

# When Does Group Invariant Learning Survive Spurious Correlations?

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# Outline

- The question (motivation)
- 3 highlights of this paper
  - Two group criteria
  - Failures of existing methods
  - New method: SCILL
- Main experimental results
- Conclusion








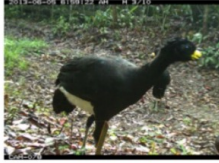
Code is available at:  
<https://github.com/Beastlyprime/group-invariant-learning>

# Motivation

When Does **Group Invariant Learning** Survive Spurious Correlations?

# Invariant learning

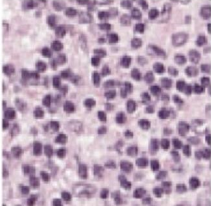
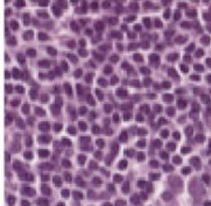
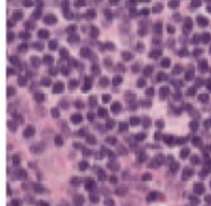
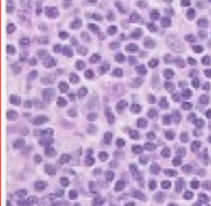
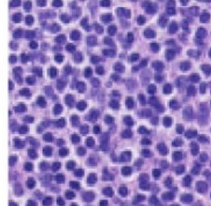
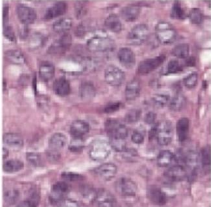
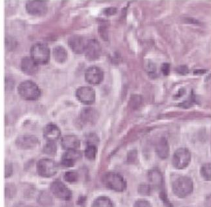
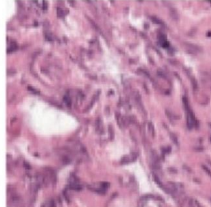
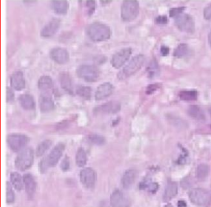
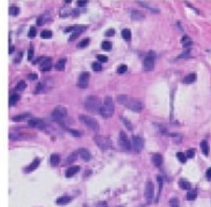
In real world applications, machine learning model encounters out-of-distribution (OOD) data

Train			Test (OOD)
$d = \text{Location 1}$	$d = \text{Location 2}$	$d = \text{Location 245}$	$d = \text{Location 246}$
			
Vulturine Guinea fowl	African Bush Elephant	...	Wild Horse
			
Cow	Cow	Southern Pig-Tailed Macaque	Great Curassow




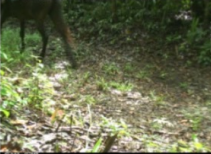



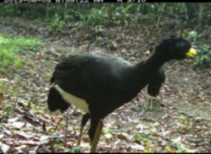
photos from new locations

# Invariant learning and environments

Invariant learning: a notable kind of method for OOD generalization

Train				Val (OOD)	Test (OOD)
	d = Hospital 1	d = Hospital 2	d = Hospital 3	d = Hospital 4	d = Hospital 5
y = Normal					
y = Tumor					

samples from different hospitals

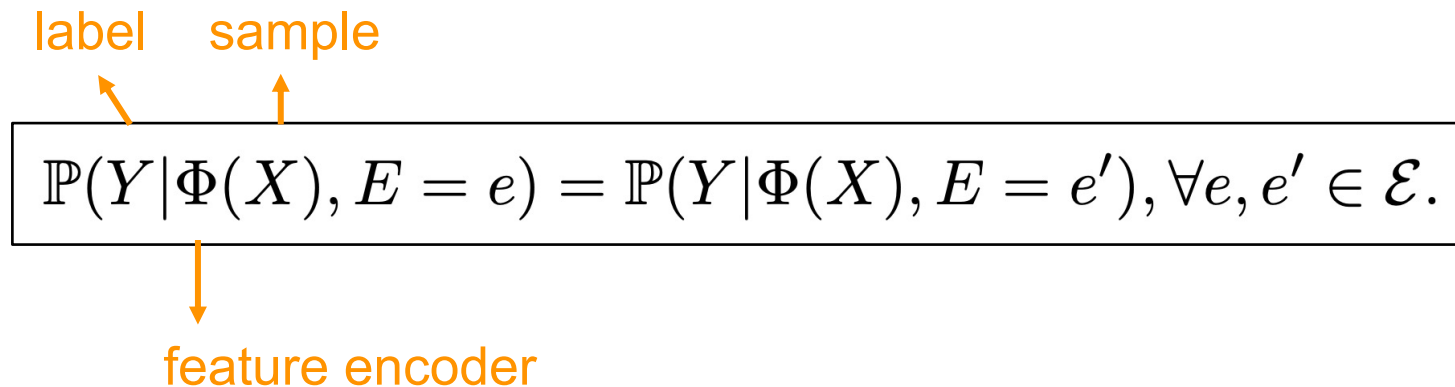
Train			Test (OOD)
$d = \text{Location 1}$	$d = \text{Location 2}$	$d = \text{Location 245}$	$d = \text{Location 246}$
 Vulturine Guineafowl	 African Bush Elephant	 ...	 Wild Horse
 Cow	 Cow	 Southern Pig-Tailed Macaque	 Great Curassow

photos from different locations

Invariant learning is designed for the case when environment labels are available

# Invariant learning and environments

Intuitively, the target is to learn the common rule on different environments



The diagram shows the equation  $\mathbb{P}(Y|\Phi(X), E = e) = \mathbb{P}(Y|\Phi(X), E = e'), \forall e, e' \in \mathcal{E}.$  enclosed in a black rectangular box. Three orange arrows point from text labels to parts of the equation: one from 'label' to  $Y$ , one from 'sample' to  $X$ , and one from 'feature encoder' to  $\Phi$ .

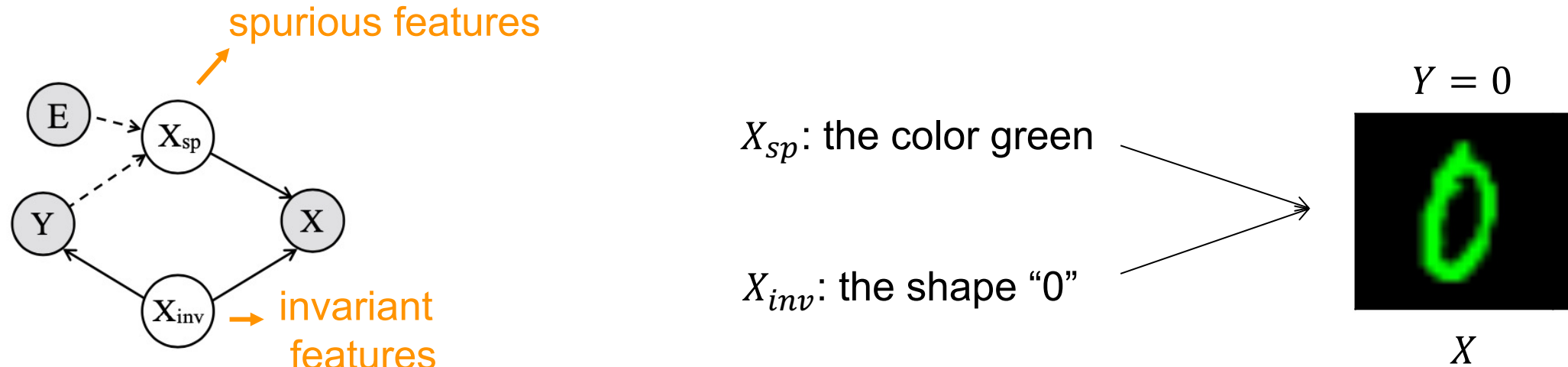
label   sample

$$\mathbb{P}(Y|\Phi(X), E = e) = \mathbb{P}(Y|\Phi(X), E = e'), \forall e, e' \in \mathcal{E}.$$

feature encoder

# Invariant learning and environments

Formally, invariance is deduced by assumptions on the data generating process



$\mathbb{P}^e(Y|X_{inv}) := \mathbb{P}(Y|X_{inv}, E = e)$  keeps invariant across different environments

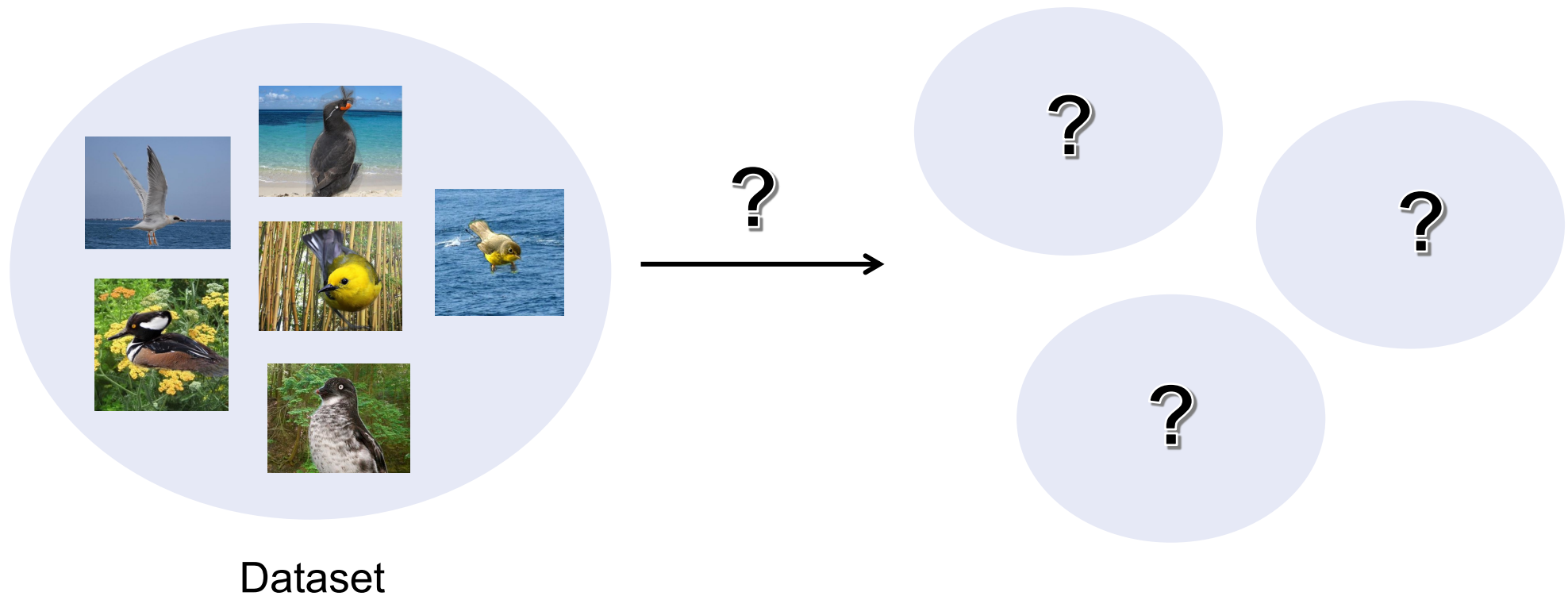
Correlation between  $Y$  and  $X_{sp}$  is **spurious**, which changes across environments

# Group invariant learning

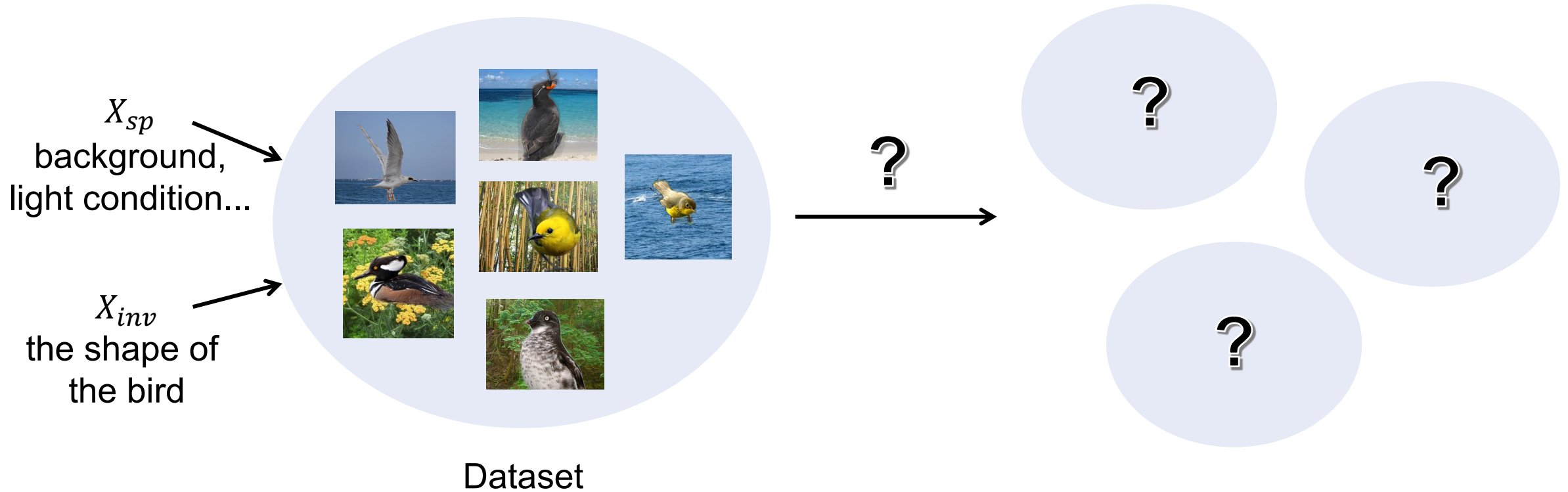
- Limitation: we need the environment labels are known
- “Group invariant learning” extend IL to the case when environments are unknown



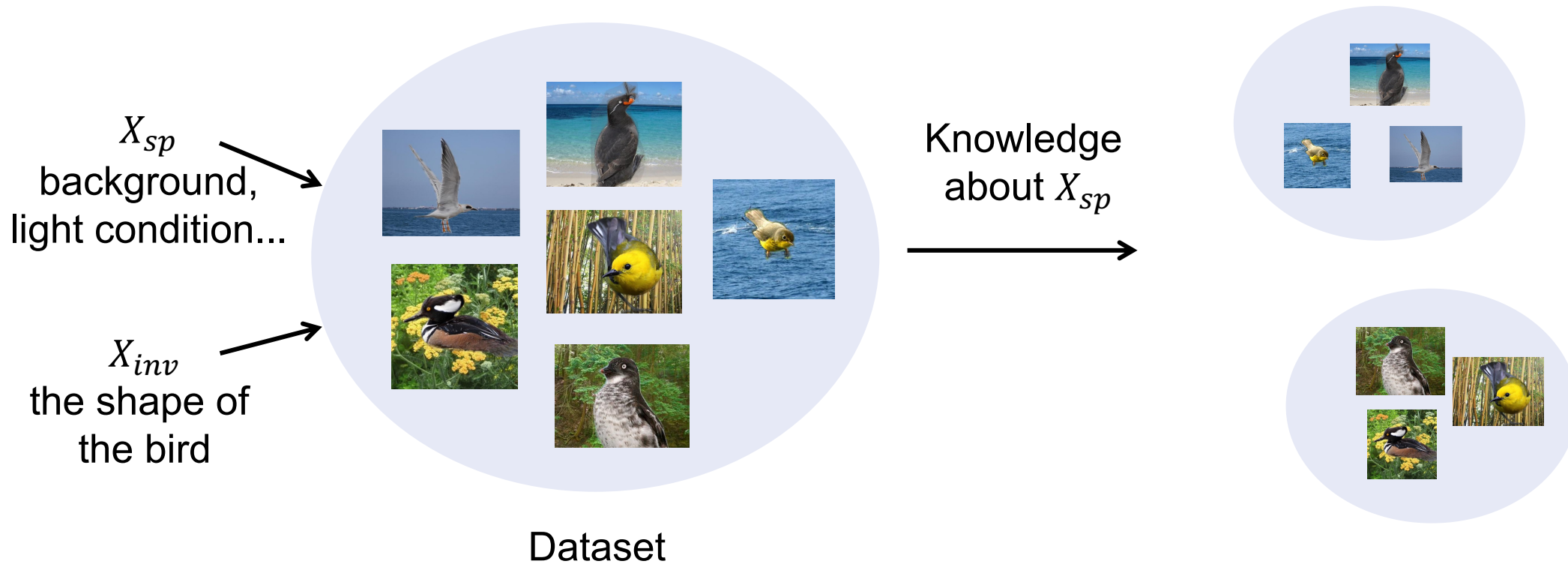
# Infer Environments for Invariant Learning



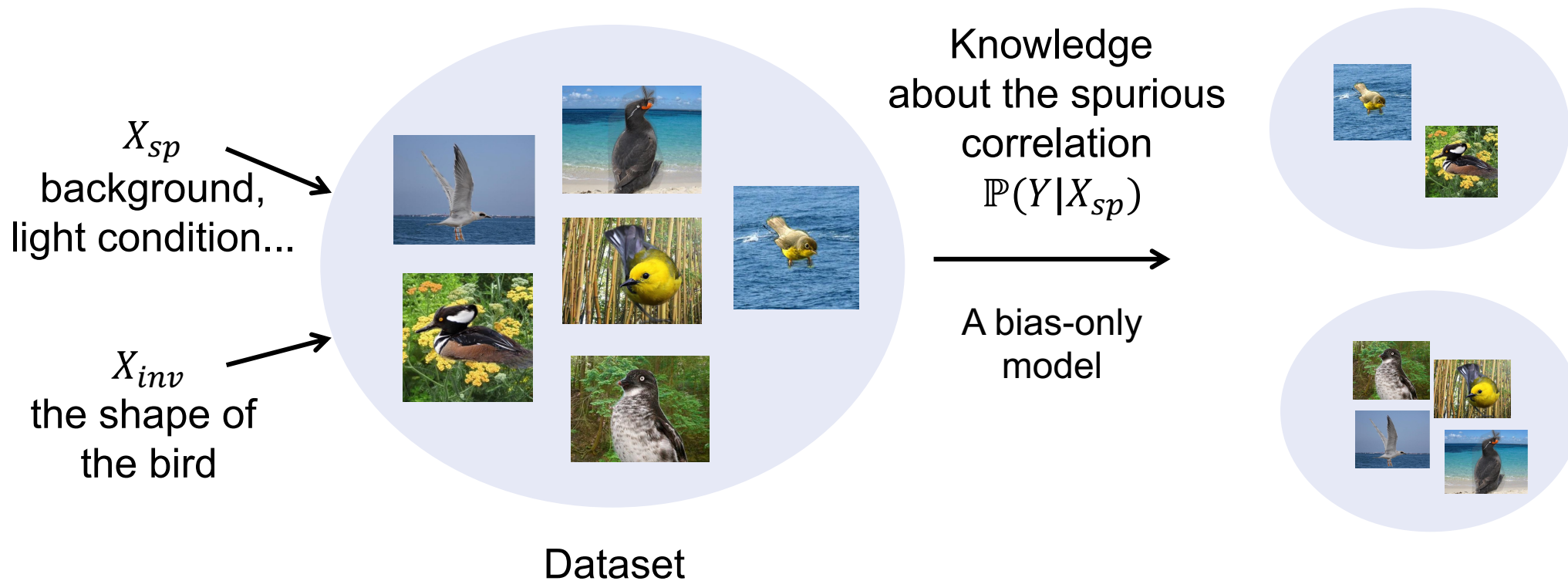
# Infer Environments for Invariant Learning



# Infer Environments for Invariant Learning

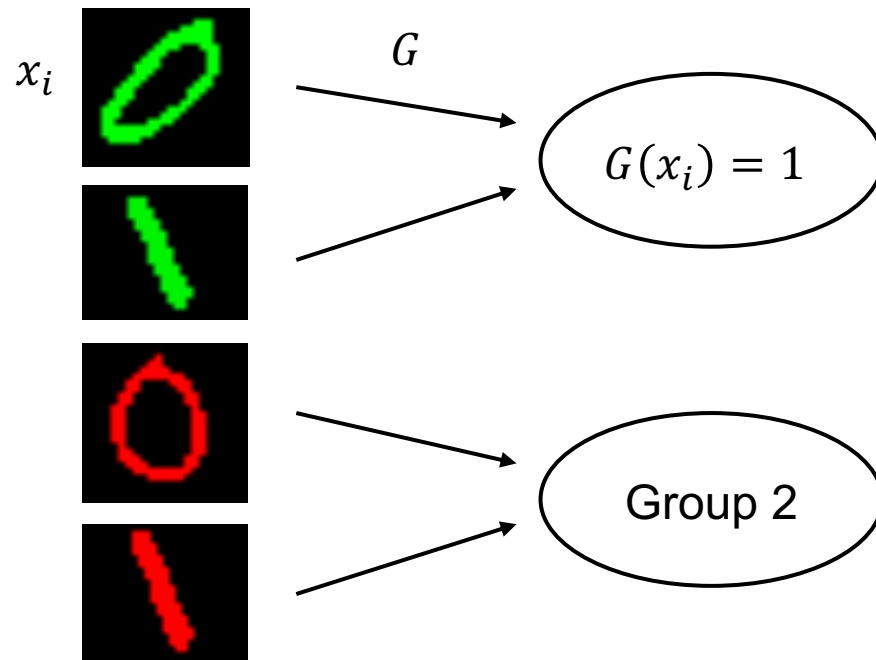


# Infer Environments for Invariant Learning



# “Groups”: posterior environments

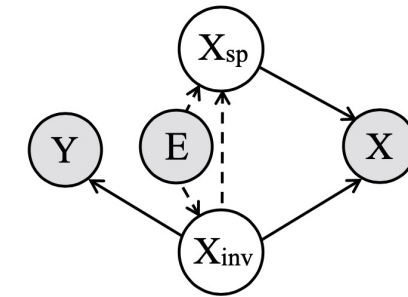
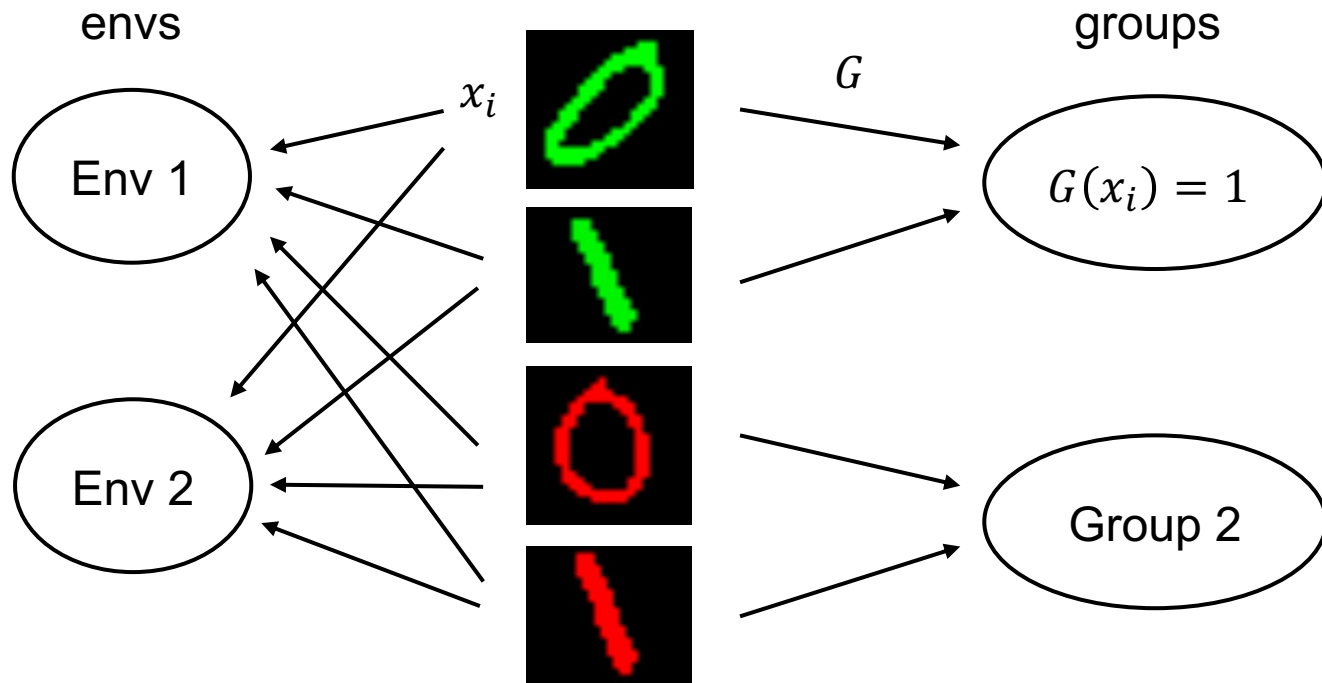
Group invariant learning extend IL to the case when environments are unknown



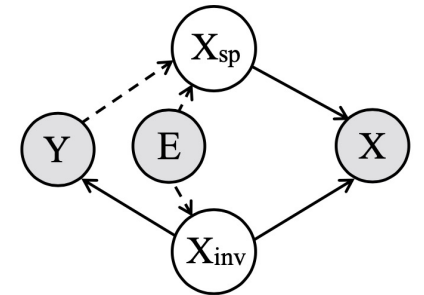
Infer posterior environments (groups)  
with knowledge about  $X_{sp}$  and  $Y$

# “Groups”: posterior environments

Groups are different from priori environments



(c) confounded [2; 25]



(d) hybrid [39; 23]

Cannot be inferred without  $X_{inv}$

We need new theory for group invariant learning

# Highlight 1: Two group criteria

- Falsity exposure criterion:  
Groups should fully expose the falsity of spurious correlations (informally)

**Criterion 4.1** (Falsity Exposure). For any  $\sigma(X_{sp})$ -measurable function  $h$  that satisfies  $\forall g, g' \in \mathcal{G}$ ,  $\mathbb{P}(Y|h(X_{sp}), g) = \mathbb{P}(Y|h(X_{sp}), g')$ , it must satisfy  $\mathbb{P}(Y|h(X_{sp})) = \mathbb{P}(Y)$ .

- Label balance criterion:  
The label proportion between groups should be the same (informally)

**Criterion 4.3** (Label Balance). For any  $g, g' \in \mathcal{G}$  and  $y, y' \in \mathcal{Y}$  with non-zero  $\mathbb{P}(Y = y|g)$ ,  $\mathbb{P}(Y = y'|g)$ ,  $\mathbb{P}(Y = y|g')$  and  $\mathbb{P}(Y = y'|g')$ , the following equation holds.

$$\mathbb{P}(Y = y|g)/\mathbb{P}(Y = y'|g) = \mathbb{P}(Y = y|g')/\mathbb{P}(Y = y'|g') \quad (2)$$

Both criterion are necessary !

## Highlight 2: Failures of existing methods


more  $\xrightarrow{\text{additional information}}$  less

Existing methods	clustering $X_{sp}$	clustering $P(Y X_{sp})$	majority/minority split
<b>Falsity exposure</b>	No guarantee	✓	?
<b>Label balance</b>	No guarantee	✗	?

- We focus on the majority/minority split (EUIL, ICML2021)
- On **some dataset** (e.g. colored-MNIST), the majority/minority split satisfy both criteria
- In the presence of **multivariate** spurious features, it fails both criteria



## Highlight 3: a new method SCILL

- Same as EILL, it relies on a reference model  $f_r$  which approximates  $\mathbb{P}^e(Y|X_{sp})$ .
- For falsity exposure:  
Construct groups such that  $Y \perp f_r(X) | g$
- For label balance:  
Attach a weight  $\omega^g(y) := \mathbb{P}(Y = y) / \mathbb{P}(Y = y | g)$  to samples in group  $g$ .
- Learning objective:  $\mathcal{L}(f) := \sum_{g \in \mathcal{G}} \tilde{\mathcal{R}}^g(f) + \lambda \cdot \text{penalty}(\{S_g(f)\}_{g \in \mathcal{G}})$   
  
Invariance penalty

# Highlight 3: a new method SCILL

- Sufficiency of SCILL:

**Theorem 5.1.** *If  $\mathcal{G}$  satisfies  $f_r^*(X) \perp\!\!\!\perp Y|g, \forall g \in \mathcal{G}$ , where  $f_r^* : \mathcal{X} \rightarrow \mathcal{Y}$  is spurious-only, i.e.  $\sigma(X_{sp})$ -measurable, and minimizes the prediction loss  $\mathcal{L}_{ce}^r = \mathbb{E}[\sum_y \mathbb{P}(Y = y|X) \log f_r(X)_y]$ , the optimal model minimizing the objective (3) satisfies SFC.*

SCILL can survive spurious correlations with an ideal reference model

# Experimental Results

## Patched-Colored-MNIST (PC-MNIST)

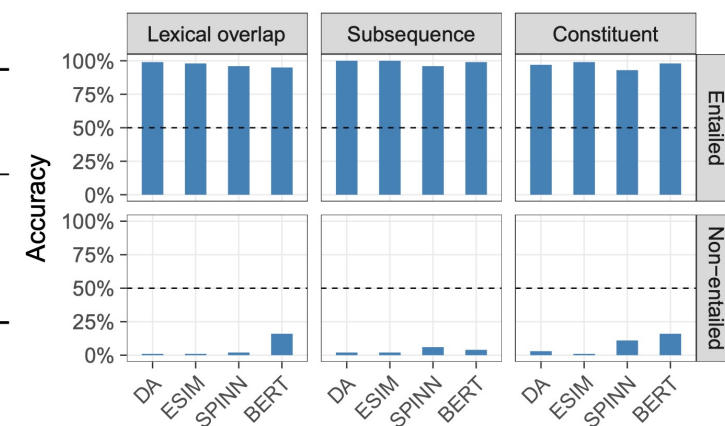


Two spurious features:  
color and patch

## MNLI-HANS

Heuristic	Supporting Cases	Contradicting Cases
Lexical overlap	2,158	261
Subsequence	1,274	72
Constituent	1,004	58

on MNLI, multiple syntactic  
features and  
the labels have spurious  
correlations.



ERM models fail on HANS

# Experimental Results

## Patched-Colored-MNIST(PC-MNIST)

Method	Penalty	ID		Oracle		TEV	
		Val	Test	Val	Test	Val	Test
ERM	-	90.22 $\pm$ 0.56	50.64 $\pm$ 0.56	89.95 $\pm$ 0.45	54.53 $\pm$ 0.60	-	-
EIL	IRM	90.21 $\pm$ 0.48	50.63 $\pm$ 0.45	78.01 $\pm$ 0.45	63.63 $\pm$ 0.71	69.81 $\pm$ 0.27	50.99 $\pm$ 0.58
	REx	90.24 $\pm$ 0.45	51.21 $\pm$ 0.64	79.10 $\pm$ 0.43	64.04 $\pm$ 0.80	70.05 $\pm$ 0.23	51.01 $\pm$ 0.68
	cMMD	90.24 $\pm$ 0.43	51.36 $\pm$ 0.61	77.27 $\pm$ 0.28	65.09 $\pm$ 0.63	70.15 $\pm$ 0.25	52.70 $\pm$ 1.40
	PGI	90.19 $\pm$ 0.46	51.07 $\pm$ 0.54	80.03 $\pm$ 1.41	64.27 $\pm$ 0.26	70.37 $\pm$ 0.14	50.64 $\pm$ 0.38
SCILL	IRM	79.65 $\pm$ 0.76	62.49 $\pm$ 0.55	71.54 $\pm$ 0.35	67.46 $\pm$ 0.19	71.54 $\pm$ 0.35	67.46 $\pm$ 0.19
	REx	80.23 $\pm$ 0.83	62.13 $\pm$ 0.99	72.59 $\pm$ 1.44	<b>67.60</b> $\pm$ 0.24	70.77 $\pm$ 0.50	67.33 $\pm$ 0.30
	cMMD	83.13 $\pm$ 0.93	59.76 $\pm$ 0.92	73.12 $\pm$ 0.47	67.49 $\pm$ 0.52	72.38 $\pm$ 0.51	<b>67.81</b> $\pm$ 0.34
	PGI	80.67 $\pm$ 1.75	<b>62.52</b> $\pm$ 0.32	71.73 $\pm$ 1.43	67.26 $\pm$ 0.14	71.35 $\pm$ 0.24	67.36 $\pm$ 0.33

Across 4 invariance penalties and 3 selection protocols, SCILL shows significant improvement

# Experimental Results

## MNLI-HANS

Method	Penalty	ID		Oracle		TEV	
		Val	Test	Val	Test	Val	Test
ERM	-	$84.12 \pm 0.15$	$64.88 \pm 3.00$	$84.12 \pm 0.15$	$64.88 \pm 3.00$	-	-
EIL	IRM	$84.01 \pm 0.08$	$65.35 \pm 0.93$	$83.82 \pm 0.17$	$66.42 \pm 0.98$	$84.01 \pm 0.08$	$65.35 \pm 0.93$
	REx	$84.10 \pm 0.13$	$65.16 \pm 0.19$	$83.91 \pm 0.20$	$66.87 \pm 2.92$	$84.00 \pm 0.48$	$66.43 \pm 1.00$
	cMMD	$83.56 \pm 0.03$	$63.22 \pm 1.76$	$83.22 \pm 0.13$	$64.25 \pm 1.63$	$83.38 \pm 0.20$	$62.72 \pm 2.03$
	PGI	$84.17 \pm 0.08$	$65.57 \pm 2.25$	$83.78 \pm 0.03$	$66.02 \pm 0.93$	$83.94 \pm 0.64$	$65.57 \pm 2.25$
SCILL	IRM	$82.75 \pm 0.17$	$69.11 \pm 1.76$	$82.56 \pm 0.33$	$68.72 \pm 1.24$	$82.67 \pm 0.14$	$69.82 \pm 1.29$
	REx	$82.68 \pm 0.28$	<b><math>69.73 \pm 1.63</math></b>	$82.59 \pm 0.22$	<b><math>71.20 \pm 1.81</math></b>	$82.56 \pm 0.33$	$69.75 \pm 1.53$
	cMMD	$82.74 \pm 0.26$	$69.15 \pm 1.39$	$82.39 \pm 0.45$	$70.77 \pm 1.40$	$82.61 \pm 0.04$	<b><math>70.92 \pm 0.79</math></b>
	PGI	$82.79 \pm 0.30$	$68.57 \pm 0.54$	$81.69 \pm 0.28$	$70.99 \pm 0.48$	$82.79 \pm 0.30$	$68.57 \pm 0.54$

Across 4 invariance penalties and 3 selection protocols, SCILL shows significant improvement

# Conclusion

- The first theoretical study on group invariant learning
- Two criteria for group invariant learning to survive spurious correlations
- Failures of existing methods on multivariate spurious features
- New method guided by the two criteria: SCILL

Code is available at:

<https://github.com/Beastlyprime/group-invariant-learning>

**Thanks for Your Attention !**