically suggest that the **enhanced** vector space alignment produced by Soft TPRs is **broadly** beneficial for deep learning models (both representation learners & downstream models).
  - Please see the Appendix in our paper for an extensive suite of experimental results.

Result #1: Structural

- Structurally, Soft TPRs are **more explicitly compositional** than baselines (as quantified by disentanglement metrics).
  - SoTA disentanglement (DCI boosts of **29%+**, **74%+** on Cars3D/MPI3D).

Table 1: FactorVAE and DCI scores. Additional results in Section C.3.3

Models	Cars3D		Shapes3D		MPI3D	
	FactorVAE score	DCI score	FactorVAE score	DCI score	FactorVAE score	DCI score
Symbolic scalar-tokened compositional representations						
Slow-VAE	0.902 ± 0.035	0.509 ± 0.027	0.950 ± 0.032	0.850 ± 0.047	0.455 ± 0.083	0.355 ± 0.027
Ada-GVAE-k	0.947 ± 0.064	*0.664 ± 0.167*	*0.973 ± 0.006*	$\mathbf{0.963 \pm 0.077}$	0.496 ± 0.095	0.343 ± 0.040
GVAE	0.877 ± 0.081	0.262 ± 0.095	0.921 ± 0.075	0.842 ± 0.040	0.378 ± 0.024	0.245 ± 0.074
ML-VAE	0.870 ± 0.052	0.216 ± 0.063	0.835 ± 0.111	0.739 ± 0.115	0.390 ± 0.026	0.251 ± 0.029
Shu	0.573 ± 0.062	0.032 ± 0.014	0.265 ± 0.043	0.017 ± 0.006	0.287 ± 0.034	0.033 ± 0.008
Symbolic vector-tokened compositional representations						
VCT	*0.966 ± 0.029*	0.382 ± 0.080	0.957 ± 0.043	0.884 ± 0.013	*0.689 ± 0.035*	*0.475 ± 0.005*
COMET	0.339 ± 0.008	0.024 ± 0.026	0.168 ± 0.005	0.002 ± 0.000	0.145 ± 0.024	0.005 ± 0.001
Fully continuous compositional representations						
Ours	$\mathbf{0.999 \pm 0.001}$	$\mathbf{0.863 \pm 0.027}$	$\mathbf{0.984 \pm 0.012}$	*0.926 ± 0.028*	$\mathbf{0.949 \pm 0.032}$	$\mathbf{0.828 \pm 0.015}$

Result #2: Representation Learner Convergence

- Soft TPRs have **faster** representation learner **convergence**.
- Representations **useful** for downstream tasks can be consistently learned with substantially **fewer** representation learner training iterations.
  - We consider the 2 standard downstream tasks used in disentanglement: FoV regression and abstract visual reasoning.
  - Note that at 100 iterations of representation learner training, Soft TPRs (in blue) achieve performance (Fig 20 & Fig 22) that is only achieved with **2 orders' magnitude** more training iterations by the most competitive baseline.

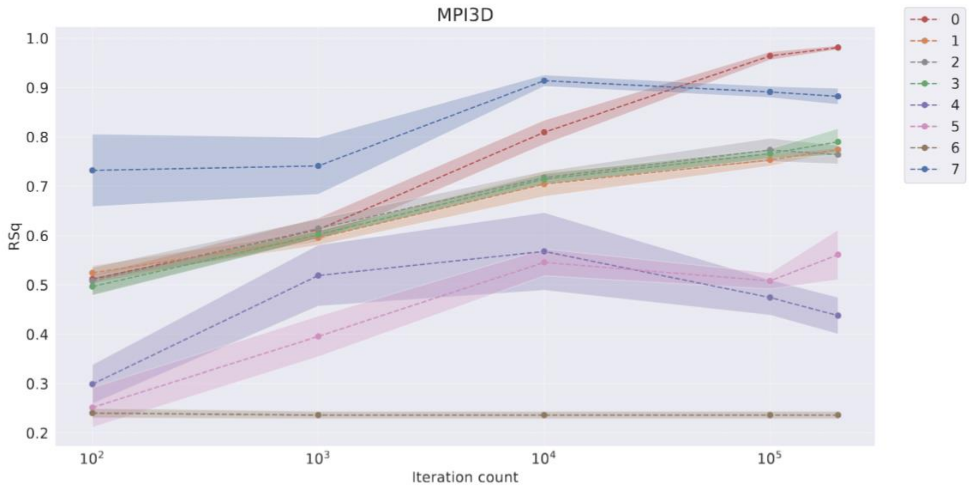


Figure 20: Convergence of representation learners as measured by FoV regression on the MPI3D dataset (dimensionality-controlled setting)

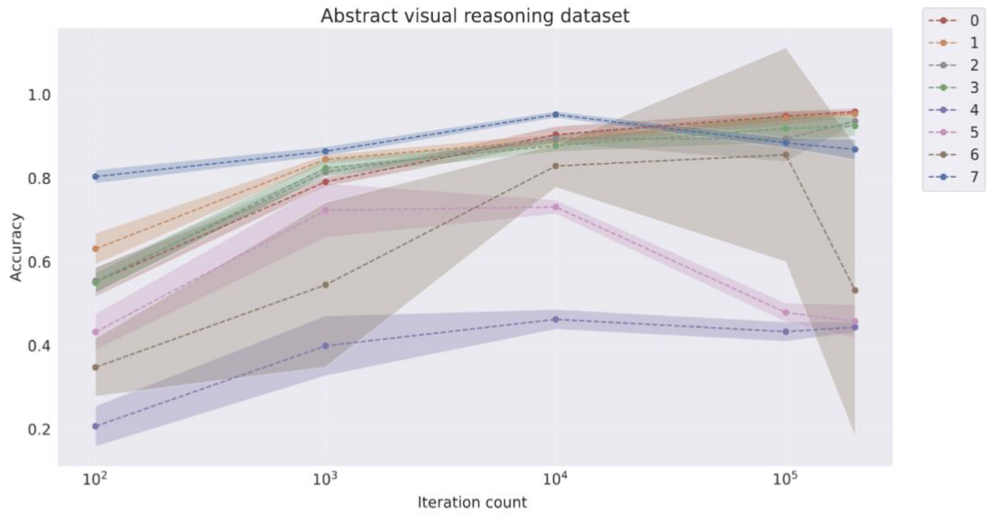


Figure 22: Convergence of representation learners as measured by classification performance on the abstract visual reasoning dataset (dimensionality-controlled setting)

Result #3: Downstream Performance

- Soft TPRs have **substantially superior** downstream **sample efficiency** (e.g., **93%+**) and **low-sample** regime performance (e.g., **138%+, 168%+**).
  - Again, we consider the 2 standard downstream tasks of FoV regression and abstract visual reasoning, a subset of results below:

Table 4: Downstream FoV R^2 scores (odd columns) and sample efficiencies (even columns) on the MPI3D dataset.

Models	100 samples	100 samples/all	250 samples	250 samples/all
	Symbolic scalar-tokened compositional representations			
Slow-VAE	0.127 ± 0.050	0.130 ± 0.051	0.152 ± 0.011	0.155 ± 0.011
Ada-GVAE-k	*0.206 ± 0.031*	0.270 ± 0.037	0.213 ± 0.023	0.279 ± 0.026
GVAE	0.181 ± 0.030	0.234 ± 0.035	0.217 ± 0.023	0.282 ± 0.027
ML-VAE	0.182 ± 0.013	0.236 ± 0.019	*0.222 ± 0.024*	0.288 ± 0.030
Shu	0.151 ± 0.016	*0.343 ± 0.024*	0.211 ± 0.026	*0.482 ± 0.075*
	Symbolic vector-tokened compositional representations			
VCT	0.086 ± 0.051	0.189 ± 0.107	0.119 ± 0.070	0.246 ± 0.137
COMET	-0.051 ± 0.015	0.000 ± 0.000	-0.042 ± 0.018	0.000 ± 0.000
	Fully continuous compositional representations			
Ours	**0.490 ± 0.068**	**0.556 ± 0.078**	**0.594 ± 0.056**	**0.665 ± 0.067**

Table 5: Abstract visual reasoning accuracy in the low-sample regime of 500 samples.

Models	Symbolic scalar-tokened
Slow-VAE	0.196 ± 0.028
Ada-GVAE-k	0.203 ± 0.007
GVAE	0.182 ± 0.013
ML-VAE	0.193 ± 0.012
Shu	0.200 ± 0.010
	Symbolic vector-tokened
VCT	*0.277 ± 0.039*
COMET	0.259 ± 0.016
	Fully continuous
Ours	**0.360 ± 0.033**

Thank you 😊

- In summary:
  1. We propose a **new framework** for learning fully continuous compositional representations (Soft TPR + Soft TPR Autoencoder)
  2. Our approach is the **first** to learn fully continuous compositional representations in the **non-formal** domain of vision
  3. Extensive empirical results highlight the far-reaching benefits of our representation's **enhanced vector space alignment**, for representational structure, representation learners, and downstream models, underscoring the necessity of reconceptualising compositional representations in a fully continuous manner.
- Please see our full paper for more details on our approach, including proofs, conceptual motivation, theory, and suggestions for future work.
- Code is available!
- Questions? Thoughts? Contact bethia.sun@unsw.edu.au

Paper

Code

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