Byzantine-tolerant federated Gaussian process regression for streaming data

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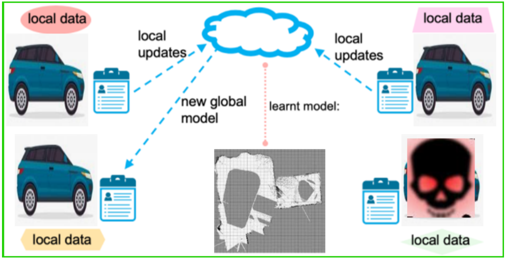
Problem formulation

Network model:

- Cloud can communicate with agents
- Agents cannot communicate with each other
- Byzantine agents send arbitrary model parameters to the cloud



Observation model:

$$y^{[i]}(t) = \quad (z^{[i]}(t)) + e^{[i]}(t)$$

- Training data $(z^{[i]}(t); y^{[i]}(t))$ arrive sequentially

Objective: Design a Byzantine-tolerant algorithm which

- Correctly learns the function
- Does not require to share local streaming data $(z^{[i]}(t); y^{[i]}(t))$

Byzantine-tolerant federated GPR

Contribution

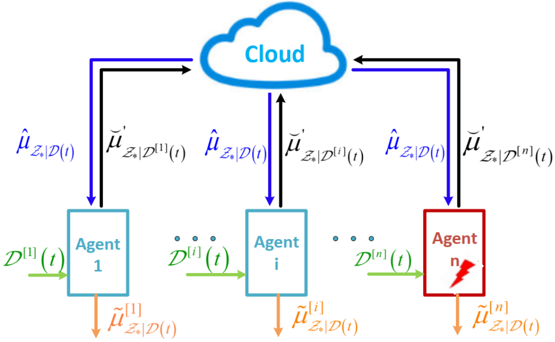
Design a Byzantine-tolerant federated Gaussian process regression (GPR) algorithm which

- Can guarantee the correct predictions and tolerate less than one quarter Byzantine agents
- Can deal with streaming data and perform on-line learning

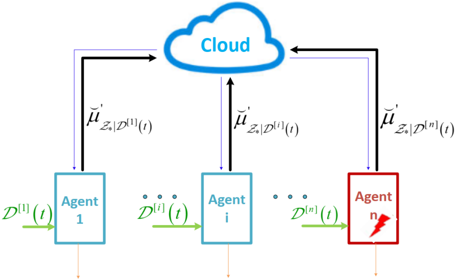
Agent-based local GPR

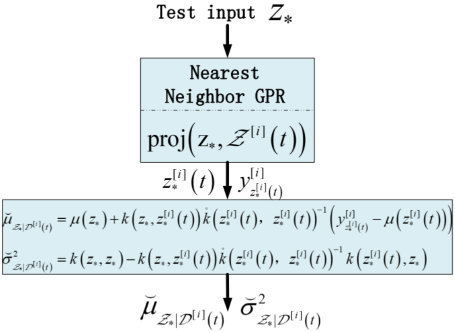
Cloud-based aggregated GPR

Agent-based fused GPR



Agent-based local GPR

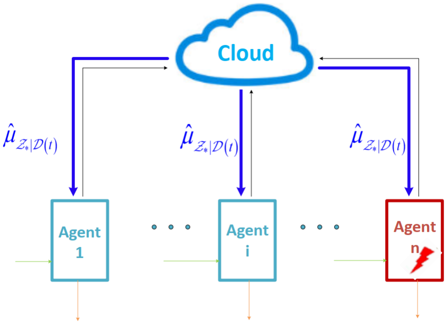


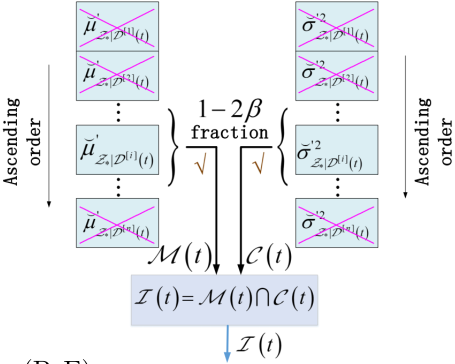


$\breve{\mu}'_{z|D^{[i]}(t)}$ $\breve{\mu}_{z|D^{[i]}(t)}$ Benign agent;
? Byzantine agent

$\breve{\sigma}'^2_{z|D^{[i]}(t)}$ $\breve{\sigma}^2_{z|D^{[i]}(t)}$ Benign agent;
? Byzantine agent

Cloud-based aggregated GPR



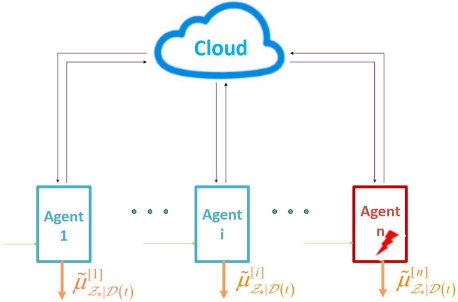


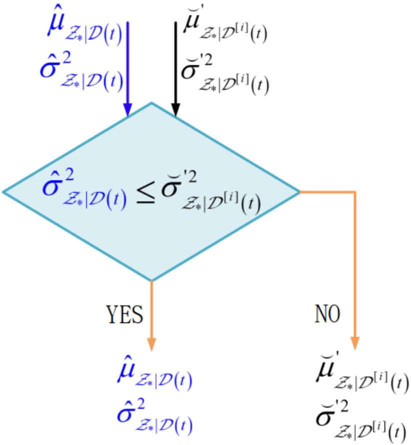
Byzantine-tolerant product of experts (PoE):

$$\hat{\mu}_{z|\mathcal{D}(t)} = \frac{\hat{\sigma}^2_{z|\mathcal{D}(t)}}{|\mathcal{I}(t)|}\sum_{i\in\mathcal{I}(t)} \breve{\mu}'_{z|\mathcal{D}^{[i]}(t)}\,\breve{\sigma}'^{-2}_{z|\mathcal{D}^{[i]}(t)},$$

$$\hat{\sigma}^2_{z|\mathcal{D}(t)} = \frac{|\mathcal{I}(t)|}{\sum_{i\in\mathcal{I}(t)} \breve{\sigma}'^{-2}_{z|\mathcal{D}^{[i]}(t)}}.$$

Agent-based fused GPR





Output: $\tilde{}^{[i]}_{z\ jD(t)}, (\tilde{}^{[i]}_{z\ jD(t)})^2$

Robustness of cloud-based aggregated GPR

Assumption

Less than one quarter of the agents are Byzantine.

Dispersion: $d^{[i]}(t) \triangleq \sup_{z \, 2 \, \mathcal{Z}} D(z; Z^{[i]}(t))$

Theorem (Cloud-based aggregated GPR: Mean)

For any $z \;\; 2 \, \mathcal{Z}$ *and* $0 < \quad < 1$*, with probability at least* $1 \quad$ *, it holds that*

$$\hat{\ }_{z \; jD(t)} \qquad (z \;\;)$$

$$(1 \quad \frac{(d^{\max}(t))}{{}^{2}_{f} + (\ {}^{\max}_{e})^{2}}) k \;\; k_{1} + \frac{{}^{2}_{f}\ \grave{}\ \ d^{\max}(t)}{{}^{2}_{f} + (\ {}^{\min}_{e})^{2}} + \mathrm{p}\overline{2 \;\; {}^{2}(\ln 2 \quad \ln \quad)} + \Delta(d^{\max}(t))$$ *where*

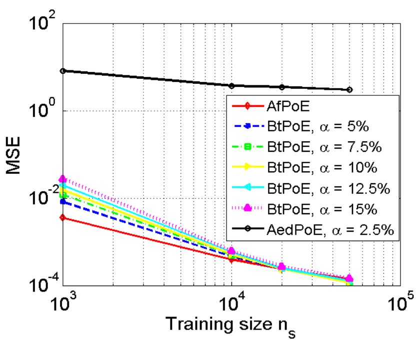
$$\Delta(s) \triangleq \frac{2 \;\; (\ \mathrm{p}\overline{2 \;\; {}^{2}(\ln(2n) \quad \ln \quad)} + \frac{{}^{2}_{f} k \;\; k_{1}}{{}^{2}_{f} + (\ {}^{\min}_{e})^{2}})}{1 \quad 4} \; \frac{{}^{4}_{f} + {}^{2}_{f}(\ {}^{\max}_{e})^{2} \qquad (s)^{2}}{{}^{2}_{f}(\ {}^{\min}_{e})^{2}}.$$

Theorem (Cloud-based aggregated GPR: Variance)

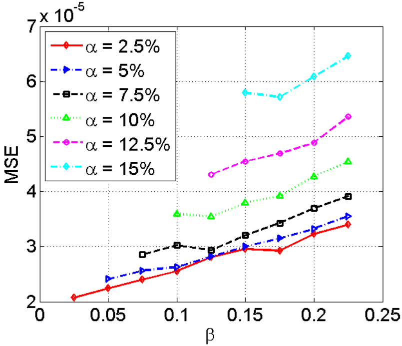
For any $z \;\; 2 \, \mathcal{Z}$*, it holds that* $\frac{{}^{2}_{f}(\ {}^{\min}_{e})^{2}}{{}^{2}_{f} + (\ {}^{\max}_{e})^{2}} \qquad \hat{\ }^{2}_{z \; jD(t)} \qquad {}^{2}_{f} \qquad \frac{(d^{\max}(t))^{2}}{{}^{2}_{f} + (\ {}^{\max}_{e})^{2}}.$

Experiments (Synthetic dataset)

Experiment 1: Prediction performance in terms of consistency and different ,



(a) Consistency evaluation



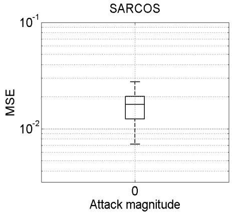
(b) Prediction performance on di erent β

Experiment 2: Prediction performance on different functions

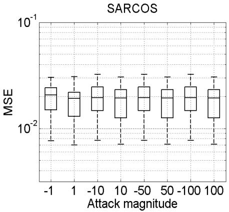
Algorithm	AfPoE	BtPoE	AedPoE
MSE (10 ^{3})	0:0049 0:007	0:0236 0:172	26:5339 0:019

Experiments (Real-world datasets)

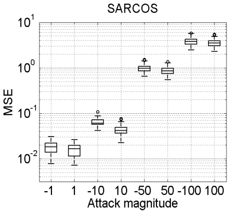
Experiment 3: Performance on different attack magnitudes



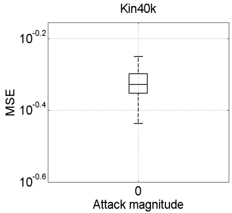
(c) Attack-free standard PoE



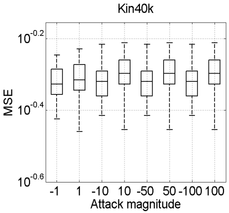
(d) Byzantine-tolerant PoE



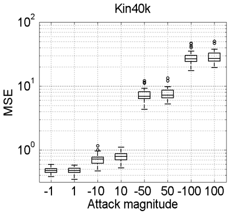
(e) Attacked standard PoE



(f) Attack-free standard PoE



(g) Byzantine-tolerant PoE



(h) Attacked standard PoE