DOCTOR

A Simple Method for Detecting Misclassification Errors

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Main Goal of this Work

Deep Neural Networks (DNNs) exhibit unwanted behaviors as they tend to be overconfident even in presence of wrong decisions.



DOCTOR is a simple method to detect whether a model's prediction is likely to be correct (accept), or not (reject).



Detection Framework



Detection Framework



* P_X be the (unknown) probability distribution over X (feature space);



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- * $P_{\widehat{Y}|X}$ be the posterior probability given a sample (model distribution).



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- ★ f_{D_n} : $X \to Y$ be the predictor.



Optimal (Oracle) Detector

For a given $x \in \mathcal{X}$,

★ $E(x) \triangleq \mathbb{1}[Y \neq f_{D_n}(x)]$ denotes the **true-error variable**;



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* The probability of misclassification w.r.t. $P_{Y|X}$ is given by

$$\underline{Pe(\mathbf{x})} \triangleq \mathbb{E}[E(\mathbf{x})|\mathbf{x}] = 1 - P_{Y|X}(f_{\mathcal{D}_n}(\mathbf{x})|\mathbf{x})$$



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Optimal (Oracle) Detector



Unfortunately, in practice $P_{Y|X}$ is not available: we need to find a way to approximate Pe(x).

DOCTOR: $D_{\alpha}(\mathbf{x}, \gamma)$

For a given $x \in \mathcal{X}$,

$$1 - \hat{g}(\mathsf{x}) \triangleq \sum_{y \in \mathcal{Y}} P_{\widehat{Y}|X}(y|\mathsf{x}) \mathbb{P}(\widehat{Y} \neq y|\mathsf{x}) = 1 - \sum_{y \in \mathcal{Y}} P_{\widehat{Y}|X}^2(y|\mathsf{x})$$

is the probability of incorrectly classifying x if it was randomly labeled according to $P_{\widehat{Y}|X}.$



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For a given $x \in \mathcal{X}$,

The self-error variable is defined as:

 $\widehat{E}(\mathsf{x}) \triangleq \mathbb{1}[\widehat{Y} \neq f_{\mathcal{D}_n}(\mathsf{x})];$



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* The probability of error classification w.r.t. $P_{\widehat{Y}|X}$ is given by

$$\widehat{Pe}(\mathsf{x}) \triangleq \mathbb{E}[\widehat{E}(\mathsf{x})|\mathsf{x}] = 1 - P_{\widehat{Y}|\mathsf{X}}(f_{D_n}(\mathsf{x})|\mathsf{x}).$$
Accept 0 /
Reject 1
$$-\frac{P_{\widehat{Y}|\mathsf{X}}(f_{D_n}(\mathsf{x})|\mathsf{x})}{\mathbb{E}[\widehat{Pe}(\mathsf{x}) > \gamma \cdot (1 - \widehat{Pe}(\mathsf{x}))]}$$

FRR versus TRR

The false rejection rate (FRR) represents the probability that a hit (sample correctly classified) is rejected, while the true rejection rate (TRR) is the probability that a miss (sample wrongly classified) is rejected.



FRR versus TRR

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AUROC

The area under the Receiver Operating Characteristic curve (ROC) depicts the relationship between TRR and FRR. The perfect detector corresponds to a score of 100%.

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The area under the Receiver Operating Characteristic curve (ROC) depicts the relationship between TRR and FRR. The perfect detector corresponds to a score of 100%.

FRR at 95% TRR

This is the probability that a hit is rejected when the TRR is at 95%.

Totally Black Box (TBB) Scenario

In TBB only the output of the last layer of the network is available, hence gradient-propagation to perform input pre-processing is not allowed.

Partially Black Box (PBB) Scenario

In PBB we allow method-specific inputs perturbations and the possibility of doing temperature scaling.

1) ODIN [Liang et al., 2018]

It comprises temperature scaling and input pre-processing via perturbation. It compares the maximum softmax probability with a threshold $\delta \in [0, 1]$ to decide whether to accept or to reject the input sample.

2) Mahalanobis distance for OOD (MHLNB) [Lee et al., 2018] It consists in calculating the distance between the input sample and training distribution. It compares the distance with a threshold $\zeta \in \mathbb{R}$ to decide whether to accept or to reject the input sample.

3) Energy Score (ENERGY) [Liu et al., 2020] It comprises the denominator of the softmax activation and it compares it with a threshold $\xi \in \mathbb{R}$.

1) Softmax Response

(SR) [Hendrycks and Gimpel, 2017, Geifman and El-Yaniv, 2017] ODIN without temperature scaling and input pre-processing.

2) Mahalanobis distance for OOD (MHLNB) [Lee et al., 2018] Mahalanobis distance without input pre-processing and with the softmax output in place of the logits.

















TinyImageNet





DATASET	METHOD	AUROC % FRR % (95 % TRR)		DATASET	METHOD	AUROC %		FRR % (95 % TRR)			
DATASET		TBB	PBB	TBB	PBB	DATASET	METHOD	TBB	PBB	TBB	PBB
	D _c	94	95.2	17.9	13.9		D	92.3	93	38.6	36.6
CIFAR10 Acc. 95%	Dj	68.5	94.8	18.6	13.4		Dj	92.2	92.8	39.7	38.4
	ODIN	93.8	94.2	18.2	18.4	Acc. 96%	ODIN	92.3	92.3	38.6	40.7
	SR	93.8	-	18.2	-		SR	92.3	-	38.6	-
	MHLNB	92.2	84.4	30.8	44.6		MHLNB	87.3	88	85.8	54.7
	ENERGY	-	91.1	-	34.7		ENERGY	-	88.9	-	49.4
	D _c	87	88.2	40.6	35.7	Amazon	Do	89.7	-	27.1	-
	Dj	84.2	87.4	40.6	36.7	Fashion Acc. 85%	D_{β}	89.7	-	26.3	-
CIFAR100	ODIN	86.9	87.1	40.5	40.7		SR	87.4	-	50.1	
Acc. 78%	SR	86.9	-	40.5	-		ENERGY	-	-	-	-
	MHLNB	82.6	50	66.7	94		D _o	68.8	-	73.2	-
	ENERGY	-	78.7	-	65.4	Amazon	Dj	68.8	-	73.2	-
	D _c	84.9	86.1	45.8	43.3	Acc. 73%	SR	67.3	-	86.6	-
Tiny	Dj	84.9	85.3	45.8	45.1		ENERGY	-	-	-	-
ImageNet	ODIN	84.9	84.9	45.8	<u>45.3</u>	-	D	84.4	-	54.2	-
Acc. 63%	SR	84.9	-	45.8	-	IMDb	Di	84.4	-	54.4	-
	MHLNB	78.4	59	82.3	86	Acc. 90%	SR	83.7	-	61.7	-
	ENERGY	-	78.2	-	63.7		ENERGY	-	-	-	-

	METHOD	AUROC % FRR % (95 % TRR)			DATASET	METHOD	AUROC %		FRR % (95 % TRR)		
DATASET	METHOD	TBB	PBB	TBB	PBB	DATASET	WETHOD	TBB	PBB	TBB	PBB
CIFAR10 Acc. 95%	De	94	95.2	17.9	13.9		D_c	92.3	93	38.6	36.6
	D_{β}	68.5	94.8	18.6	13.4		D_j	92.2	92.8	39.7	38.4
	ODIN	93.8	94.2	18.2	18.4	SVHN	ODIN	92.3	92.3	38.6	40.7
	SR	93.8	-	18.2	-	ACC: 9070	SR	92.3	-	38.6	-
	MHLNB	92.2	84.4	30.8	44.6		MHLNB	87.3	88	85.8	54.7
	ENERGY	-	91.1	-	34.7		ENERGY	-	88.9	-	49.4
	Do	87	88.2	40.6	35.7	Amazon Fashion Acc. 85%	De	89.7	-	27.1	-
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CIFAR100	ODIN	86.9	87.1	40.5	40.7		SR	87.4	-	50.1	
Acc. 78%	SR	86.9	-	40.5	-		ENERGY	-	-	-	
	MHLNB	82.6	50	66.7	94		De	68.8	-	73.2	-
	ENERGY	-	78.7	-		Amazon	Dj	68.8	-	73.2	-
	D_{c}	84.9	86.1	45.8	43.3	Acc. 73%	SR	67.3	-	86.6	-
Tiny	Dj	84.9	85.3	45.8	45.1		ENERGY	-	-	-	-
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Acc. 63%	SR	84.9	-	45.8	-		Di	84.4	-	54.4	-
	MHLNB	78.4	59	82.3	86	Acc. 90%	SR	83.7	-	61.7	-
	ENERGY	-	78.2	-	63.7		ENERGY	-	-	-	-

Overall Results: TBB & PBB

♦

DATASET	METHOD	AUROC % FRR % (95 % TRR)		DATASET	METHOD	AUROC %		FRR % (95 % TRR)			
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	D	94	95.2	17.9	13.9		D_c	92.3	93	38.6	36.6
	Dj	68.5	94.8	18.6	13.4		D_{β}	92.2	92.8	39.7	38.4
	ODIN	93.8	94.2	18.2	18.4		ODIN	92.3	92.3	38.6	40.7
	SR	93.8	-	18.2	-		SR	92.3	-	38.6	-
	MHLNB	92.2	84.4	30.8	44.6		MHLNB	87.3	88	85.8	54.7
	ENERGY	-	91.1	-	34.7		ENERGY	-	88.9	-	49.4
	D_c	87	88.2	40.6	35.7		De	89.7	-	27.1	-
	Di	84.2	87.4	40.6	36.7		D_{β}	89.7	-	26.3	-
	ODIN	86.9	87.1	40.5	40.7		SR	87.4	-	50.1	
	SR	86.9	-	40.5	-		ENERGY	-	-	-	-
	MHLNB	82.6	50	66.7	94		D.	68.8	-	73.2	-
	ENERGY	-	78.7	-	65.4		Dj	68.8	-	73.2	-
	D_c	84.9	86.1	45.8	43.3		SR	67.3	-	86.6	-
	Dj	84.9	85.3	45.8	45.1		ENERGY	-	-	-	-
	ODIN	84.9	84.9	45.8	45.3		D.	84.4	-	54.2	-
	SR	84.9	-	45.8	-		Di	84.4	-	54.4	-
	MHLNB	78.4	59	82.3	86		SR	83.7	-	61.7	-
	ENERGY	-	78.2	-	63.7		ENERGY	-	-	-	-

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	D_{c}	94	95.2	17.9	13.9	SVHN Acc. 96%	Do	92.3	93	38.6	36.6
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	ODIN	93.8	94.2	18.2	18.4		ODIN	92.3	92.3	38.6	40.7
	SR	93.8	-	18.2	-		SR	92.3	-	38.6	-
	MHLNB	92.2	84.4	30.8	44.6		MHLNB	87.3	88	85.8	54.7
	ENERGY	-	91.1	-	34.7		ENERGY	-	88.9	-	49.4
	D_{c}	87	88.2	40.6	35.7		De	89.7	-	27.1	-
	Da	84.2	87.4	40.6	36.7		Dj	89.7	-	26.3	-
	ODIN	86.9	87.1	40.5	40.7		SR	87.4	-	50.1	
	SR	86.9	-	40.5	-		ENERGY	-	-	-	-
	MHLNB	82.6	50	66.7	94		D.	68.8	- 1	73.2	-
	ENERGY	-	78.7	-	65.4		Dj	68.8	-	73.2	-
	D_c	84.9	86.1	45.8	43.3		SR	67.3	-	86.6	-
	Dj	84.9	85.3	45.8	45.1		ENERGY	-	-	-	-
	ODIN	84.9	84.9	45.8	45.3		D.	84.4	-	54.2	-
	SR	84.9	-	45.8	-		Di	84.4	-	54.4	_
	MHLNB	78.4	59	82.3	86		SR	83.7	-	61.7	-
	ENERGY	-	78.2	-	63.7		ENERGY	-	-	-	-

Overall Results: TBB & PBB

We observe a reduction of up 4% of FRR in the PBB scenario.

	METHOD	AUR	AUROC %		6 (95 % TRR)			1		1	
DATASET	METHOD	твв	PBB	TBB	PBB	DATASET	METHOD	AUR	DC %	FRR 9	6 (95 % TRR)
		I						TBB	PBB	TBB	PBB
	D _c	94	95.2	17.9	13.9						\square
CIFAR10	D_{β}	68.5	94.8	18.6	(13.4)		D _e	92.3	93	38.6	(36.6)
Acc. 95%	ODIN	93.8	94.2	18.2	18.4	SVHN	D_{β}	92.2	92.8	39.7	38.4
	SR	93.8		18.2	-	Acc. 96%	ODIN	92.3	92.3	38.6	<u>40.7</u>
	MHINB	92.2	84.4	30.8	44.6		SR	92.3	-	38.6	-
	ENERGY		91.1	-	34.7		MHLNB	87.3	88	85.8	54.7
	Literioi		51.1		0		ENERGY	-	88.9	-	49.4
CIEAR100	D	87	88.2	40.6	(35.7)	Amazon Fashion Acc. 85%	D	89.7	-	27.1	
	Dj	84.2	87.4	40.6	36.7		Dj	89.7	-	26.3	-
Acc. 78%	ODIN	86.9	87.1	40.5	40.7		SR	87.4	-	50.1	
/ 10/0	SR	86.9	-	40.5	-		ENERGY	-	-	-	-
	MHLNB	82.6	50	66.7	94		D	68.8	-	73.2	-
	ENERGY	-	78.7	-	65.4	Amazon	Dj	68.8	-	73.2	-
				45.0		Acc. 73%	SR	67.3	-	86.6	-
	D _e	84.9	86.1	45.8	43.3		ENERGY	-	-	-	-
Tiny	Dj	84.9	85.3	45.8	45.1		D	84.4		54.2	
ImageNet	ODIN	84.9	84.9	45.8	<u>45.3</u>	IMDb	D.	84.4	-	54.4	
Acc. 63%	SR	84.9	-	45.8	-	Acc. 90%	SP	83.7		61.7	
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	ENERGY	-	78.2	-	63.7	·	ENERGY	-	-	-	-

Misclassification Detection in Presence of Out-Of-Distribution (OOD) Samples

- DOCTOR is not tuned for OOD detection (differently from ODIN).
- ★ We test ODIN and DOCTOR when one sample to reject out of five (♣), three (♦), or two (♠) is OOD.

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- DOCTOR is not tuned for OOD detection (differently from ODIN).
- We test ODIN and DOCTOR when one sample to reject out of five (♣), three (◊), or two (♠) is OOD.

DATASET-	DATASET-		AUR			FRR % (95 % TRR)					
In	Out	D_a	D_{β}	ODIN	ENERGY	D_a	D_{β}	ODIN	ENERGY		
CIFAR10	iSUN	95.4 / 0.1	95.1 / 0.1	94.6 / 0.1	92.4 / 0	14 / 0.5	13.5 / 0.4	17.2 / 0.3	32.2 / 0.1		
	Tiny (res)	95.2 / 0.1	94.9 / 0	94.6 / 0.1	92.3 / 0.1	14 / 0.4	14 / 0.5	17.8 / 0.4	32.2 / 0.1		
CIFAR10	iSUN	95.5 / 0.1	95.3 / 0.1	94.9 / 0.1	92.9 / 0	14.4 / 0.6	13.4 / 0.2	16.8 / 0.5	27 / 1		
\diamond	Tiny (res)	95.4 / 0.1	95 / 0.1	94.8 / 0.1	92.8 / 0	15 / 0.1	14.8 / 0.7	17 / 0.5	28.8 / 1.9		
CIFAR10	iSUN	95.6 / 0.1	95.6 / 0	95.4 / 0	93.6 / 0.1	15.1 / 0.1	13.6 / 0.5	16.1 / 0.2	25.1 / 0.2		
	Tiny (res)	95.5 / 0.1	95.2 / 0.1	95.1 / 0.1	93.5 / 0	14.7 / 0.3	14.8 / 0.5	17.1 / 0.4	25.6 / 0.3		

Table 2. Results in terms of mean / standard deviation.

Misclassification Detection in Presence of Out-Of-Distribution (OOD) Samples

DOCTOR is not tuned for OOD detection (differently from ODIN).

★ We test ODIN and DOCTOR when one sample to reject out of five (♣), three (◊), or two (♠) is OOD.

DATASET-	DATASET-		AUR	DC %	FRR % (95 % TRR)				
In	Out	D_a	Dj	ODIN	ENERGY	D_a	D_{j}	ODIN	ENERGY
CIFAR10	iSUN	95.4 / 0.1	95.1 / 0.1	94.6 / 0.1	92.4 / 0	14 / 0.5	13.5 / 0.4	17.2 / 0.3	32.2 / 0.1
*	Tiny (res)	95.2 / 0.1	94.9 / 0	94.6 / 0.1	92.3 / 0.1	14 / 0.4	14 / 0.5	17.8 / 0.4	32.2 / 0.1
CIFAR10	iSUN	95.5 / 0.1	95.3 / 0.1	94.9 / 0.1	92.9 / 0	14.4 / 0.6	13.4 / 0.2	16.8 / 0.5	27 / 1
\diamond	Tiny (res)	95.4 / 0.1	95 / 0.1	94.8 / 0.1	92.8 / 0	15 / 0.1	14.8 / 0.7	17 / 0.5	28.8 / 1.9
CIFAR10	iSUN	95.6 / 0.1	95.6 / 0	95.4 / 0	93.6 / 0.1	15.1 / 0.1	13.6 / 0.5	16.1 / 0.2	25.1 / 0.2
•	Tiny (res)	95.5 / 0.1	95.2 / 0.1	95.1 / 0.1	93.5 / 0	14.7 / 0.3	14.8 / 0.5	17.1 / 0.4	25.6 / 0.3

Table 2. Results in terms of mean / standard deviation.

Thanks for your attention. See you soon at the poster session.



References i

- Geifman, Y. and El-Yaniv, R. (2017).

Selective classification for deep neural networks.

In Guyon, I., von Luxburg, U., Bengio, S., Wallach, H. M., Fergus, R., Vishwanathan, S. V. N., and Garnett, R., editors, <u>Advances in Neural</u> <u>Information Processing Systems 30: Annual Conference on Neural</u> <u>Information Processing Systems 2017, December 4-9, 2017, Long</u> <u>Beach, CA, USA, pages 4878–4887.</u>

- Hendrycks, D. and Gimpel, K. (2017).

A baseline for detecting misclassified and out-of-distribution examples in neural networks.

In 5th International Conference on Learning Representations, ICLR 2017, Toulon, France, April 24-26, 2017, Conference Track Proceedings.

References ii

Lee, K., Lee, K., Lee, H., and Shin, J. (2018). A simple unified framework for detecting out-of-distribution samples and adversarial attacks.

In Advances in Neural Information Processing Systems 31: Annual Conference on Neural Information Processing Systems 2018, NeurIPS 2018, December 3-8, 2018, Montréal, Canada, pages 7167–7177.

Liang, S., Li, Y., and Srikant, R. (2018).

Enhancing the reliability of out-of-distribution image detection in neural networks.

In <u>6th</u> International Conference on Learning Representations, ICLR 2018, Vancouver, BC, Canada, April 30 - May 3, 2018, Conference Track Proceedings.

Liu, W., Wang, X., Owens, J. D., and Li, Y. (2020). **Energy-based out-of-distribution detection.** In Larochelle, H., Ranzato, M., Hadsell, R., Balcan, M., and Lin, H., editors, <u>Advances in Neural Information Processing Systems 33</u>: <u>Annual Conference on Neural Information Processing Systems 2020</u>, NeurIPS 2020, December 6-12, 2020, virtual.

Supplementary: Optimal (Oracle) Discriminator

- ★ $E \triangleq \mathbb{1}[Y \neq f_{D_n}(X)]$ denotes the error variable corresponding to f_{D_n}
- ★ $x \in X$ and $y \in Y$ drawn from the unknown distribution p_{XY}
- ★ $p_{XY}(x, y) \equiv P_E(1)p_{XY|E}(x, y|1) + P_E(0)p_{XY|E}(x, y|0)$
- ★ $p_X(x) \equiv P_E(1)p_{X|E}(x|1) + P_E(0)p_{X|E}(x|0)$
- Pe(x) ≜ E[E(x)|x] = 1 P_{Y|X}(f_{D_n}(x)|x) is the probability of error classification w.r.t. P_{Y|X}

$$\begin{split} D(\mathbf{x}, \gamma) &= \mathbb{1}[p_{X|E}(\mathbf{x}|1) > \gamma \cdot p_{X|E}(\mathbf{x}|0)] \\ &= \mathbb{1}[P_{E|X}(1|\mathbf{x})P_E(0) > \gamma \cdot (1 - P_{E|X}(1|\mathbf{x}))P_E(1)] \\ &= \mathbb{1}[\operatorname{Pe}(\mathbf{x})P_E(0) > \gamma \cdot (1 - \operatorname{Pe}(\mathbf{x}))P_E(1)] \\ &= \mathbb{1}[\operatorname{Pe}(\mathbf{x}) > \gamma' \cdot (1 - \operatorname{Pe}(\mathbf{x}))], \end{split}$$
where $\gamma' = \frac{P_E(1)}{P_E(0)}.$