

## What Distributions are Robust to Indiscriminate Poisoning Attacks for Linear Learners?

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## **Machine Learning Pipeline**



## **Indiscriminate Poisoning Attacks**



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## **Indiscriminate Poisoning Attacks**





architecture, possible defenses

on some datasets while ineffective on others

#### Are these attacks always effective without defenses?

[1]: Koh et al., "Stronger Data Poisoning Attacks Break Data Sanitization Defenses", Machine Learning 2021.

[2]: Suya et al., "Model-Targeted Poisoning Attacks with Provable Convergence", ICML 2021

[3]: Steinhard et al., "Certified Defenses Against Data Poisoning Attacks", N(eur)IPS 2017

## **Evaluation without Defenses**

#### **Datasets to train Linear SVM**

MNIST I-7

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Digits of "I" and "7" from MNIST





**Filtered Enron** 



Collection of Spam emails Filter our 3% near boundary points from **Enron** 

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Results of more models and datasets are in the paper.

## **Best Attack Effectiveness Varies**



Are some datasets (e.g., MNIST I-7) just robust to state-of-the-art poisoning attacks or inherently robust to any poisoning attacks?

## **Understanding Optimal Attacks**

**Theorem I (Informal)**: optimal finite-sample poisoning attacks are consistent estimators of optimal distributional poisoning attacks if:

- I) hypothesis class satisfies uniform convergence
- 2) surrogate loss for model training is strongly-convex
- 3) risk of the model is Lipchitz continuous.

Finite-sample optimal poisoning attacks (practice): relevant to practical applications

Generate poisoned dataset to maximize risk



Distributional optimal poisoning attacks (theory): convenient for analysis

Generate poisoned distribution to maximize risk

Useful to study distributional optimal attacks as they still connect to finite-sample attacks in practice!

## Using Maximum Poisoning Ratio

**Theorem 2 (Informal)**: for convex hypothesis class, optimal distributional poisoning is achieved with maximum poisoning ratio  $\epsilon$  if either condition is satisfied:

I) clean data points are not filtered during training

2) For any model  $\theta$ , there is a distribution  $\mu$  such that gradient w.r.t.  $\mu$  is 0.

When studying distributional optimal poisoning attacks, we can use the maximum poisoning ratio!

### **Characterize Optimal Attacks in I-D Gaussian**



Linear SVM on I-D two Gaussian mixtures

Poisoning points are in constraint set [-u, u]with **constraint size** 2u. Goal

Analyze the impact of distributional properties on optimal poisoning attacks that have maximum risk on clean distribution

## **Distributional Factors on Optimal Attack**

**Theorem 3 (Informal)**: distributions with smaller  $|\gamma_1 - \gamma_2|/\sigma$  (separability ratio) and larger 2u (larger constraint size) are inherently more vulnerable to poisoning attacks and vice-versa.

 $|\gamma_1 - \gamma_2|/\sigma$ : small ratio implies more near-boundary points and more prone to misclassifications

Larger constraint size 2u: moves the decision boundary more with poisoning points

## Projected Separability Ratio (Sep/SD)

Projected Separability Ratio  $|\gamma_1 - \gamma_2|/\sigma$ : compute by projecting onto  $w_c$ , name as Sep/SD



Projected Separability (Sep) Projected Standard Deviation (SD)

Lower Sep/SD: more vulnerable

Higher Sep/SD: less vulnerable

## **Projected Constraint Size Ratio (Sep/Size)**

Projected constraint size 2u: project C onto  $w_c$ , name as Size (use Sep/Size to compare different datasets)



### **Correlation of Factors to the Upper Bound**

**Theorem 4 (Informal)**: training models with monotone non-decreasing loss w.r.t the (negative) margin, the maximum risk from any poisoning is upper bounded by the loss on the clean distribution and the loss w.r.t. the projected constraint size, for the given clean model.

Lower loss on clean distribution  $\rightarrow$  higher average margin, higher Sep/SD, inherently less vulnerable

Lower projected constraint size  $\rightarrow$  loss is small, inherently less vulnerable

### Negative Correlation of Factors to Empirical Vulnerability



#### Less vulnerable datasets (e.g., MNIST 1-7) have higher Sep/SD and Sep/Size (smaller Size), and vice versa!

### Implications: Improved Robustness from Better Representations



#### **Better Features Reduce Attack Effectiveness**



Measures error increase from state-of-the-art attacks at 3% poisoning ratio.

R-X: ResNet18 model on CIFAR10 dataset trained for X epochs.

LeNet: fully trained simple CNN

Binary classification: "Truck" vs "Ship"

# Main Takeaways

Distributions with high class-wise separability and low projected constraint size are inherently robust to indiscriminate poisoning attacks.

Learning better feature representations can improve resistance to poisoning attacks.

Updated paper: https://arxiv.org/abs/2307.01073





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