Byzantine-tolerant federated Gaussian process regression for streaming data

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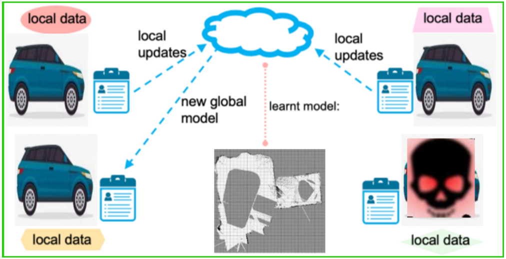
November 28, 2022

2022 Conference on Neural Information Processing Systems
New Orleans, LA

Problem formulation

Network model:

- Cloud can communicate with agents
- Agents cannot communicate with each other
- Byzantine agents send arbitrary model parameters to the cloud



Observation model:

$$y^{[i]}(t) = \eta(\boldsymbol{z}^{[i]}(t)) + e^{[i]}(t)$$

- Training data $(\boldsymbol{z}^{[i]}(t), y^{[i]}(t))$ arrive sequentially

Objective: Design a Byzantine-tolerant algorithm which

- Correctly learns the function η
- Does not require to share local streaming data $(z^{[i]}(t), y^{[i]}(t))$

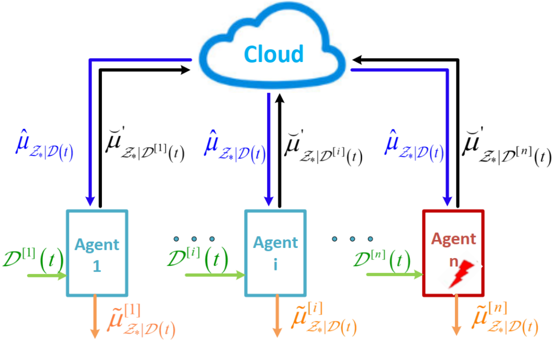
Byzantine-tolerant federated GPR

Contribution

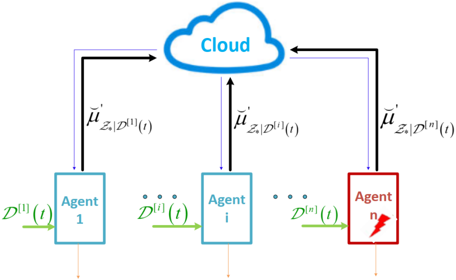
Design a Byzantine-tolerant federated Gaussian process regression (GPR) algorithm which

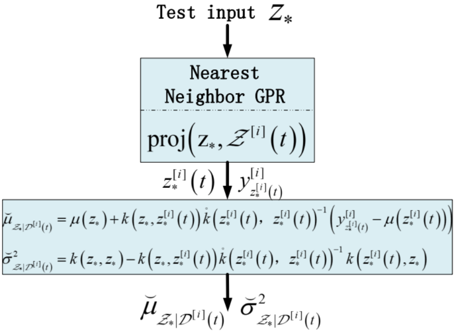
- Can guarantee the correct predictions and tolerate less than one quarter Byzantine agents
- Can deal with streaming data and perform on-line learning

- Agent-based local GPR
- Cloud-based aggregated GPR
- Agent-based fused GPR



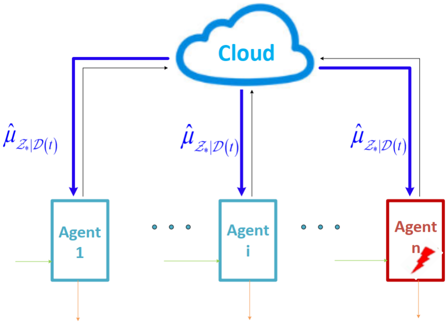
Agent-based local GPR

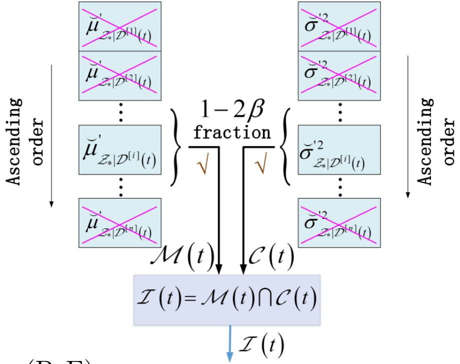




- $\breve{\mu}'_{\boldsymbol{z}_*|\mathcal{D}^{[i]}(t)} \leftarrow \begin{cases} \breve{\mu}_{\boldsymbol{z}_*|\mathcal{D}^{[i]}(t)} & \text{Benign agent,} \\ \star & \text{Byzantine agent} \end{cases}$
- $\breve{\sigma}'^2_{\boldsymbol{z}_*|\mathcal{D}^{[i]}(t)} \leftarrow \begin{cases} \breve{\sigma}^2_{\boldsymbol{z}_*|\mathcal{D}^{[i]}(t)} & \text{Benign agent,} \\ \star & \text{Byzantine agent} \end{cases}$

Cloud-based aggregated GPR



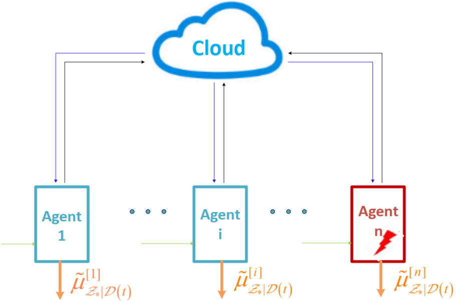


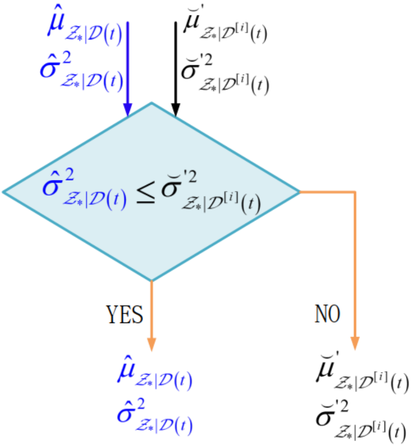
- Byzantine-tolerant product of experts (PoE):

$$\hat{\mu}_{\boldsymbol{z}_*|\mathcal{D}(t)} = \frac{\hat{\sigma}^2_{\boldsymbol{z}_*|\mathcal{D}(t)}}{|\mathcal{I}(t)|} \sum_{i \in \mathcal{I}(t)} \breve{\mu}'_{\boldsymbol{z}_*|\mathcal{D}^{[i]}(t)} \breve{\sigma}'^{-2}_{\boldsymbol{z}_*|\mathcal{D}^{[i]}(t)},$$

$$\hat{\sigma}^2_{\boldsymbol{z}_*|\mathcal{D}(t)} = \frac{|\mathcal{I}(t)|}{\sum_{i \in \mathcal{I}(t)} \breve{\sigma}'^{-2}_{\boldsymbol{z}_*|\mathcal{D}^{[i]}(t)}}.$$

Agent-based fused GPR





- Output: $\tilde{\mu}^{[i]}_{\boldsymbol{z}_*|\mathcal{D}(t)}$, $(\tilde{\sigma}^{[i]}_{\boldsymbol{z}_*|\mathcal{D}(t)})^2$

Robustness of cloud-based aggregated GPR

Assumption

Less than one quarter of the agents are Byzantine.

Dispersion: $d^{[i]}(t) \triangleq \sup_{\boldsymbol{z}\in\boldsymbol{\mathcal{Z}}} D(\boldsymbol{z}, \mathcal{Z}^{[i]}(t))$

Theorem (Cloud-based aggregated GPR: Mean)

For any $\boldsymbol{z}_* \in \boldsymbol{\mathcal{Z}}$ *and* $0 < \delta < 1$, *with probability at least* $1-\delta$, *it holds that*
$\left|\hat{\mu}_{\boldsymbol{z}_*|\mathcal{D}(t)} - \eta(\boldsymbol{z}_*)\right| \leq$
$(1 - \frac{\kappa(d^{\max}(t))}{\sigma_f^2+(\sigma_e^{\max})^2})\|\eta\|_\infty + \frac{\sigma_f^2\ell_\eta d^{\max}(t)}{\sigma_f^2+(\sigma_e^{\min})^2} + \sqrt{2\sigma^2(\ln 2 - \ln\delta)} + \Delta(d^{\max}(t))$ *where*

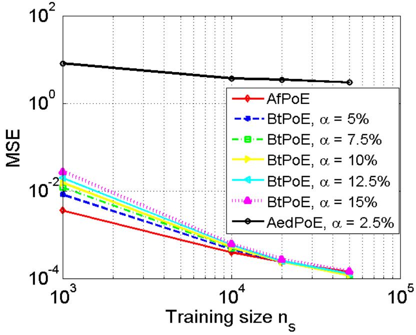
$$\Delta(s) \triangleq \frac{2\alpha(\sqrt{2\sigma^2(\ln(2n)-\ln\delta)}+\frac{\sigma_f^2\|\eta\|_\infty}{\sigma_f^2+(\sigma_e^{\min})^2})}{1-4\beta}\frac{\sigma_f^4+\sigma_f^2(\sigma_e^{\max})^2-\kappa(s)^2}{\sigma_f^2(\sigma_e^{\min})^2}.$$

Theorem (Cloud-based aggregated GPR: Variance)

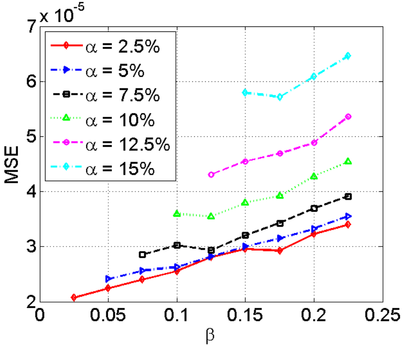
For any $\boldsymbol{z}_* \in \boldsymbol{\mathcal{Z}}$, *it holds that* $\frac{\sigma_f^2(\sigma_e^{\min})^2}{\sigma_f^2+(\sigma_e^{\max})^2} \leq \hat{\sigma}^2_{\boldsymbol{z}_*|\mathcal{D}(t)} \leq \sigma_f^2 - \frac{\kappa(d^{\max}(t))^2}{\sigma_f^2+(\sigma_e^{\max})^2}$.

Experiments (Synthetic dataset)

Experiment 1: Prediction performance in terms of consistency and different α, β



(a) Consistency evaluation



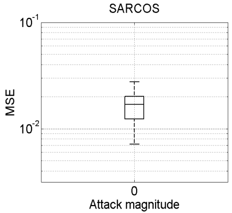
(b) Prediction performance on different β

Experiment 2: Prediction performance on different functions

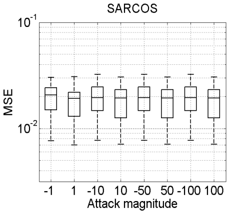
Algorithm	AfPoE	BtPoE	AedPoE
MSE ($\times 10^{-3}$)	0.0049 ± 0.007	0.0236 ± 0.172	26.5339 ± 0.019

Experiments (Real-world datasets)

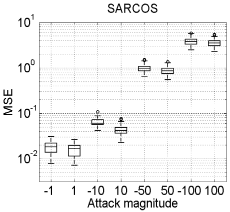
Experiment 3: Performance on different attack magnitudes



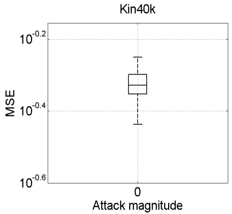
(c) Attack-free standard PoE



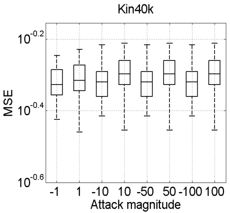
(d) Byzantine-tolerant PoE



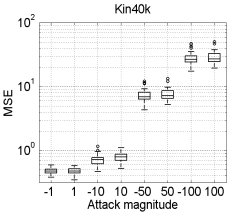
(e) Attacked standard PoE



(f) Attack-free standard PoE



(g) Byzantine-tolerant PoE



(h) Attacked standard PoE

Conclusion

- Design a Byzantine-tolerant federated GPR algorithm
- Derive the upper bounds on the prediction errors and the lower and upper bounds of the predictive variances
- Demonstrate the robustness of Byzantine-tolerant GPR algorithm through experiments

Thank you