

Distributed Lion for Communication Efficient Distributed Training

The update of lion is **binary**
Lion has **only one** optimizer state

Algorithm 1 AdamW Optimizer

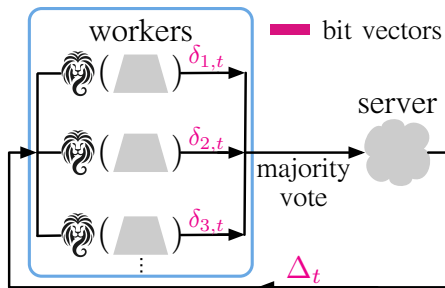
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given  $\beta_1, \beta_2, \epsilon, \lambda, \eta, f$ 
initialize  $\theta_0, m_0 \leftarrow 0, v_0 \leftarrow 0, t \leftarrow 0$ 
while  $\theta_t$  not converged do
   $t \leftarrow t + 1$ 
   $g_t \leftarrow \nabla_x f(\theta_{t-1})$ 
  update EMA of  $g_t$  and  $g_t^2$ 
   $m_t \leftarrow \beta_1 m_{t-1} + (1 - \beta_1)g_t$ 
   $v_t \leftarrow \beta_2 v_{t-1} + (1 - \beta_2)g_t^2$ 
  bias correction
   $\hat{m}_t \leftarrow m_t / (1 - \beta_1^t)$ 
   $\hat{v}_t \leftarrow v_t / (1 - \beta_2^t)$ 
  update model parameters
   $\theta_t \leftarrow \theta_{t-1} - \eta(\hat{m}_t / (\sqrt{\hat{v}_t} + \epsilon) + \lambda \theta_{t-1})$ 
end while
return  $\theta_t$ 
  
```

Algorithm 2 Lion Optimizer

```

given  $\beta_1, \beta_2, \lambda, \eta, f$ 
initialize  $\theta_0, m_0 \leftarrow 0$ 
while  $\theta_t$  not converged do
   $g_t \leftarrow \nabla_x f(\theta_{t-1})$ 
  update model parameters
   $c_t \leftarrow \beta_1 m_{t-1} + (1 - \beta_1)g_t$ 
   $\theta_t \leftarrow \theta_{t-1} - \eta \text{sign}(c_t) + \lambda \theta_{t-1}$ 
  update EMA of  $g_t$ 
   $m_t \leftarrow \beta_2 m_{t-1} + (1 - \beta_2)g_t$ 
end while
return  $\theta_t$ 
  
```



Each worker keep tracks of its own optimizer state

Algorithm 1 Distributed Lion Training

Inputs: Initial parameters $x_0 \in \mathbb{R}^d$, datasets $\{D_1, \dots, D_N\}$, loss function f , learning rate ϵ , hyper-parameters $\beta_1, \beta_2, \lambda$, and the weight decay λ .

Initialization: $t = 0, \forall i, m_{i,0} = 0$, and $x_{i,0} = x_0$.

while not converged **do**

Worker-side: Each worker i samples a batch $\xi_{i,t} \in D_i$, computes the following, and sends $\delta_{i,t}$ to the server:

$$\begin{aligned}
 &\text{if } t > 0, \quad x_{i,t} \leftarrow x_{i,t-1} - \epsilon(\Delta_{t-1} + \lambda x_{i,t-1}) \\
 &\delta_{i,t} \leftarrow \text{sign}(\beta_1 m_{i,t} + (1 - \beta_1)\nabla_x f(x_{i,t}; \xi_{i,t})) \\
 &m_{i,t+1} \leftarrow \beta_2 m_{i,t} + (1 - \beta_2)\nabla_x f(x_{i,t}; \xi_{i,t}).
 \end{aligned}$$

Server-side: The server computes the aggregated update Δ_t and broadcast it to all workers:

$$\Delta_t = \begin{cases} \frac{1}{N} \left(\sum_{i=1}^N \delta_{i,t} \right) & \text{(Averaging)} \\ \text{sign} \left(\sum_{i=1}^N \delta_{i,t} \right) & \text{(Majority Vote)} \end{cases} \quad \text{and } t \leftarrow t + 1.$$

end while

Minimum bandwidth requirements of different methods for a model with d parameters and n workers:

Method	Bandwidth Requirement	
	Worker→Server	Server→Worker
Global Lion/AdamW	32d	32d
TermGrad (Wen et al., 2017)	1.5d	$\log(2n + 1)d$
DGC (Lin et al., 2017)	$(1 - \eta)32d$	32d
Distributed Lion-Avg	d	$\log(n)d$
Distributed Lion-MaVo	d	d

Theorem 3.6 (Majority Vote). Assumptions 3.1, 3.2, and 3.3 hold, consider the Majority vote scheme in Algorithm 1, $\beta_1, \beta_2 \in (0, 1)$, and $\beta_2 > \beta_1$, and $\sigma \leq 2\sqrt{d}\beta_1\beta_2^2\|\nabla f(x_0)\|, 1 \leq t \leq T$, and $\epsilon, \lambda > 0$. Let $(x_{i,t})_{i \geq 0}$ be generated by Majority Vote, and it is in Phase II: $\|\lambda x_{i,t}\|_\infty \leq 1$ for all t .

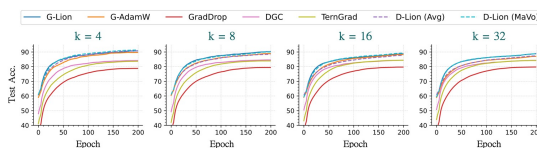
We have

$$\frac{1}{T} \sum_{t=1}^T \mathbb{E}[S(x_t)] \leq \frac{f(x_0) - f^*}{T\epsilon} + \frac{2D\beta_1\beta_2\sqrt{d}\|\nabla f(x_0)\|}{T(1 - \beta_2)} + \frac{4\beta_1 L \epsilon d}{1 - \beta_2} + \frac{2\sqrt{d}\sigma(1 + \sqrt{C}) + 2\rho}{\sqrt{N}} + 2L \epsilon d,$$

where $C = \beta_1^2(1 - \beta_2)\frac{1}{1 - \beta_2} + (1 - \beta_1)^2$, and $D = \max\{1, \sigma / (2\sqrt{d}\beta_1\beta_2^2\|\nabla f(x_0)\|)\}$,

$$\rho_t[k] = \begin{cases} 0 & \text{if } \mathbb{E}[\text{sign}(\hat{m}_{t+1}^{(k)})] = 0, \\ \mathbb{E}[\hat{m}_{t+1}^{(k)}] / \mathbb{E}[\text{sign}(\hat{m}_{t+1}^{(k)})] & \text{otherwise} \end{cases}$$

, and $\rho = \max_{1 \leq t \leq T} \|\rho_t\|$.

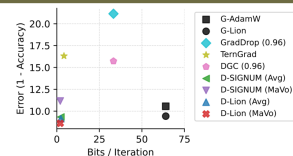


Performance of Distributed Lion v.s. baseline distributed optimizers on CIFAR-10 with 4, 8, 16, and 32 workers, each worker at each step runs on a local batch with size 32.

Method	Image Classification		Language Modeling	
	ViT-S/16	ViT-B/16	GPT-2++ (350M)	GPT-2++ (760M)
AdamW	79.74	80.94	18.43	14.70
G-Lion	79.82	80.99	18.35	14.66
D-Lion (MaVo)	79.69	80.79	18.37	14.66
D-Lion (Avg)	80.11	81.13	18.39	14.69

Results on ImageNet classification and OpenWebText language modeling. For ImageNet experiments, we report the Top-1 accuracy. For language modeling experiments, we report the validation perplexity.

Test Error v.s. Communication Bits per Iteration (closer to the lower-left is better).



Performance of G-Lion, G-AdamW, GradDrop, DGC, TermGrad, and D-Lion (Avg/MaVo) v.s. the number of workers k .

