

Unsupervised Protein-Ligand Binding Energy Prediction via Neural Euler's Rotation Equation

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Structure-based drug discovery Binding energy prediction (small molecule/antibody ligands)





Current binding energy prediction approach Supervised and unsupervised approaches







Our method: unsupervised energy-based model Learn binding free energy from crystal structures (data-driven)



(No binding affinity labels)

Binding energy = -13.6

Binding energy = -15.2







Our method: unsupervised energy-based model Learn binding free energy from crystal structures (data-driven)

- **Intuition:** crystal structures should be the local minimum of the energy landscape
- Suppose $E_{\theta}(x)$ is the energy of a protein complex
- Its likelihood is $p(x) \propto \exp(-E_{\theta}(x))$
- Minimizing the energy of crystal structures = maximizing their likelihood (standard objective in generative models / protein language models)
- Q1: How to parameterize $E_{\theta}(x)$?
- Q2: What's the training objective?

The space of all possible protein complexes

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Binding energy = -15.2







Energy-based model (EBM) architecture Requirement: E(X) is SE(3)-invariant and differentiable w.r.t. X







Training EBMs with SE(3) denoising score matching Perturbing input complexes with rigid transformation noises



Random rigid

DSM loss $\ell =$ $\|\tilde{\omega} - \nabla_{\omega} \log p(\omega)\|^{2} + \|\tilde{t} - \nabla_{t} \log p(t)\|^{2}$





Neural Euler's rotation equation (NERE) Infer rotation R from gradient $\nabla_x E_{\theta}(x)$ (force)



- The torque applied to the ligand $\tau = \sum_{i} (x_i x_i)$
- Angular velocity $\omega = I^{-1} \tau \Delta t$ for an infinitesimal time Δt
- Rotation matrix *R* is the exponential of the following matrix $W(\omega) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$

$$(-\mu) \times \nabla_{x_i} E_{\theta}(x)$$

(Euler's rotation equation) Angular acceleration of the ligand $\alpha = I^{-1}\tau$, where I is the inertia matrix



Results: protein-ligand binding Log-likelihood is strongly correlated with binding affinity

- Training set: 5237 protein-ligand complexes in PDBBind refined set (without using binding affinity data)
- Test set: 285 complexes from PDBBind (core) Measure the Pearson correlation between predicted and true affinity
- Supervised models (TankBind, IGN, KDeep) are trained on ~18000 binding affinity data in PDBBind (general subset)
- SE(3) DSM outperforms MM/GBSA and other unsupervised models like Gaussian DSM and contrastive learning





Results: antibody-antigen binding Unsupervised models outperform supervised methods

- Training set: 3416 complexes from Structure Antibody Database (SAbDab).
- Test set: 566 complexes from SAbDab that have binding affinity labels
- We compare with physical potentials (AP_PISA, ZRANK), protein language models (ESM-IF, ESM-1v), and a supervised neural network trained on SKEMPI binding affinity data
- We outperform supervised baseline because we can leverage more unlabeled antibody-antigen complexes



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Conclusion & acknowledgements Towards unsupervised models for protein-ligand binding

- 1. Formulate binding affinity prediction as a generative modeling problem
 - Train the generative model using SE(3) denoising score matching (DSM)
- 2. Propose a simple equivariant rotation prediction module for SE(3) DSM
 - Embed Euler's rotation equation into neural networks (adding physical prior)

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Main contribution

