Demographic Parity Constrained Minimax Optimal Regression under Linear Model

Kazuto Fukuchi

University of Tsukuba / RIKEN AIP

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Fair Regression

- Let X be non-sensitive features on $\mathbb{R}^d$, let S be a sensitive feature on $[M]$ where $M \geq 2$.
- With some noise ξ, let Y be outcome on $\mathbb{R}$, defined as

$$Y = f^*(X, S) + \xi.$$

- **(Goal)** Given n i.i.d. copies of (X, S, Y), construct an accurate and fair regressor f.

Fairness: demographic parity (Pedreshi et al. 2008)

$$\mathbb{P}\{f(X, S) \in E | S = s\} = \mathbb{P}\{f(X, S) \in E | S = s'\}.$$

Accuracy

- Fair Bayes-optimal regressor:
  $f^*_{\mathrm{DP}} = \arg\min_{f \in \mathcal{F}_{\mathrm{DP}}(\mu_\cdot)} \mathbf{E}[(f(X, S) - f^*(X, S))^2]$
- Deviation from f^*_{DP}:

$$\mathcal{E}(f; P) = \mathbf{E}[(f(X, S) - f^*_{\mathrm{DP}}(X, S))^2].$$

Minimax optimality in Fair Regression

Research question

What is the best algorithm for fair regression?

- Best algortihm = minimax optimal.
- Minimax optimal = atteining the minimum regression error under the worst-case scenario.

Models

- Chzhen et al. (2022) presents the sole study demonstrating minimax optimality in fair regression.
- Contrast with Chzhen et al. (2022): our model accounts for a broader source of discrimination.

	Outcome	Non-sensitive features
Chzhen et al. (2022)	$Y = \langle \beta^*, X \rangle + b_S + \xi$	$X \sim N(0, \Sigma)$
Ours	$Y = \langle \beta^*_S, X \rangle + \xi$	$X \sim N(\mu_S, \sigma_X^2 I)$

Differences from Chzhen et al. (2022) and Challenge

	partial coefficients	intercept	non-sensitive features
Chzhen et al. (2022)		✓	
ours	✓	✓	✓

(Challenge)

- Variability in partial coefficients leads to varied outcome variances against S. Our model poses a challenge of addressing variance disparities.
- Our model introduces bias via non-sensitive features. Mitigating this bias is an additional challenge.

Summary

- Examining the minimax optimality of regression with a constraint of demographic parity.
- Our model poses the following additional challenges compared to the existing results of Chzhen et al. (2022):
  - Mitigating outcome's variance disparity.
  - Addressing bias through non-sensitive features.
- Revealing the minimax optimal error rate as $\sigma_\xi^2 B^2 dM/n$.

Check out our poster and arXiv paper.