Excess Capacity and Backdoor Poisoning

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TTI Chicago

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1 Introduction

2 Backdoor Setting

3 Memorization Capacity

4 Main Results

Image: A matched black





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Figure: Poisoned data

Adversary's goal – Cause \hat{h} to make a mistake on new data with the patch/trigger added.

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• "Roughly balanced" binary classification problem – i.e., $\Pr_{(x,y)\sim\mathcal{D}}[y=+1]\in[1/50, \frac{49}{50}].$

• Assume the learner is using ERM on the 0-1 loss.

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Definition (Patch Functions (See Definition 1))

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- We denote classes of patch functions using the notation $\mathcal{F}_{adv}(\mathcal{X})$, and classes of consistent patch functions using the notation $\mathcal{F}_{adv}(\mathcal{X}, h^*)$.
- \bullet Convention $\mathcal{F}_{\mathsf{adv}}$ always contains the identity function.

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Question

Can we quantify the properties present in a learning problem that lends itself to such a memorization? (This section)

Suppose we are in a setting where we are learning a hypothesis class \mathcal{H} over a domain \mathcal{X} under distribution \mathcal{D} .

We say we can *memorize* k *irrelevant* subsets from a family $C \subseteq 2^{\mathcal{X}}$ atop a fixed h if we can find k nonempty sets $X_1, \ldots, X_k \in C$ satisfying $\mu_{\mathcal{D}}(X_i) = 0$ for all $i \in [k]$ such that for all $b \in \{\pm 1\}^k$, there exists a classifier $\hat{h} \in \mathcal{H}$ satisfying:

• For all $x \in X_i$, we have $\widehat{h}(x) = b_i$.

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We say we can *memorize* k *irrelevant* subsets from a family $C \subseteq 2^{\mathcal{X}}$ atop a fixed h if we can find k nonempty sets $X_1, \ldots, X_k \in C$ satisfying $\mu_{\mathcal{D}}(X_i) = 0$ for all $i \in [k]$ such that for all $b \in \{\pm 1\}^k$, there exists a classifier $\hat{h} \in \mathcal{H}$ satisfying:

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We define $\operatorname{mcap}_{\mathcal{X},\mathcal{D}}(h,\mathcal{C})$ to be the maximum number of sets from \mathcal{C} we can memorize for a fixed *h*. We omit the argument \mathcal{C} when \mathcal{C} is the set of all measurable subsets of \mathcal{X} . Finally, we define $\operatorname{mcap}_{\mathcal{X},\mathcal{D}}(\mathcal{H}) \coloneqq \sup_{h \in \mathcal{H}} \operatorname{mcap}_{\mathcal{X},\mathcal{D}}(h)$.

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We have:

$$\Pr_{x \sim \mathcal{D}}\left[\widehat{h}(x) = h^*(x)\right] = 1$$

and:

$$\Pr_{x \sim \mathcal{D}} \left[\widehat{h}(\mathsf{patch}\,(x)) = t \right] = 1$$

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Theorem (Informal Restatement of Theorems 9 and 10)

Memorization capacity with respect to images of valid attacks dictates the number of backdoor attacks simultaneously possible in a learning problem.

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Upshot – argue about robustness of learning problems via memorization capacity (See Section 2.3.1).

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$$h^*(x) = \operatorname{sign}(\langle w, x \rangle)$$
 where $||w|| \le 1/\gamma$

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$$\mathcal{H} = \{h(x) : h(x) = \operatorname{sign}(\langle w, x \rangle), w \in \mathbb{R}^d\}$$

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$$\mathcal{X} = \mathbb{R}^d$$
 and $\text{Supp}(\mathcal{D}) = \left\{ x : x = Ay, y \in \mathbb{R}^{d-k} \right\}$

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Then, $\mathsf{mcap}_{\mathcal{X},\mathcal{D}}\left(h^*,\mathcal{C}(\mathcal{F}_{\mathsf{adv}})
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Image: A matrix and a matrix

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Theorem (Informal Restatement of Theorem 14)

If it is possible to (agnostically) learn an adversarially robust classifier on a clean dataset, then there exists an algorithm that can announce whether a training set is corrupted by backdoor examples.

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If it is possible to (agnostically) learn an adversarially robust classifier on a clean dataset, then there exists an algorithm that can announce whether a training set is corrupted by backdoor examples.

Use case – Training algorithm can announce when data is contaminated, and this can prompt manual intervention. See Section 3.1.1 for numerical trials.

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TL;DR – robust generalization and filtering are roughly statistically equivalent.

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TL;DR – robust generalization and filtering are roughly statistically equivalent.Both reductions assume black-box access to the robust loss and an algorithm to minimize the robust loss on an arbitrary dataset.

Conclusion

We have:

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- Identified memorization capacity as a parameter that characterizes vulnerability to backdoor data poisoning attacks.
- Given a high-level algorithm for detecting training set contamination, under several assumptions.
- Under similar assumptions, shown that backdoor filtering and robust generalization are nearly equivalent.

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- For what problems do there exist backdoor-robust learning algorithms?
- Thank you!