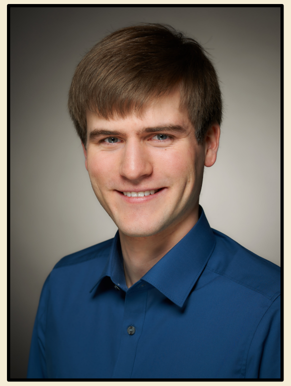
Neural Persistence Dynamics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University of Applied Sciences, Salzburg, Austria

Neural Persistence Dynamics @rkwitt1982 Sebastian Zeng

Motivation

- Observation of a coherently moving flock of birds, understood as an evolving 3D point cloud $\mathcal{P} = \{\mathbf{x}^k\}_{k=1}^K$:

*

* proprietary data (2024).

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- The `D'Orsogna` model [D'Orsogna et al. '06] describes the dynamics of individual entities $\mathbf{x}^k$

$$m\ddot{\mathbf{x}}^k = \left(\alpha - \beta \|\dot{\mathbf{x}}^k\|^2\right) \dot{\mathbf{x}}^k - \frac{1}{K} \nabla_{\mathbf{x}^k} \sum_{l \neq k} \underbrace{U\left(\|\mathbf{x}^k - \mathbf{x}^l\|, C_r, l_r\right)}_{\text{Attraction \& Repulsion}}.$$

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- Solving the **inverse problem**, i.e., predicting $\boldsymbol{\beta} = (m, \alpha, C_r, l_r)$, is inherently difficult due to:
  - the large number of observed entities, and
  - the difficulty of identifying individual motion trajectories $\mathbf{x}^k(t)$.

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> - To predict the **models parameters** β, understanding the evolving **behavioral patterns** of a collective is key.
> - Hence, we learn the **dynamics in the topology** of time evolving **point clouds**.

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  - the large number of observed entities, and
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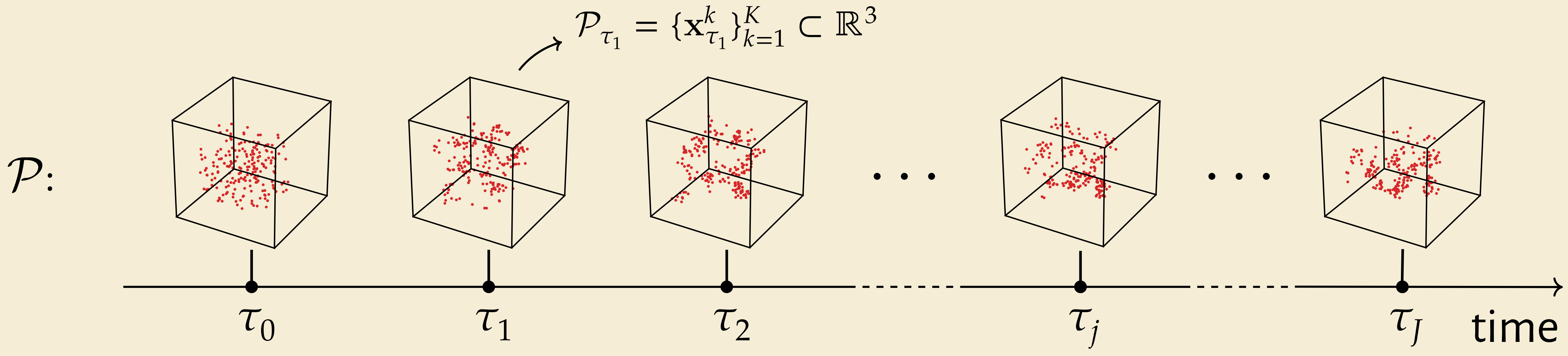
Problem setting

- For learning, we consider datasets of N time-evolving 3D point clouds $\mathcal{P}^1, \ldots, \mathcal{P}^N$; e.g.,



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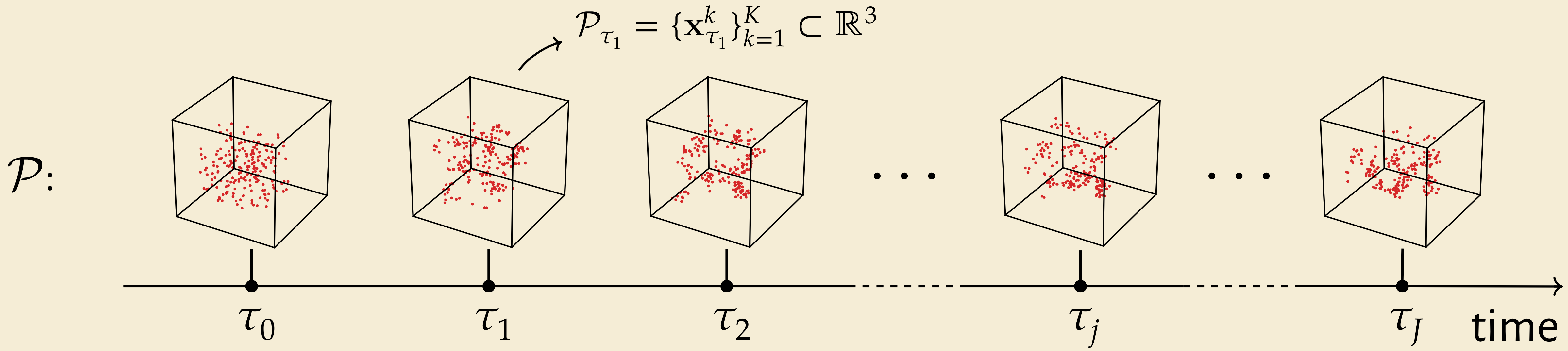

- We assume…
  (1) individual trajectories of points $\mathbf{x}^k$ are governed by a coupled equation of motion

$$\ddot{\mathbf{x}}^k = f_{\boldsymbol{\beta}}\left(\{\mathbf{x}^k\}_{l=1}^K, \dot{\mathbf{x}}^k\right)$$

  (2) $\boldsymbol{\beta}$ control such motions and specify (local) interactions among neighboring points, and

  (3) the dynamics in the topology of the point clouds are determined by a simpler latent process $\mathbf{Z}$.

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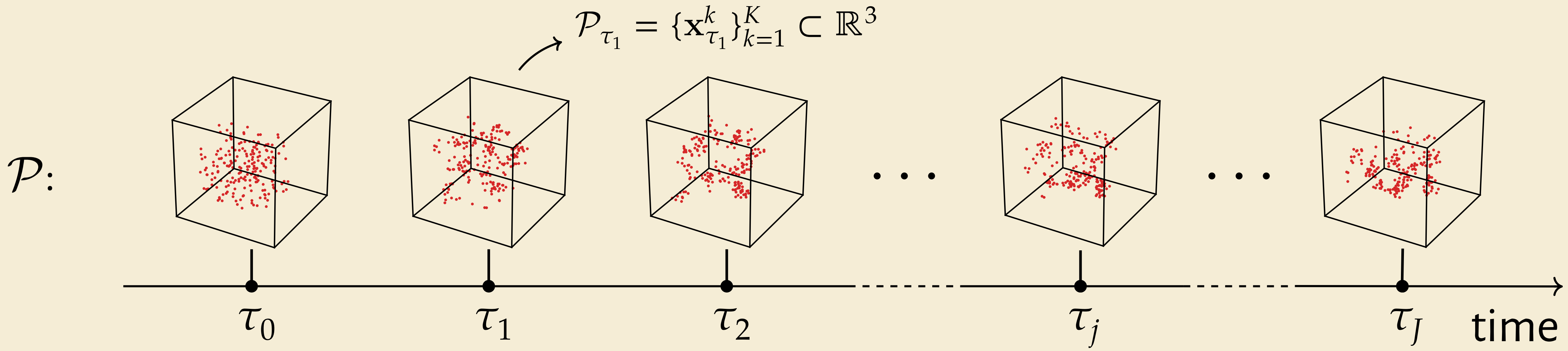

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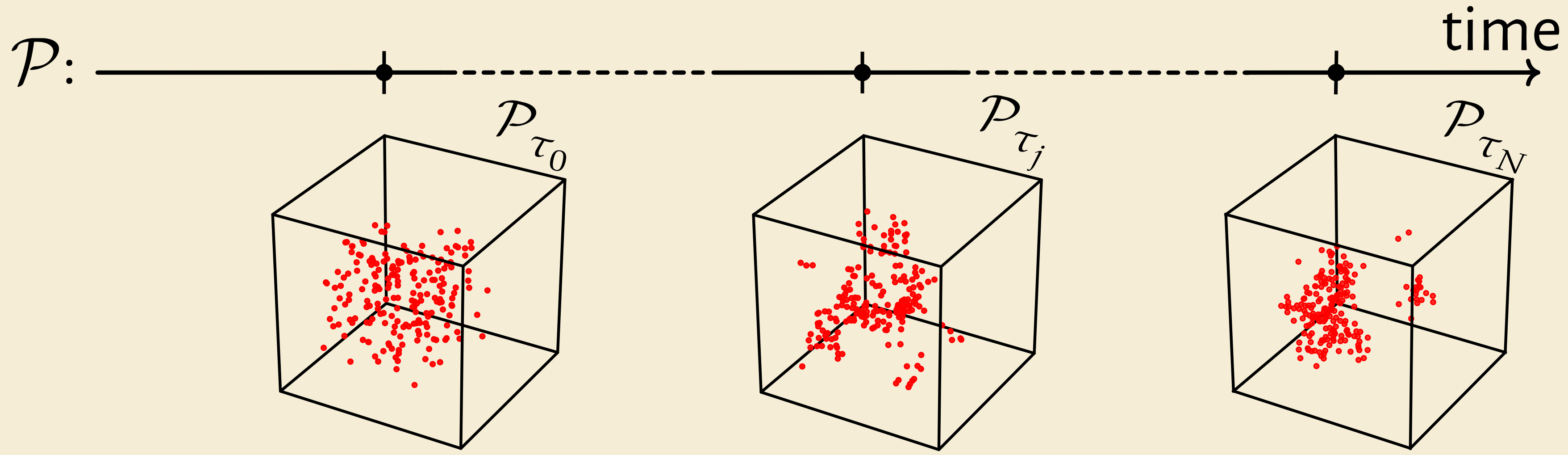
We seek to learn $\mathbf{Z}$ and thus predict $\boldsymbol{\beta}$!

Neural persistence dynamics

- For each sequence $\mathcal{P}$, we pre-compute topological features **per time point**, by...



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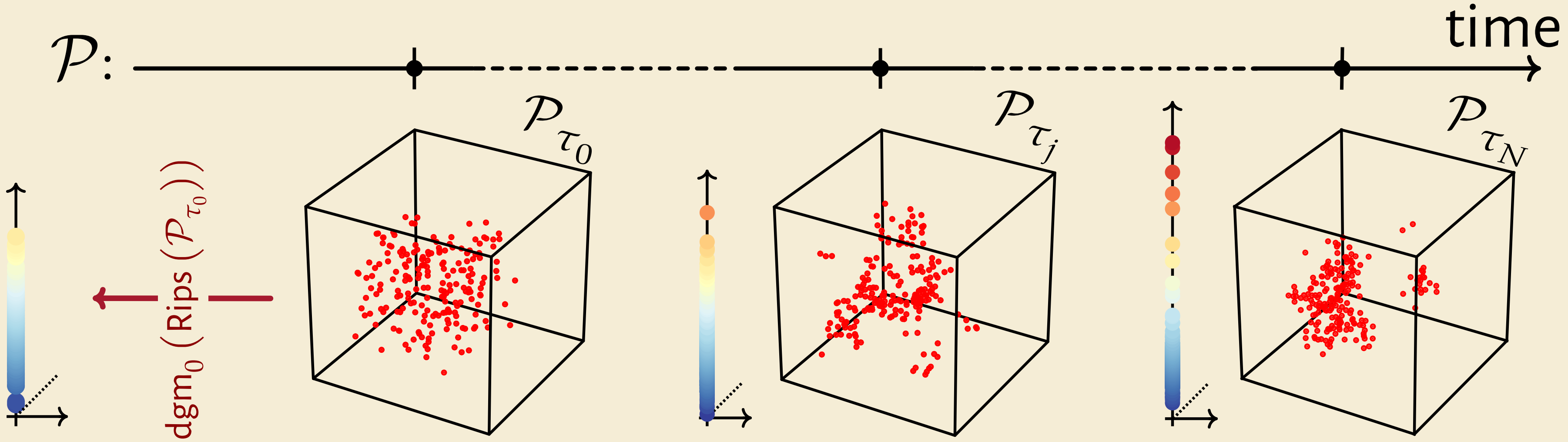

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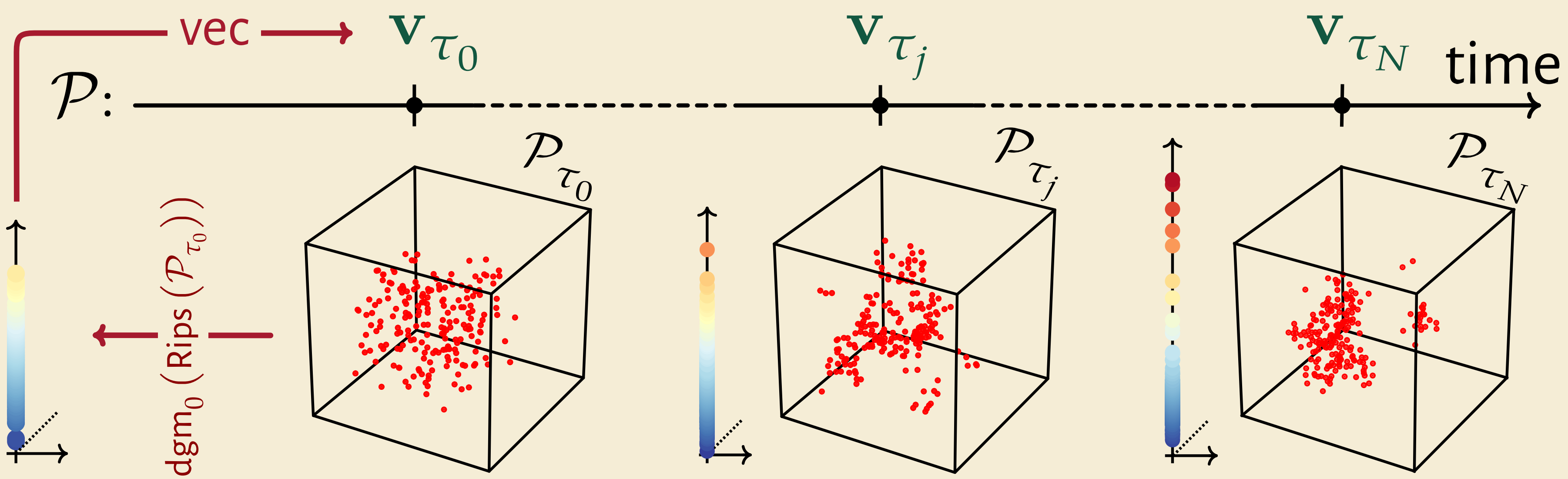

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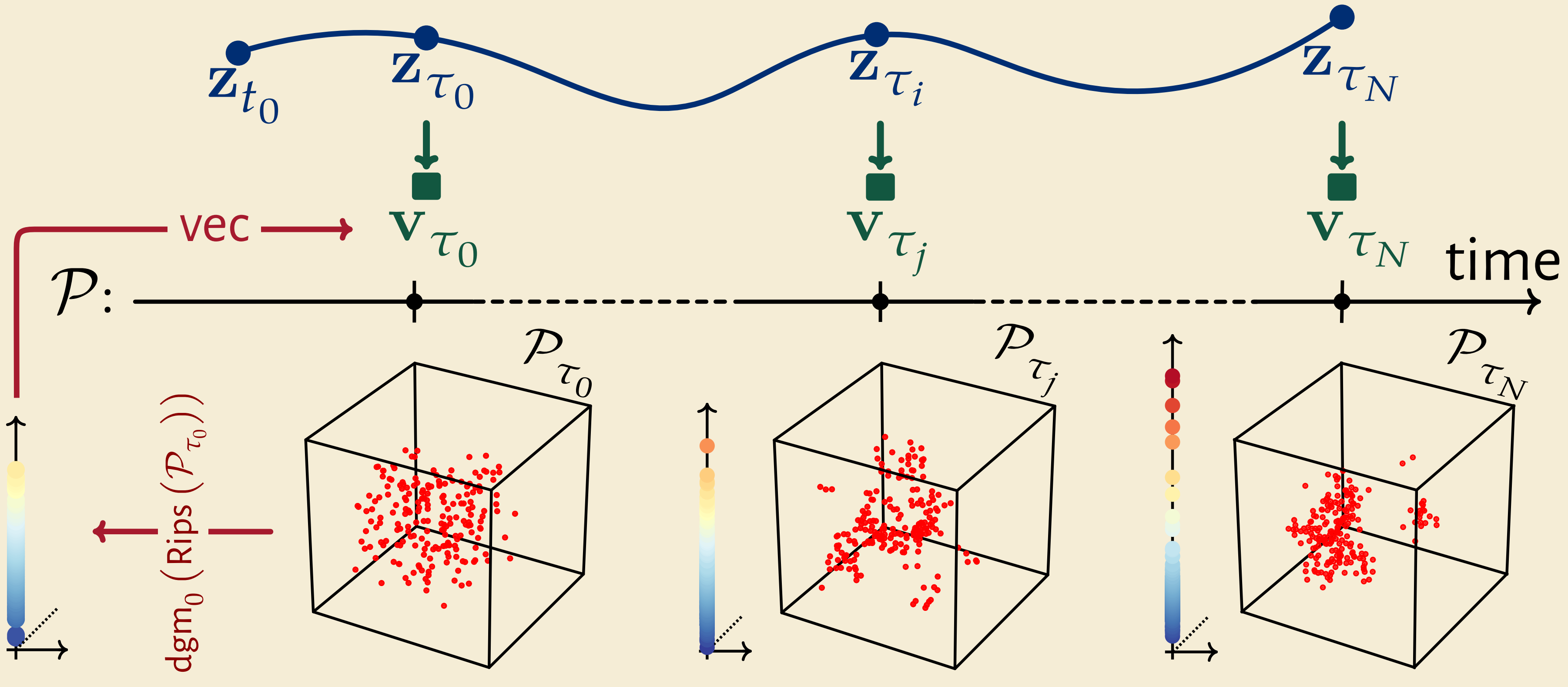

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- **Prior works** predominantly extracted **one** topological summary over time.
- **We** learn a latent process $\mathbf{Z}$ whose paths $\{\mathbf{z}_{\tau_j}\}_j$ can **(i)** reproduce the vectorizations, <u>and</u> **(ii)** serve as input for predicting β.

A model incarnation

- $\boldsymbol{Z}$ is modeled via a neural ODE by Rubanova et al. ’19 and learned in a variational Bayes regime.

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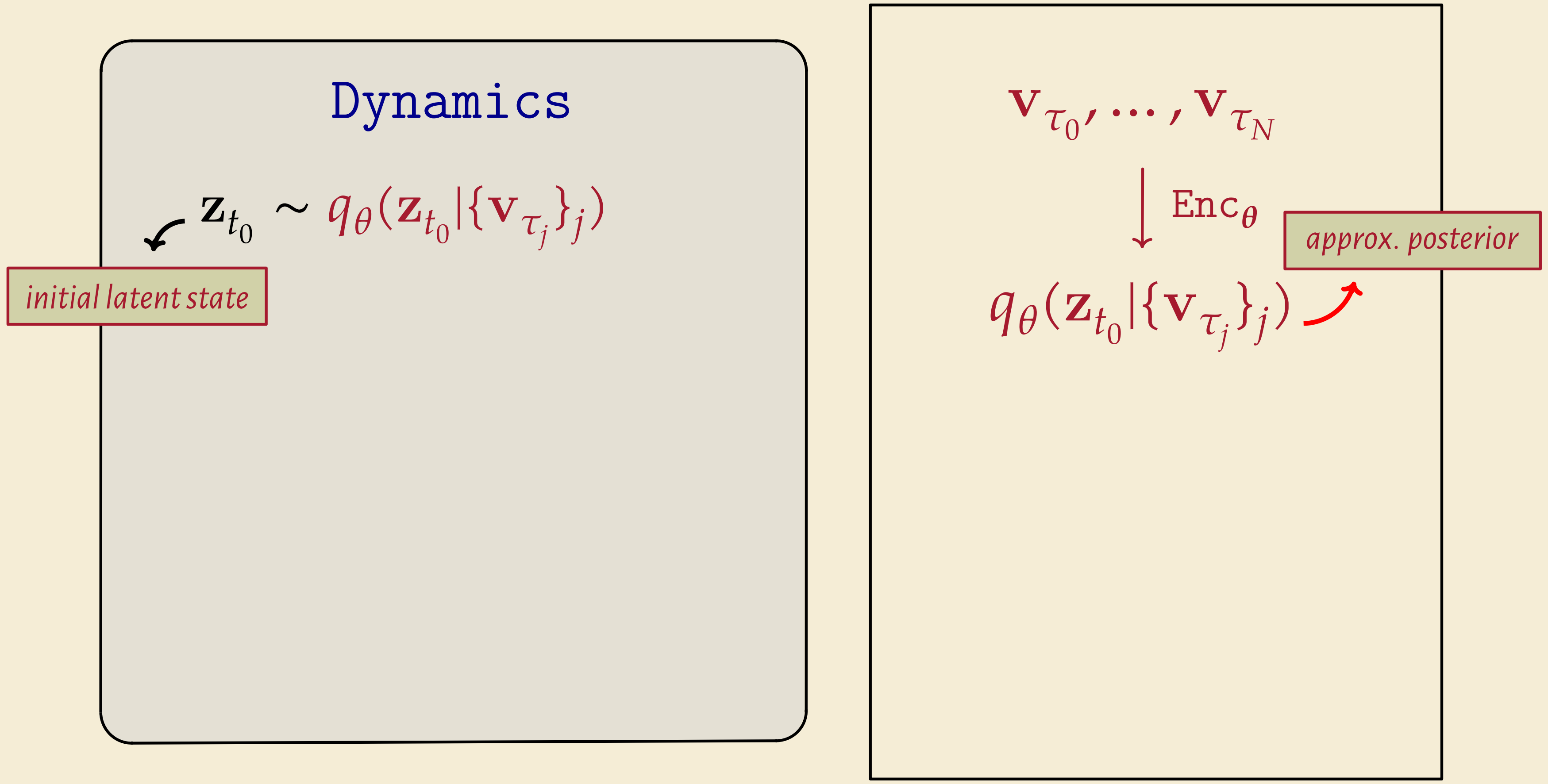

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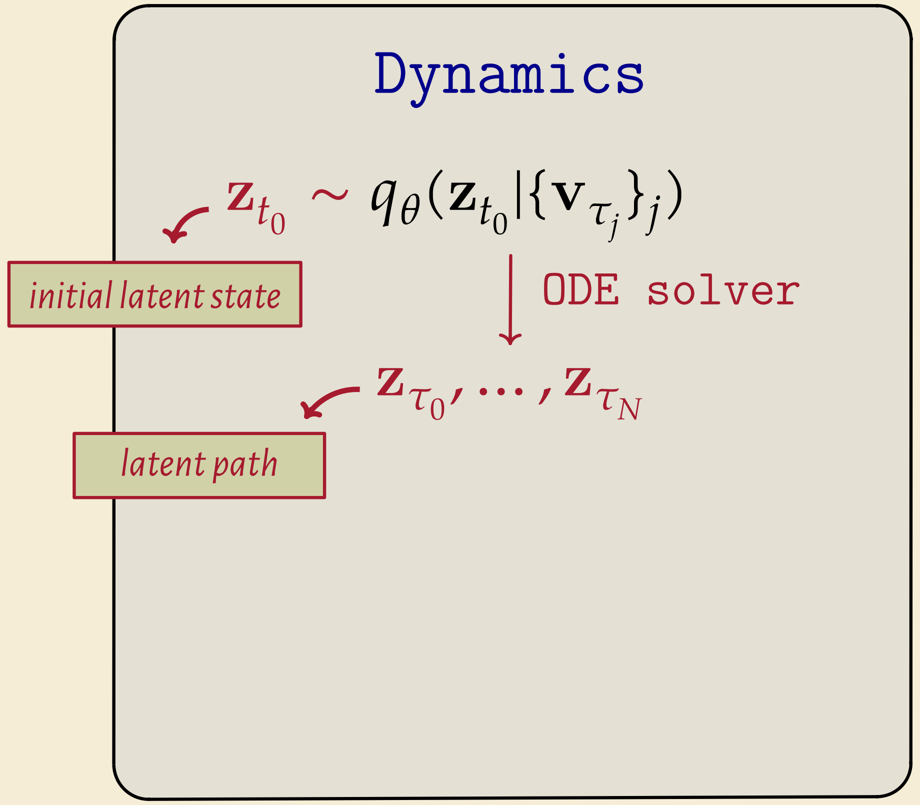

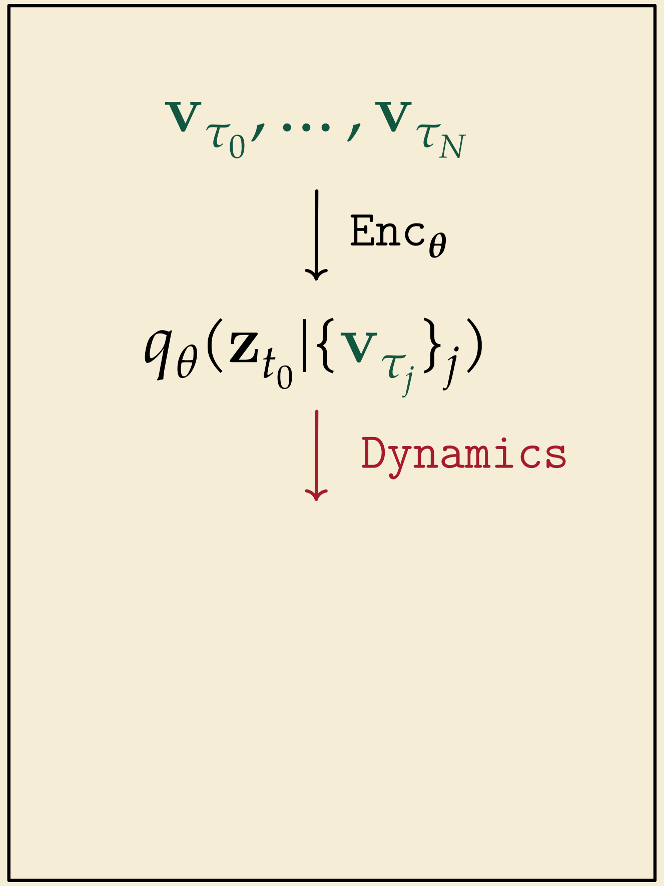

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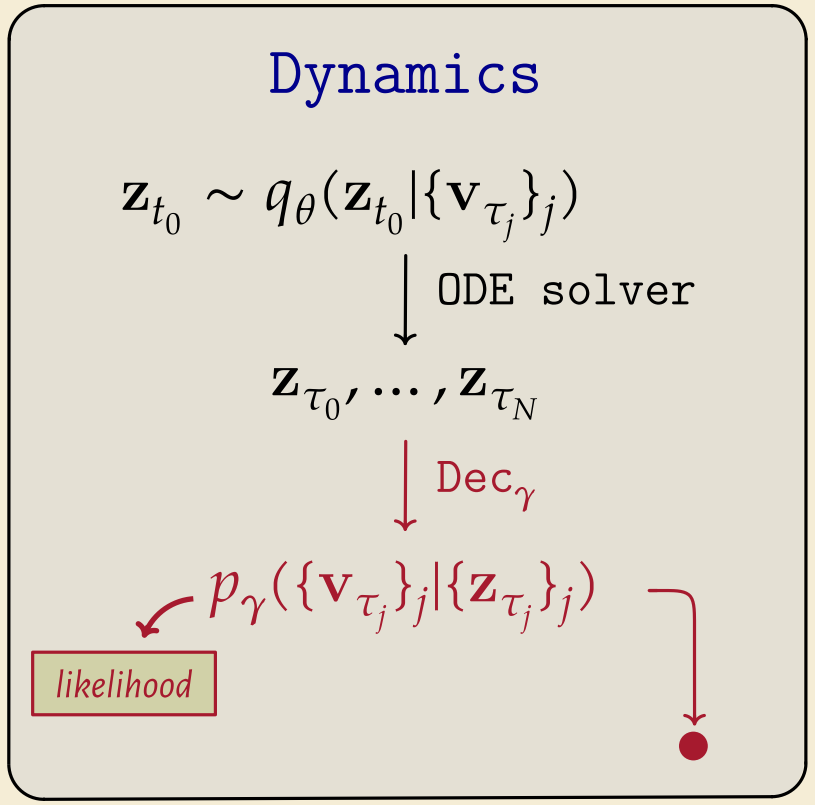

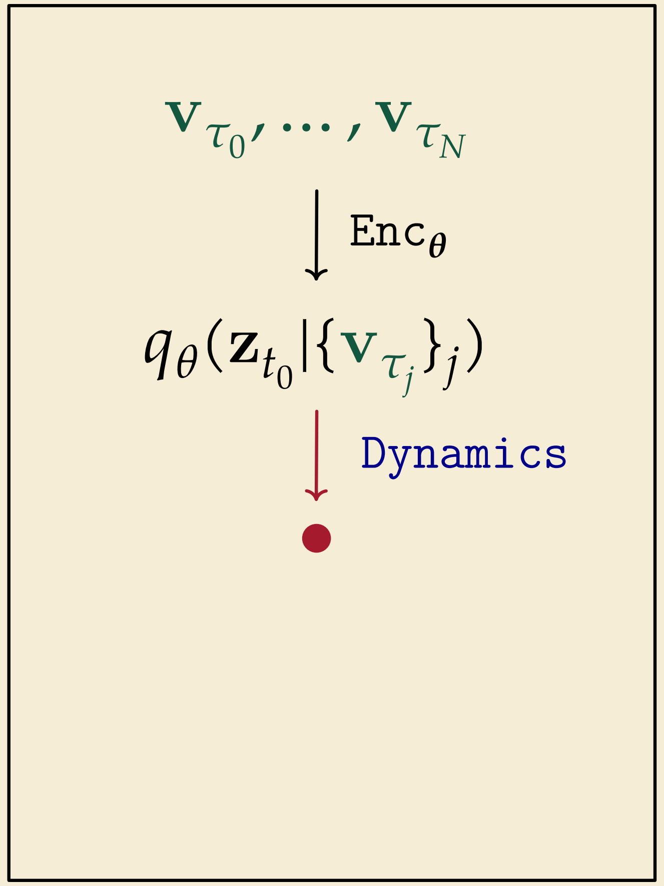

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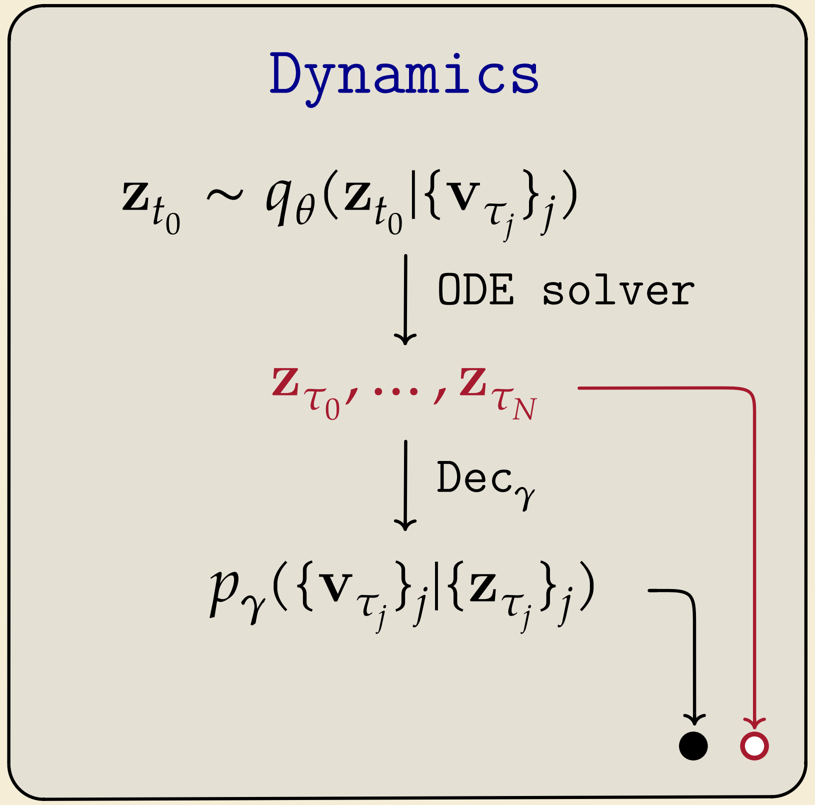

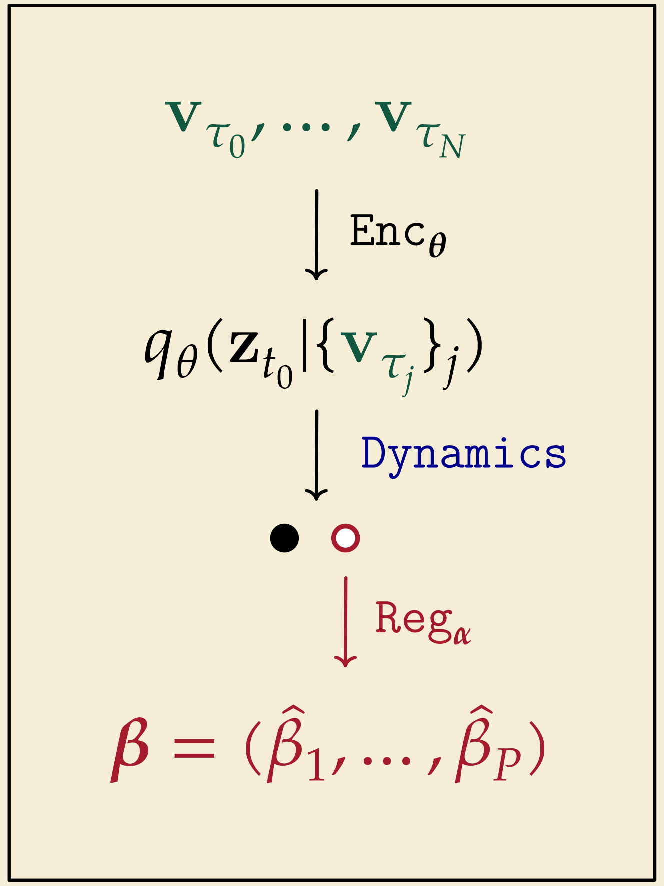

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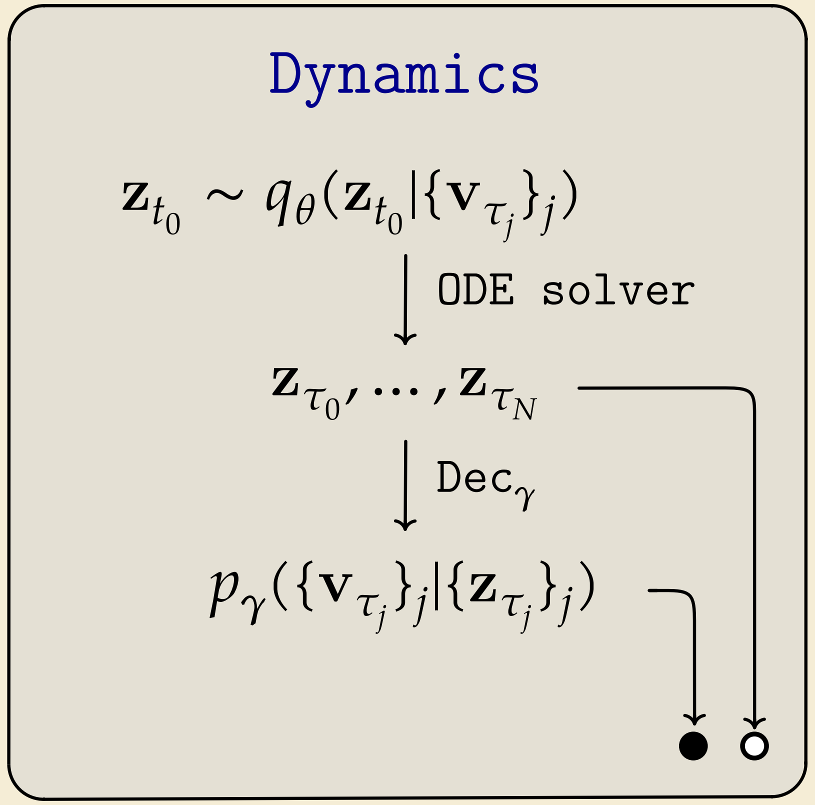

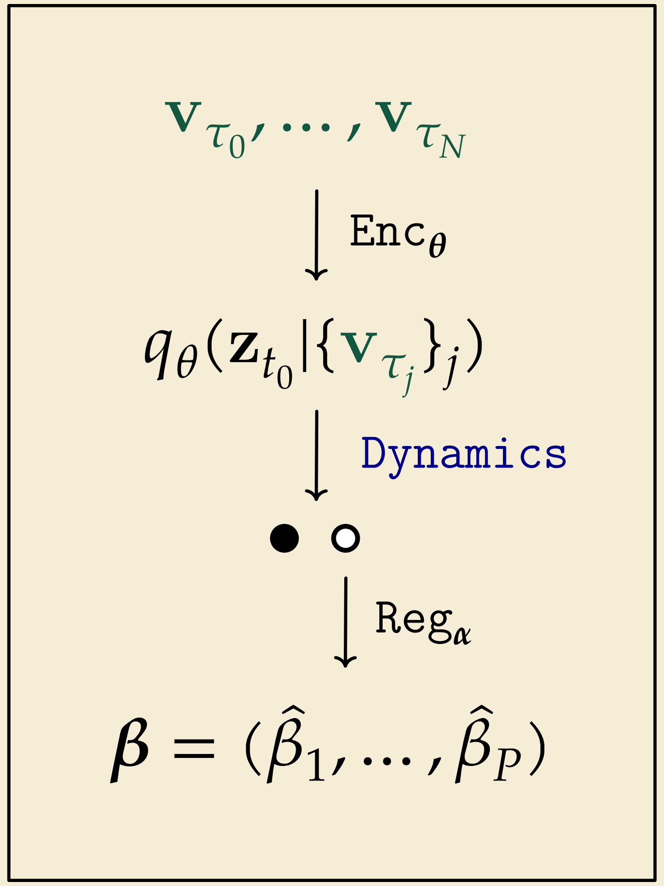

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- The model is trained upon choosing a prior $p(\mathbf{z}_{t_0})$ and maximizing (ELBO − loss_{aux}), i.e.,

$$\boldsymbol{\theta}, \boldsymbol{\gamma}, \boldsymbol{\alpha} = \underset{\boldsymbol{\theta}, \boldsymbol{\gamma}, \boldsymbol{\alpha}}{\arg\max} \underbrace{\mathbb{E}_{\mathbf{z}_{t_0} \sim q_{\boldsymbol{\theta}}} \left[\sum_j \log p_\gamma \left(\mathbf{v}_{\tau_j} | \mathbf{z}_{\tau_j} \right) \right] - \mathcal{D}_{\text{KL}} \left(q_{\boldsymbol{\theta}} \left(\mathbf{z}_{t_0} | \{\mathbf{v}_{\tau_j}\}_j \right) \| \, p \left(\mathbf{z}_{t_0} \right) \right)}_{\text{ELBO}} - \underbrace{\text{loss}_{\text{aux}} \left(\mathtt{Reg}_{\boldsymbol{\alpha}}(\{\mathbf{z}_{\tau_j}\}_j), \boldsymbol{\beta} \right)}_{\text{auxiliary loss}} .$$

Some results

		⌀ VE ↑	⌀ SMAPE ↓
dorsogna-10k	**Ours**	**0.851**±0.008	**0.097**±0.005
	PSK	0.828±0.016	0.096±0.006
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- Overall, ***Neural Persistence Dynamics (Ours)*** largely outperforms the state-of-the-art in all tasks.

In summary, ***Neural Persistence Dynamics***...

(1) scales to a **large number** of observation sequences,

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(3) **outperforms** the state-of-the-art across numerous regression tasks.

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Thanks for your attention!

Come see us at our **poster**.

Fr. 13 Dec 11 a.m. PST – 2 p.m. PST @ Poster Session 5

Full source code is available!

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