

Friendship paradox

You have more friends than I do
(at least on average)



A



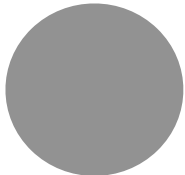
B

A rectangular box containing a gray circular placeholder on the left and three horizontal lines on the right for text input.A rectangular box containing a gray circular placeholder on the left and three horizontal lines on the right for text input.A rectangular box containing a gray circular placeholder on the left and three horizontal lines on the right for text input.A rectangular box containing a gray circular placeholder on the left and three horizontal lines on the right for text input.

In which friend list am I
likely to appear?

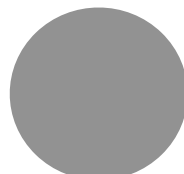
A

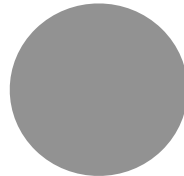




B









**3 times more likely
to appear in the
B's friend list**

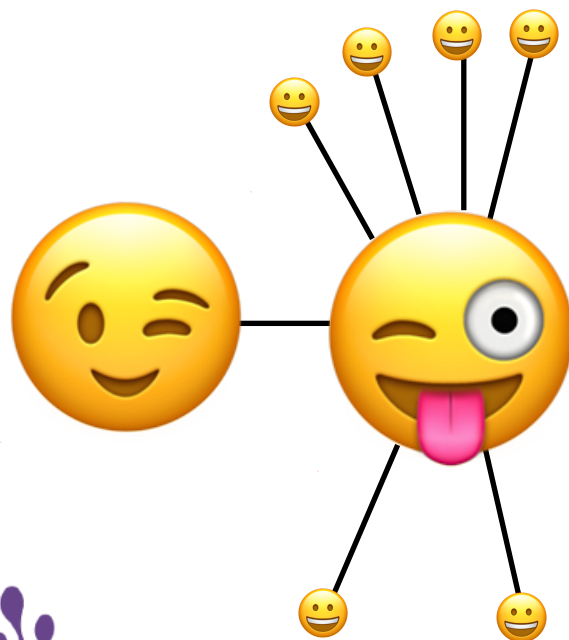
Friendship paradox

Your friends have more friends than you do
(on average)

...because more friends someone has,
more likely the someone appears in your friend list.



Can introduce *biases* in graph embeddings



Residual2Vec: Debiasing graph embedding using random graphs



arXiv.org

<https://arxiv.org/abs/2110.07654>



 **Sadamori Kojaku**



Jisung Yoon

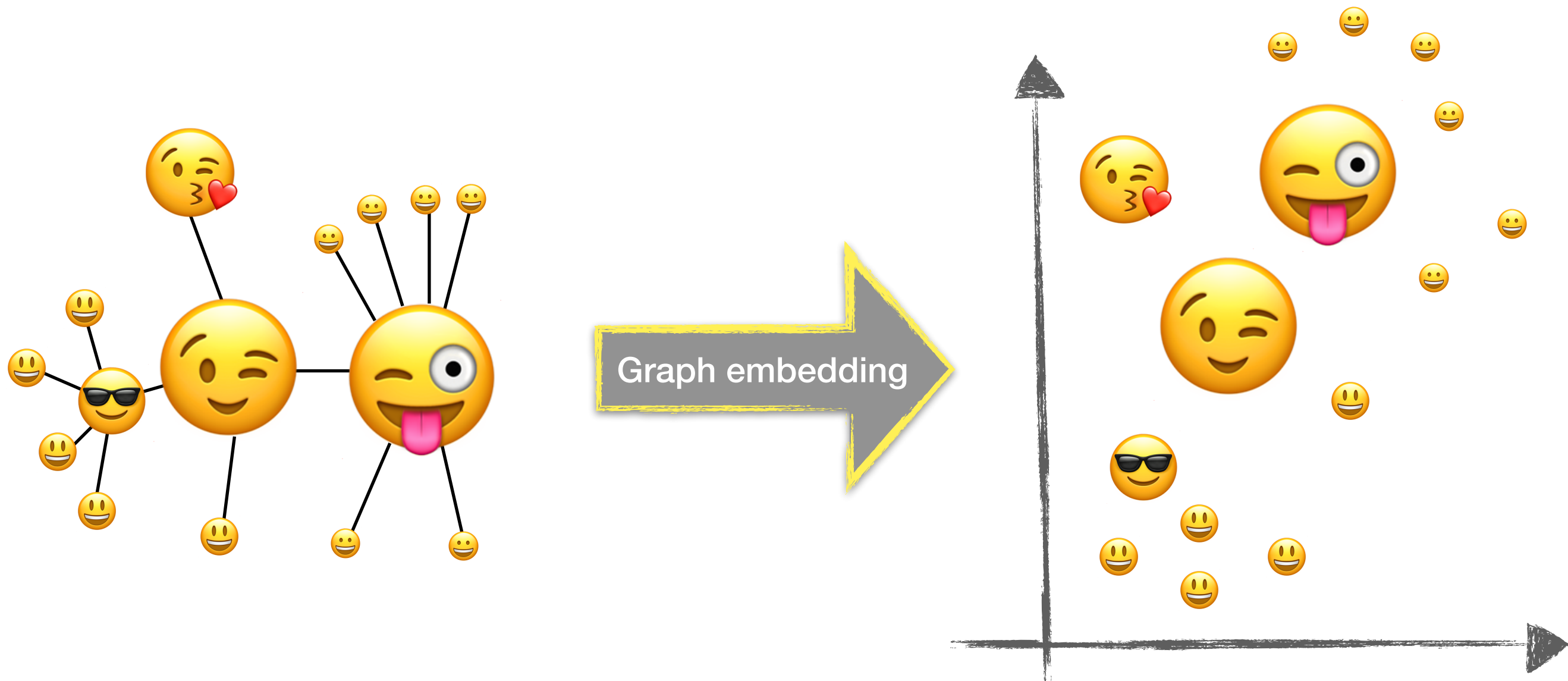


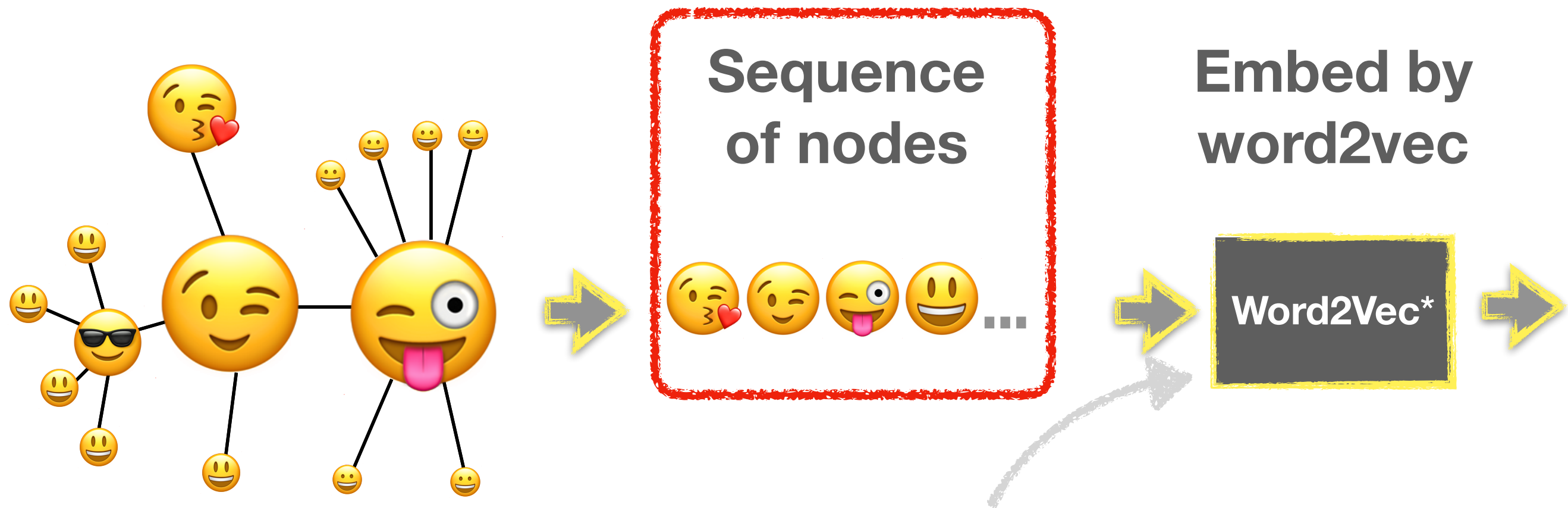
Isabel Constantino



Yong-Yeol Ahn

Graph embedding

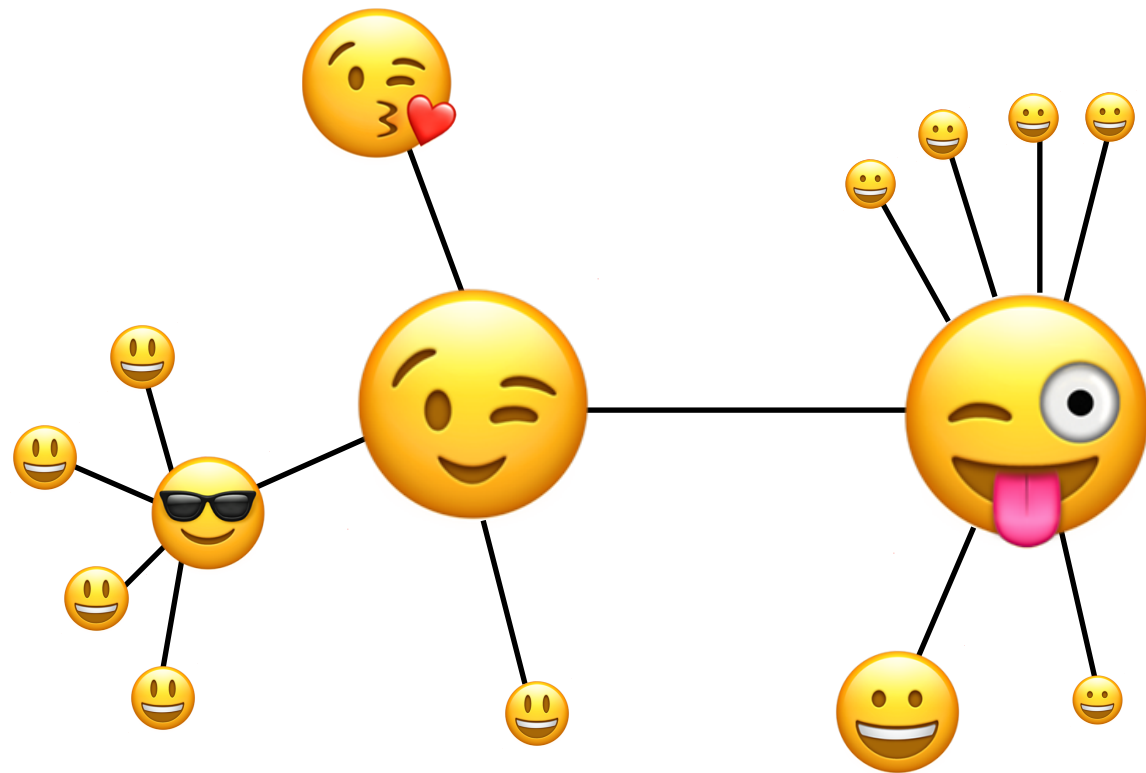




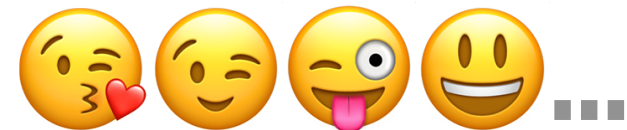
Word embedding

signal.
 nary code with which the present
 ls may take various forms, all of
 e property that the symbol (or
 representing each number (or sign
 differs from the ones representi
 er and the next higher number (o
 litude) in only one digit (or puls
 Because this code in its primar
 built up from the conventional
 a sort of reflection process and l
 rms may in turn be built up fro
 form in similar fashion, the c
 , which has as yet no recognized
 ated in this specification and
 s the "reflected binary code."
 a receiver station, reflected binar

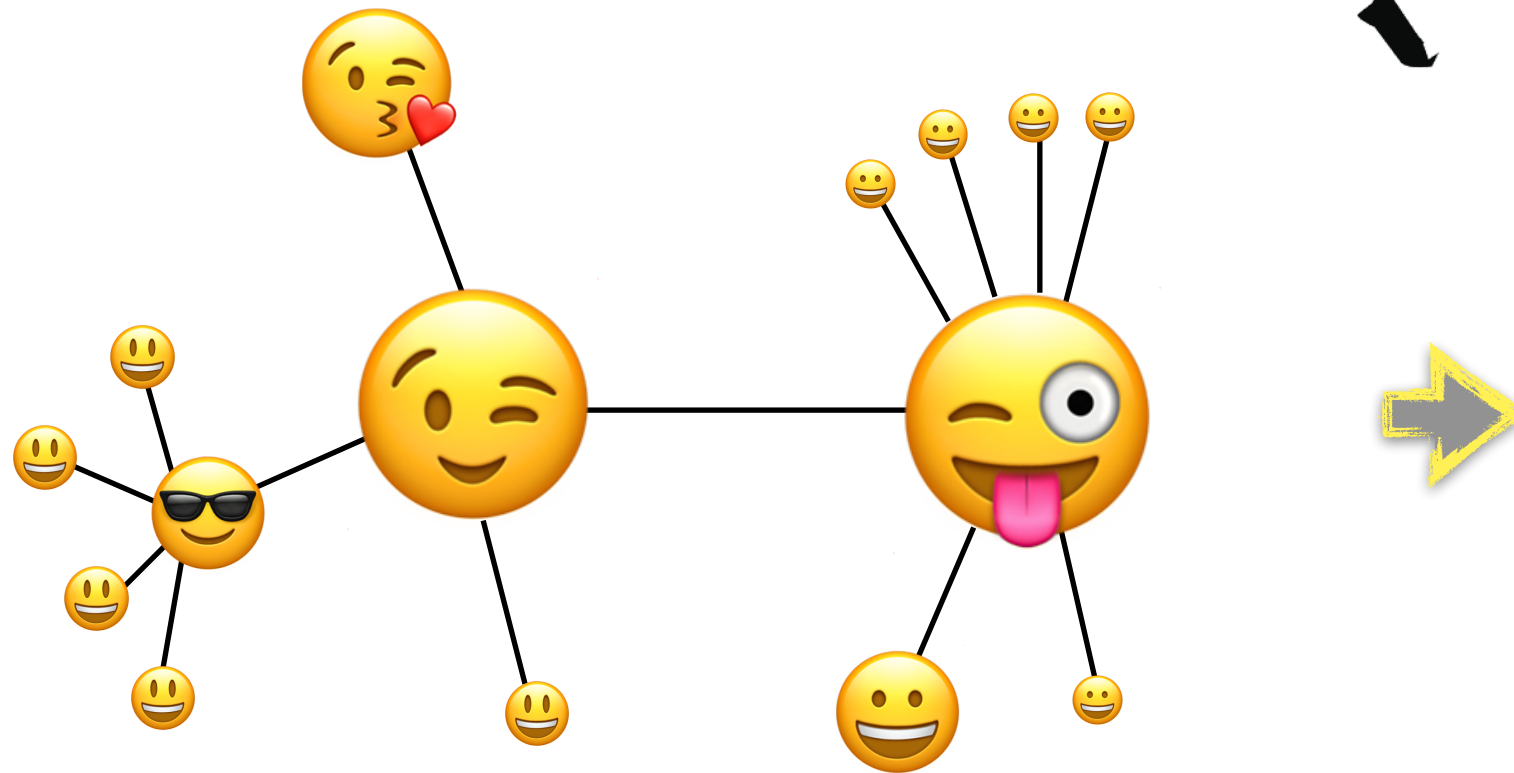
How to generate the sequence from a graph?

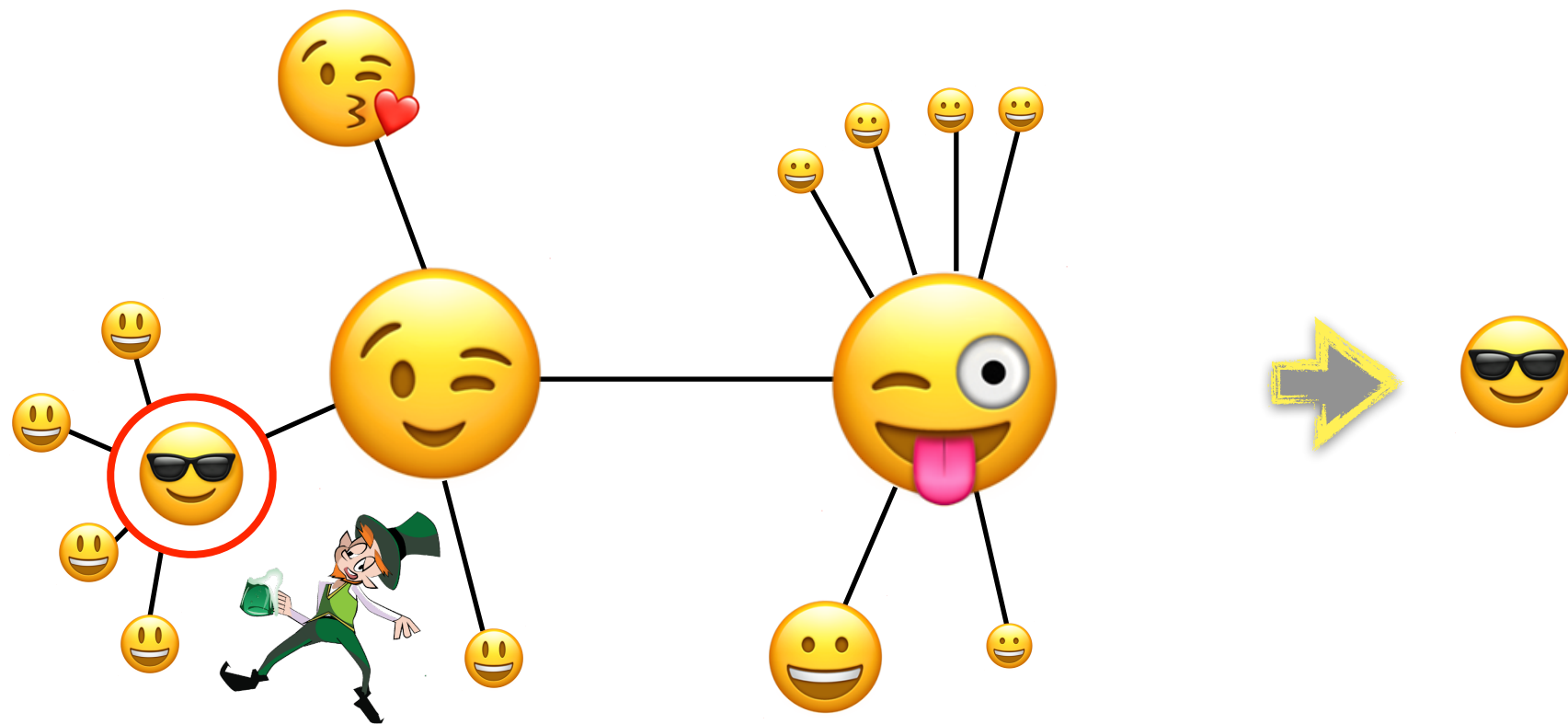


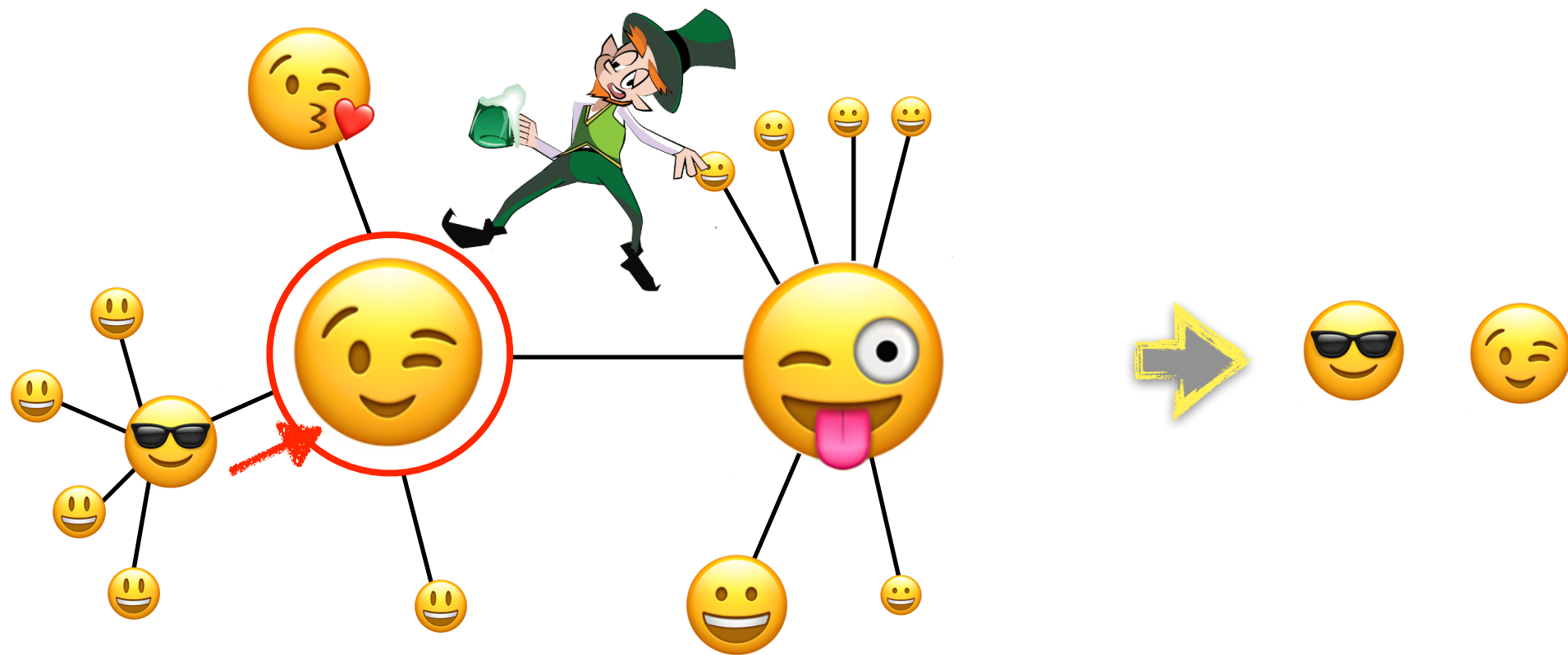
**Sequence
of nodes**

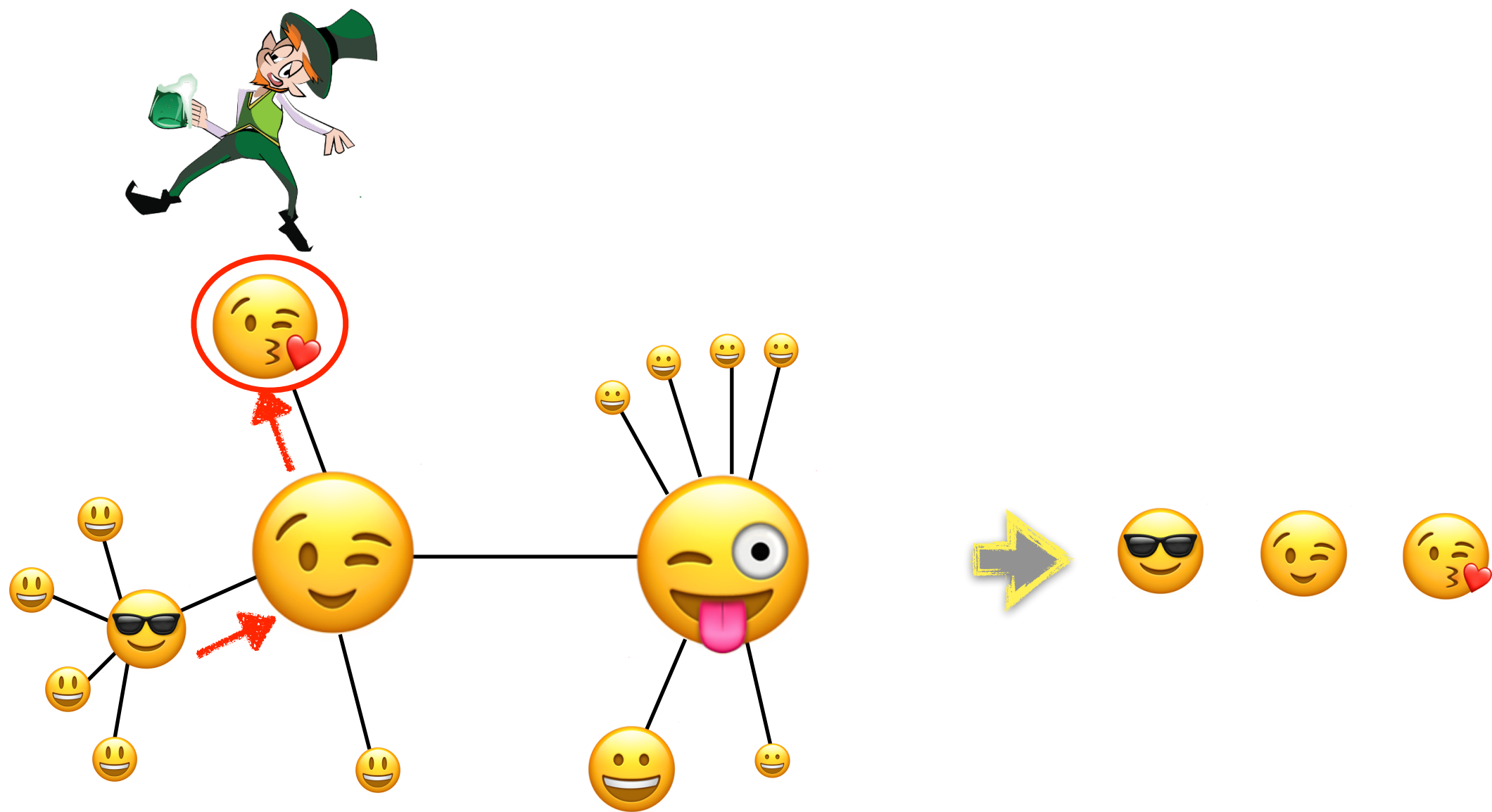


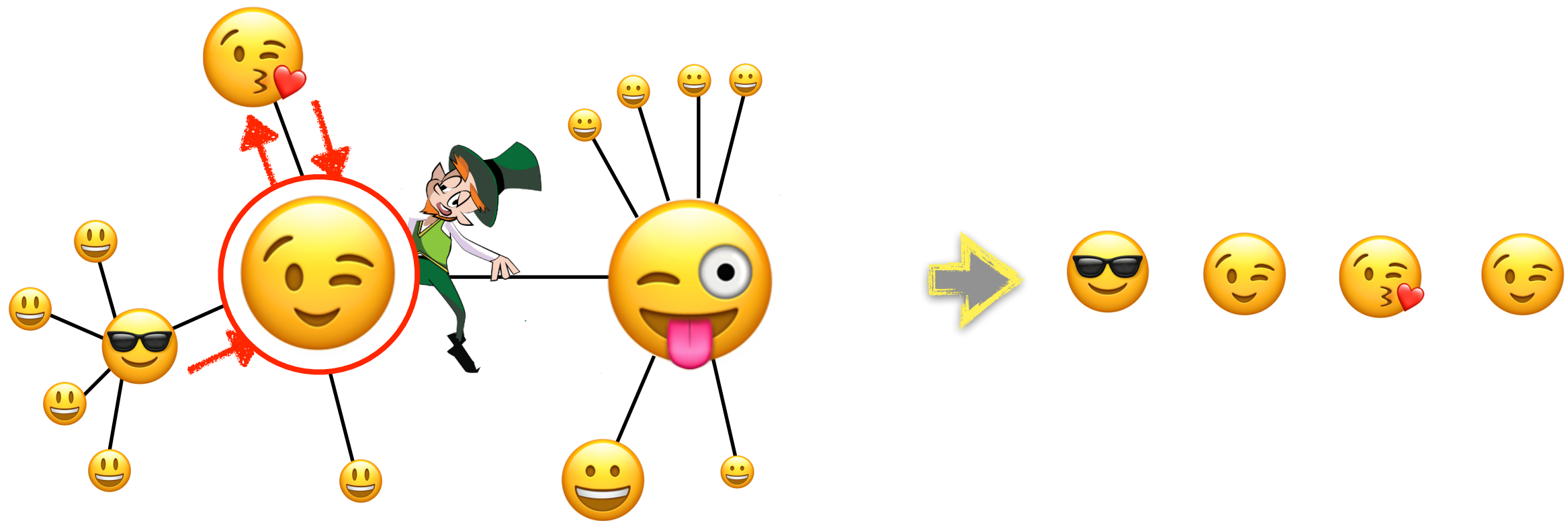
Random walker

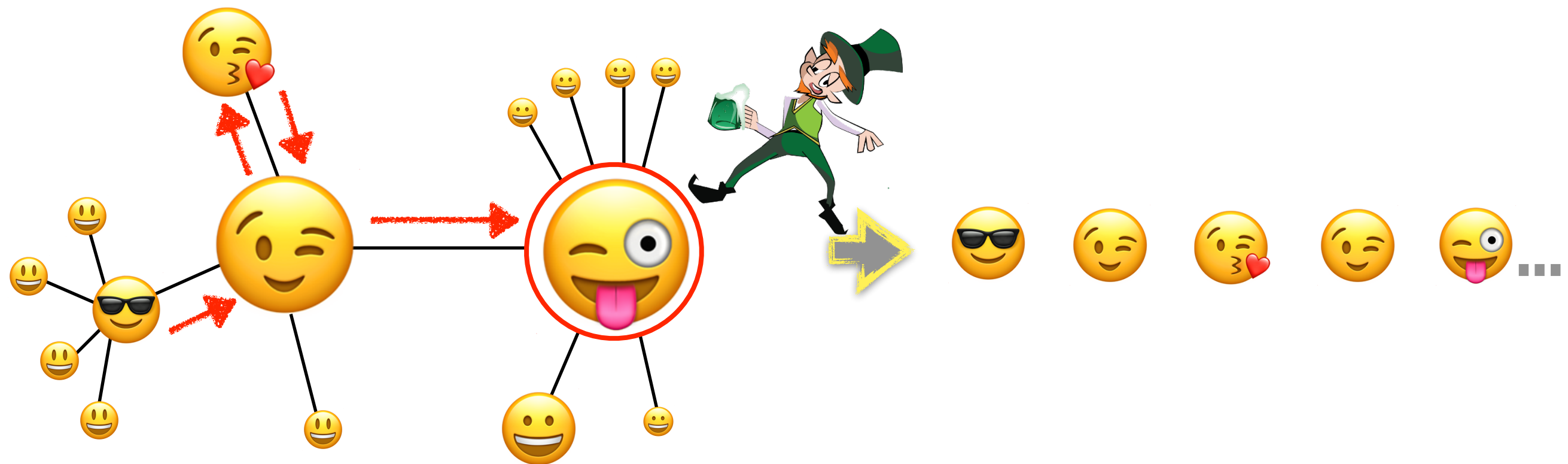




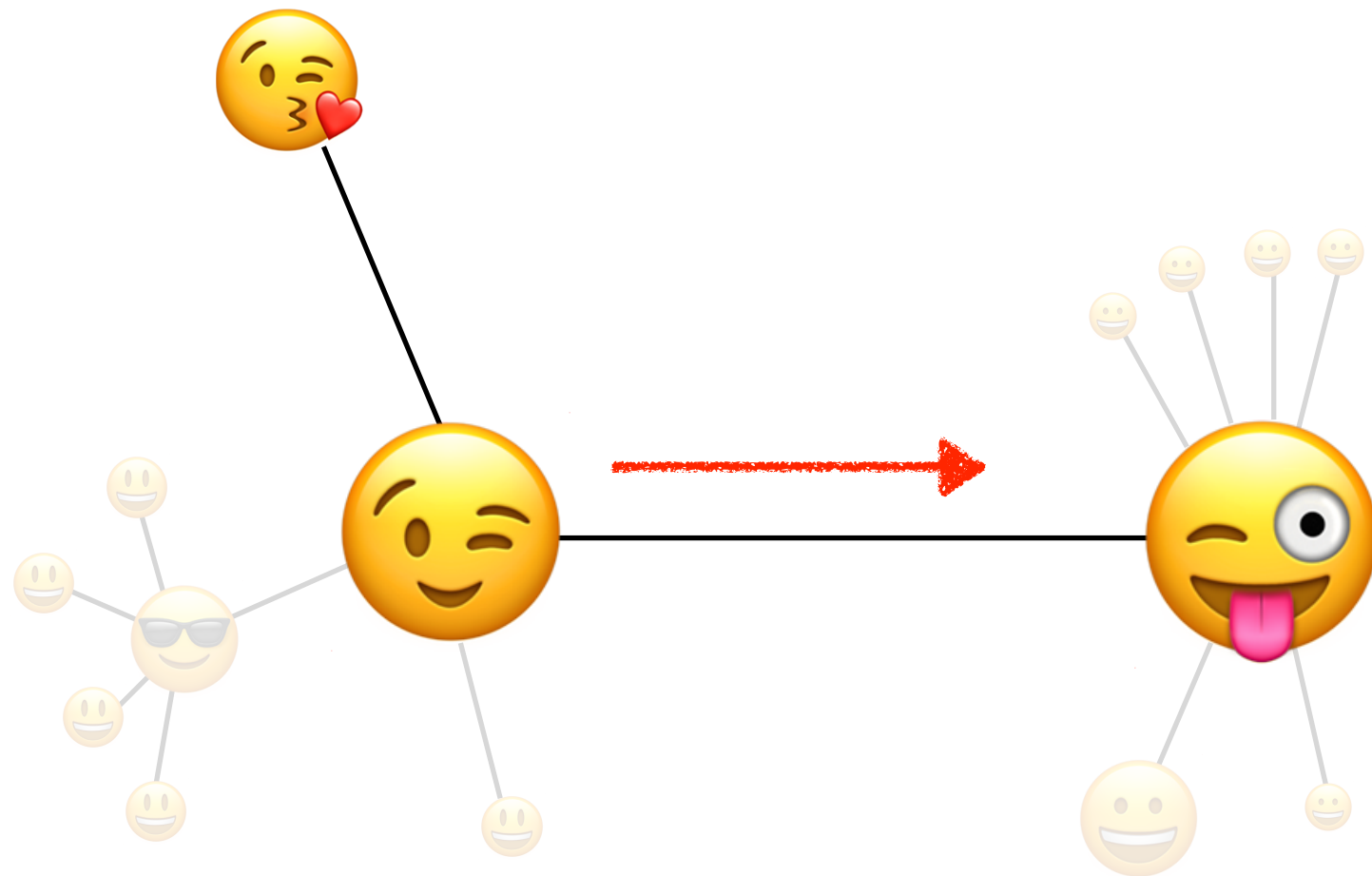








Friendship paradox

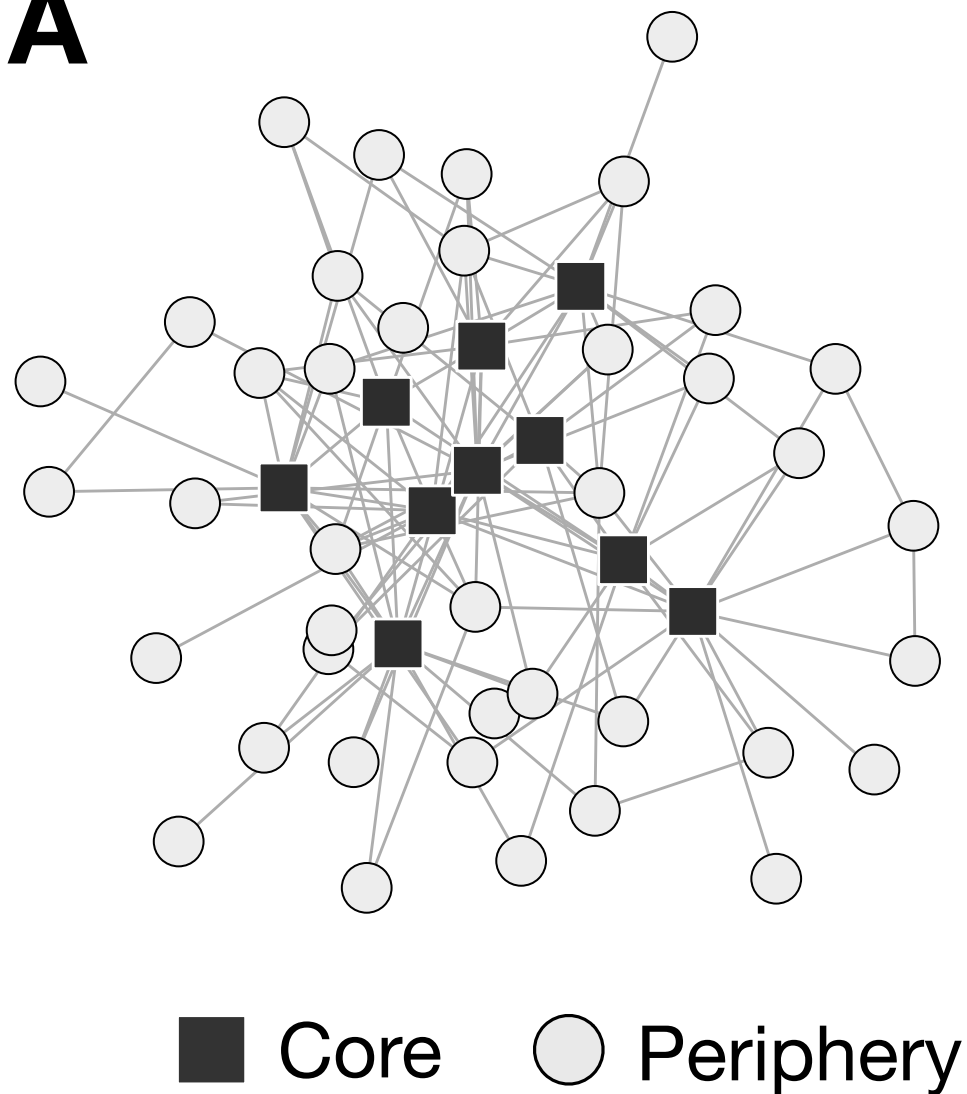


The walker is likely to visit *a node with many friends*.

Following edges is a *biased* sampling that preferentially leads the walker to nodes with many neighbors.

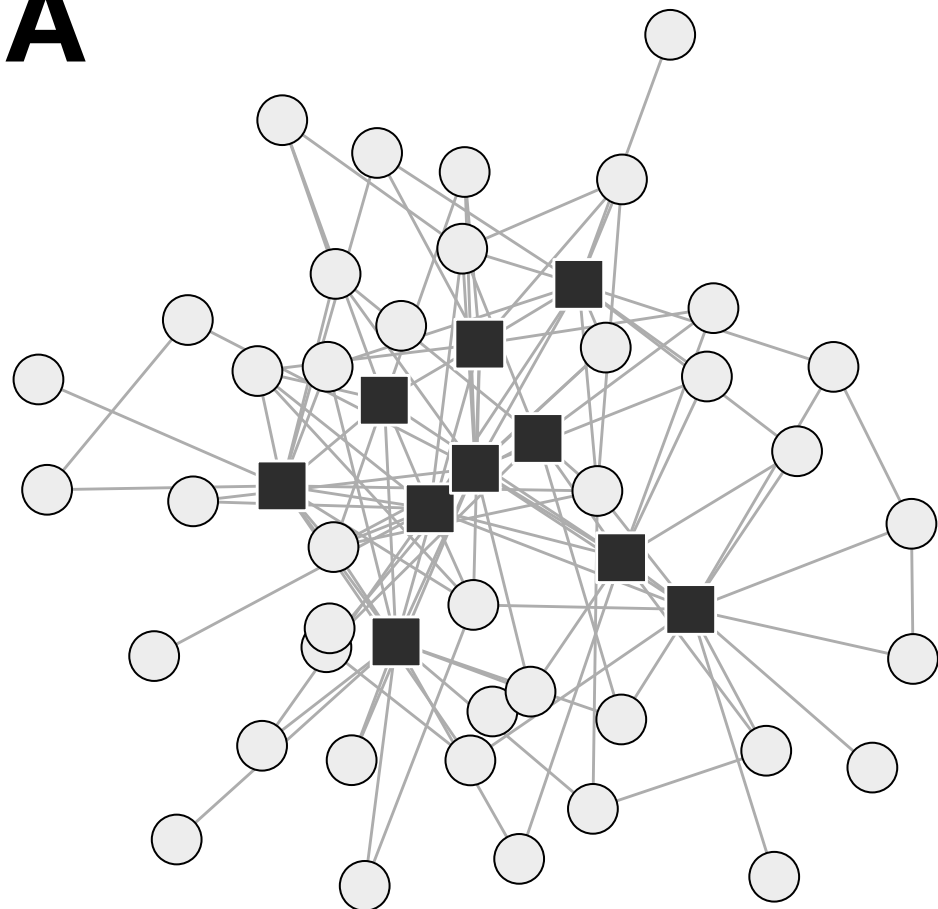
Toy example

A



Toy example

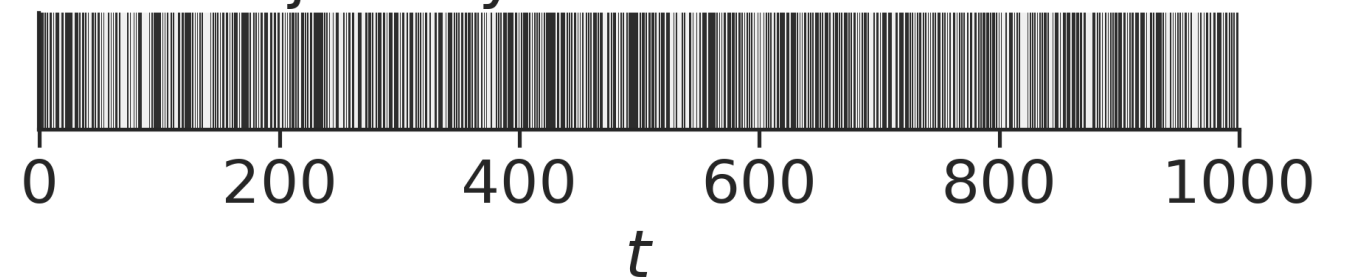
A



■ Core ○ Periphery

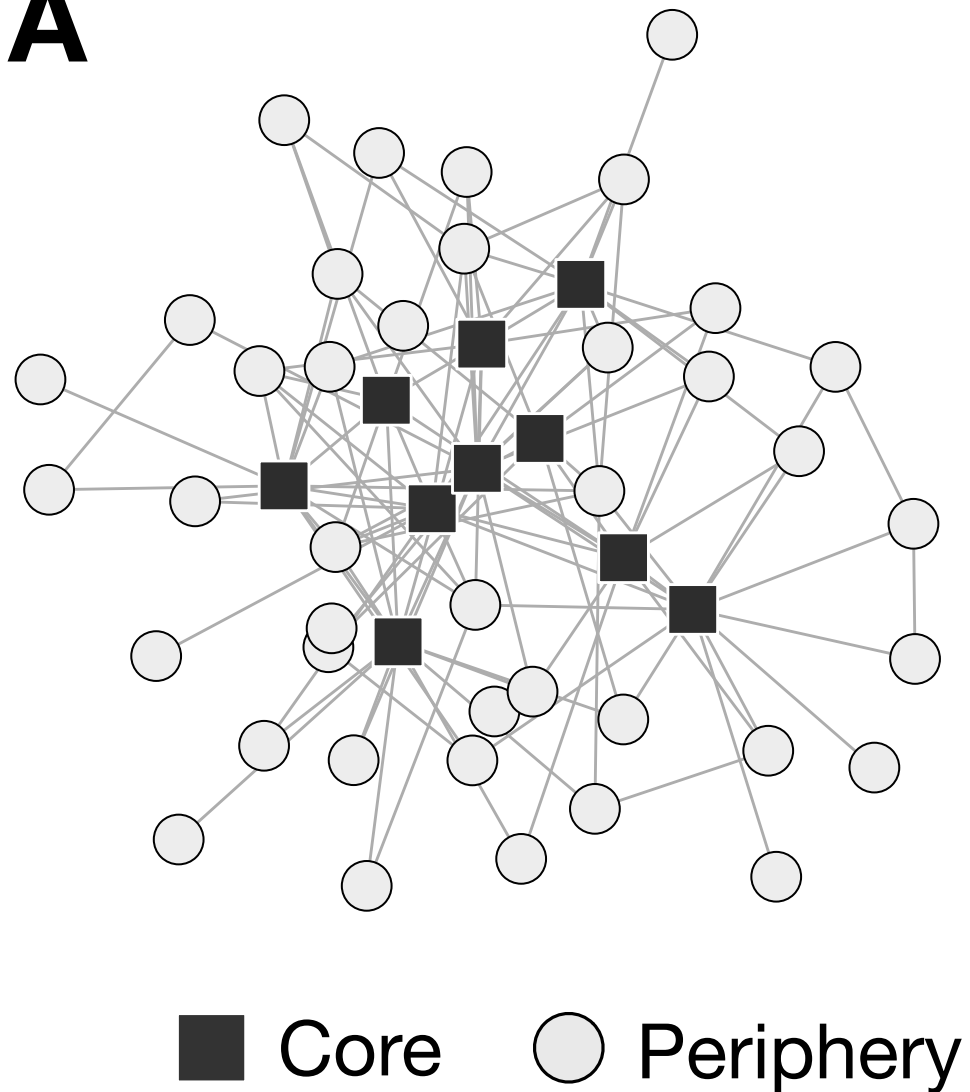
B

Trajectory of random walks

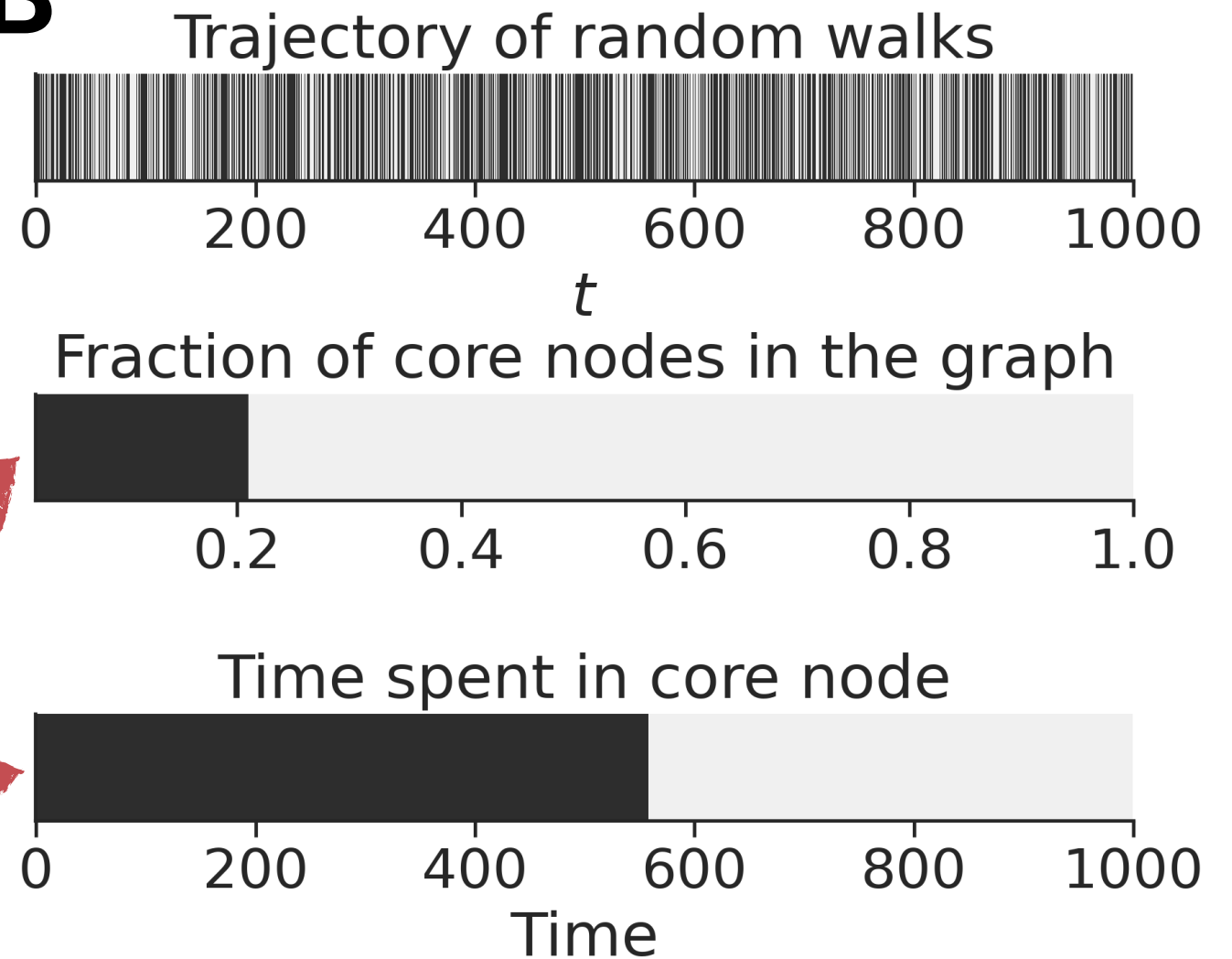


Toy example

A



B

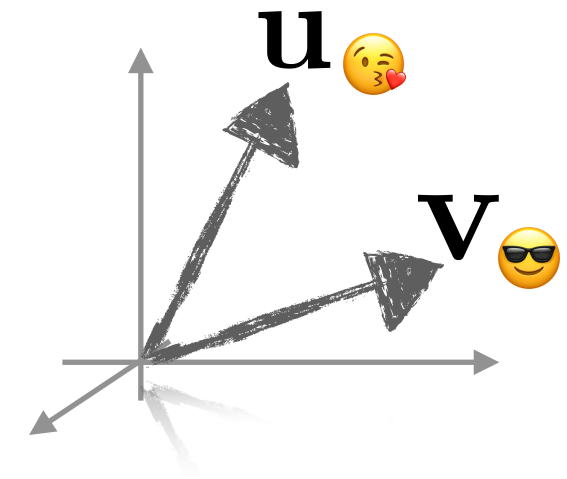
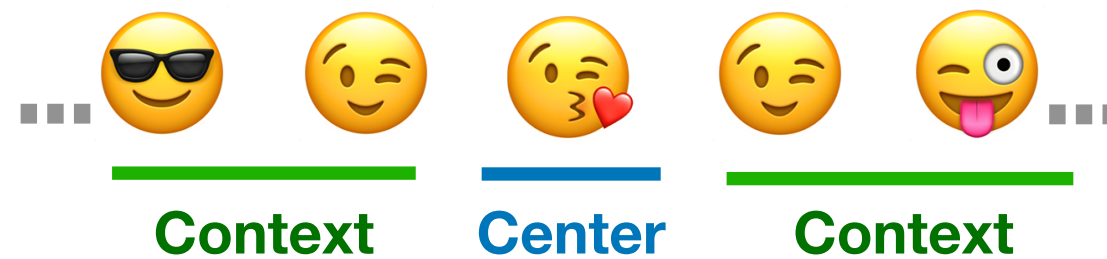


**Core nodes are overrepresented
due to *the friendship paradox!***

**Does the sampling bias have
negative impact?**

**word2vec trained with negative
sampling has an overlooked built-
in debiasing feature!**

**We demonstrate how to leverage
this feature to debias other biases!**



$$P_{w2v} \left(\text{Context} \mid \text{Center} \right) = \frac{1}{Z} \exp \left(\mathbf{u}^\top \mathbf{v} \right)$$

$$Z = \sum_j \exp \left(\mathbf{u}^\top \mathbf{v}_j \right)$$

Negative Sampling is used for training word2vec*

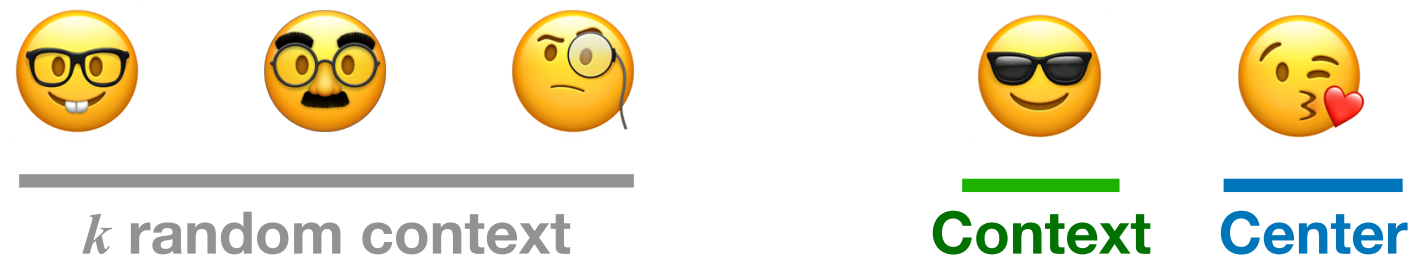
Simplified

Noise Contrastive Estimation (NCE)

*Mikolov, Tomas, Ilya Sutskever, Kai Chen, Greg Corrado, and Jeffrey Dean. 2013. "InDistributed Representations of Words and Phrases and Their Compositionality." In *Advances in Neural Information Processing Systems*, 1389–99.

Sadamori Kojaku, Jisung Yoon, Isabel Constantino, and Yong-Yeol Ahn. Residual2Vec: Debiasing graph embedding with random graphs. NeurIPS (2021)

💪 Noise Contrastive Estimation (NCE) 💪



Noise distribution

$$\sim P_0(\text{🧐}) \propto \{\text{Frequency of 🧐}\}^\gamma$$

*Gutmann, Michael, and Aapo Hyvärinen. 2010. "Noise-Contrastive Estimation: A New Estimation Principle for Unnormalized Statistical Models." In *Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics*, edited by Yee Whye Teh and Mike Titterton, 9:297–304. Proceedings of Machine Learning Research. Chia Laguna Resort, Sardinia, Italy: PMLR.

🦾 Noise Contrastive Estimation (NCE) 🦾



$$Y_{\text{👓}} = Y_{\text{🧔}} = Y_{\text{🤔}} = 0 \quad Y_{\text{😎}} = 1$$

$$P(Y_{\text{😎}} = 1 \mid \text{😎}) \propto \frac{P(\text{😎} \mid Y_{\text{😎}} = 1)}{P(\text{😎})} P(Y_{\text{😎}} = 1)$$

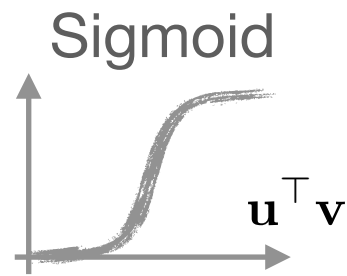
Posterior
Likelihood
Prior

$$P(\text{😎} \mid Y = 1) = \begin{cases} P_{w2v}(\text{😎} \mid \text{😘}) & (Y = 1) \\ P_0(\text{😎}) & (Y = 0) \end{cases} \quad P(Y = 1) = \frac{1}{k + 1}$$

*Gutmann, Michael, and Aapo Hyvärinen. 2010. "Noise-Contrastive Estimation: A New Estimation Principle for Unnormalized Statistical Models." In *Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics*, edited by Yee Whye Teh and Mike Titterton, 9:297–304. Proceedings of Machine Learning Research. Chia Laguna Resort, Sardinia, Italy: PMLR.

Sadamori Kojaku, Jisung Yoon, Isabel Constantino, and Yong-Yeol Ahn. Residual2Vec: Debiasing graph embedding with random graphs. NeurIPS (2021)

💪 Noise Contrastive Estimation (NCE) 💪



$$P(Y_{\text{😎}} = 1 | \text{😎}) = \frac{1}{1 + \exp(-\mathbf{u}_{\text{😘}}^{\top} \mathbf{v}_{\text{😎}} + \ln P_0(\text{😎}) + c)}$$

NCE is asymptotically unbiased for

$$P_{w2v}(\underbrace{\text{😎}}_{\text{Context}} | \underbrace{\text{😘}}_{\text{Center}}) = \frac{1}{Z} \exp(\mathbf{u}_{\text{😘}}^{\top} \mathbf{v}_{\text{😎}})$$

*Gutmann, Michael, and Aapo Hyvärinen. 2010. "Noise-Contrastive Estimation: A New Estimation Principle for Unnormalized Statistical Models." In *Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics*, edited by Yee Whye Teh and Mike Titterton, 9:297–304. Proceedings of Machine Learning Research. Chia Laguna Resort, Sardinia, Italy: PMLR.

Noise Contrastive Estimation (NCE)

$$P(Y_{\text{😎}} = 1 | \text{😎})$$

$$= \frac{1}{1 + \exp(-\mathbf{u}_{\text{😎}}^\top \mathbf{v}_{\text{😎}} + \ln P_0(\text{😎}) + c)}$$

asymptotically
unbiased for

$$P_{w2v}(\text{😎} | \text{😘}) = \frac{1}{Z} \exp(\mathbf{u}_{\text{😎}}^\top \mathbf{v}_{\text{😘}})$$

Also unbiased for
a general model

$$P(Y = 1 | x)$$

$$= \frac{1}{1 + \exp(-f(x) + \ln P_0(x) + c)}$$

asymptotically
unbiased for

$$P(x) = \frac{1}{Z} \exp(f(x))$$

*Gutmann, Michael, and Aapo Hyvärinen. 2010. "Noise-Contrastive Estimation: A New Estimation Principle for Unnormalized Statistical Models." In *Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics*, edited by Yee Whye Teh and Mike Titterton, 9:297–304. Proceedings of Machine Learning Research. Chia Laguna Resort, Sardinia, Italy: PMLR.

Noise Contrastive Estimation (NCE)

$$P(Y = 1|x) = \frac{1}{1 + \exp(-\underline{f(x)} + \ln P_0(x) + c)}$$

asymptotically
unbiased for

$$P(x) = \frac{1}{Z} \exp(f(x))$$

Negative sampling*

$$P(Y = 1|x)$$

$$= \frac{1}{1 + \exp(-f(x) + \cancel{\ln P_0(x)} + c)}$$

$$= \frac{1}{1 + \exp[-(\underline{f(x)} + \ln P_0(x)) + \ln P_0(x) + c]}$$

$f'(x)$

asymptotically
unbiased for

$$P(x) = \frac{1}{Z'} \exp(f'(x))$$

$$P(x) = \frac{1}{Z'} P_0(x) \exp(f(x))$$

word2vec trained with Skip-gram **Negative sampling**
(SGNS) is asymptotically unbiased for

$$P_{w2v}(x|y) = \frac{1}{Z} \frac{1}{Z'} P_0(x) \exp(f(u_x^T y))$$

Baseline probability

Residual from the baseline

$\sim P_0(\text{😎}) \propto \{\text{Frequency of } \text{😎}\}^\gamma$

Information other than frequency

Built-in debiasing feature for frequency bias!



$$P_{w2v}(\text{😎} | \text{😘}) = \frac{1}{Z} \underline{P_0(\text{😎})} \exp(\mathbf{u}_{\text{😎}}^\top \mathbf{v}_{\text{😘}})$$

$$\sim P_0(\text{😎}) \propto \{\text{Frequency of } \text{😎}\}^\gamma$$

Sampling bias due to *the friendship paradox**

$$\left\{ \begin{array}{l} \text{Frequency of } \text{😎} \\ \text{in the sentence} \end{array} \right\} \propto \left\{ \begin{array}{l} \# \text{ neighbors of } \text{😎} \\ \text{in the graph} \end{array} \right\}$$

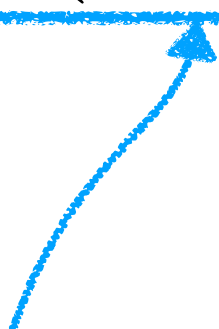
The friendship paradox has *no effect* thanks to *the built-in debiasing feature* of SGNS word2vec if $\gamma = 1$.



*Masuda, Naoki, Mason A. Porter, and Renaud Lambiotte. "Random walks and diffusion on networks." *Physics reports* 716 (2017): 1-58.

Sadamori Kojaku, Jisung Yoon, Isabel Constantino, and Yong-Yeol Ahn. Residual2Vec: Debiasing graph embedding with random graphs. NeurIPS (2021)

Residual2Vec

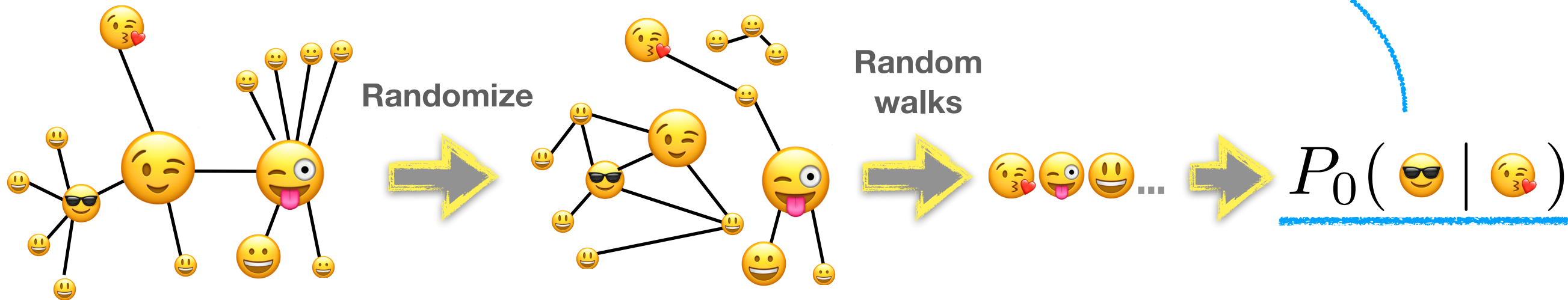
$$P_{r2v}(\text{😎} | \text{😘}) = \frac{1}{Z} \underbrace{P_0(\text{😎} | \text{😘})}_{\text{baseline}} \exp(\mathbf{u}_{\text{😘}}^\top \mathbf{v}_{\text{😎}})$$


**Model the baseline
explicitly to control bias**

Residual2Vec

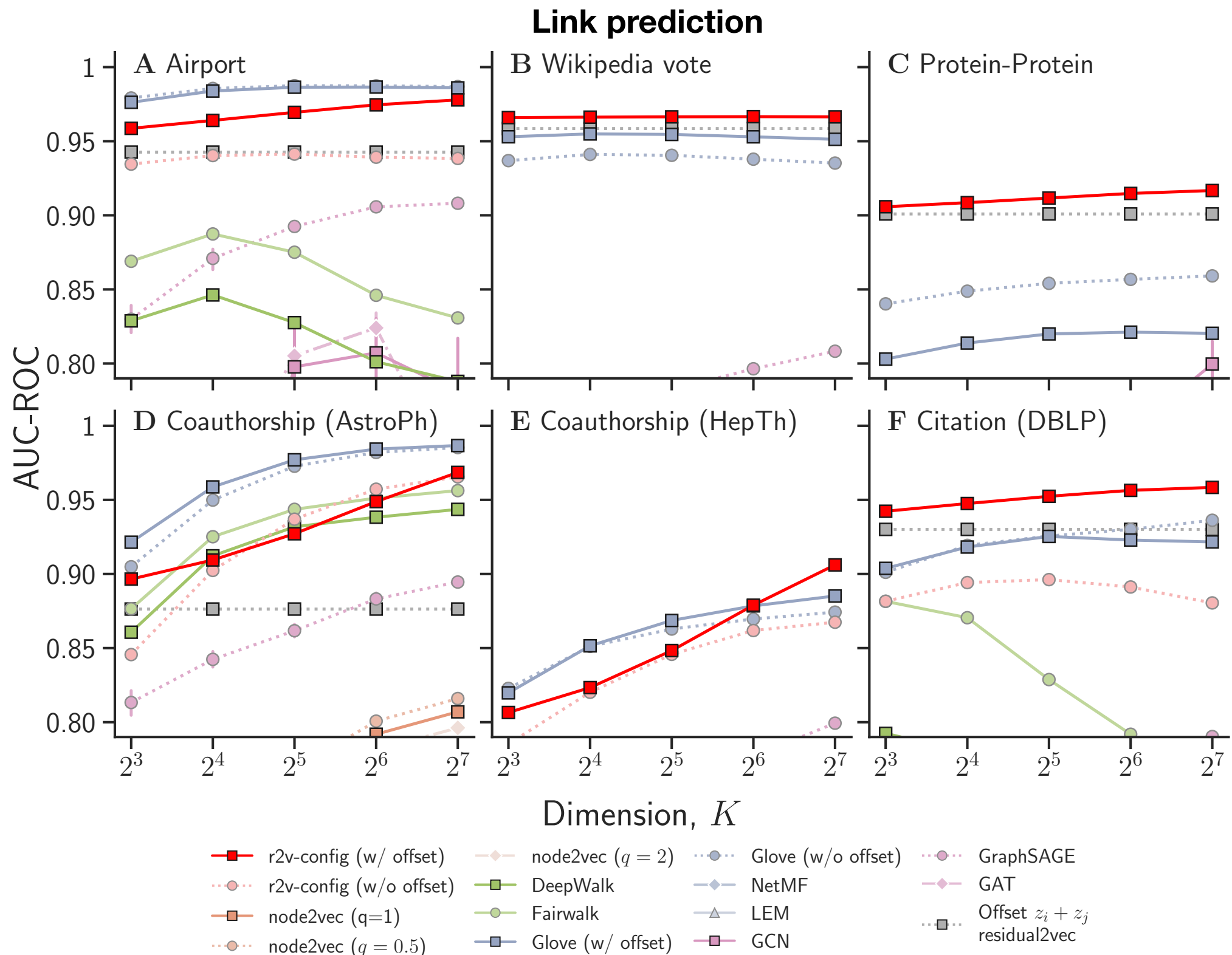
How can we model the baseline?

$$P_{r2v}(\text{😎} \mid \text{😘}) = \frac{1}{Z} \underbrace{P_0(\text{😎} \mid \text{😘})}_{\text{baseline}} \exp(\mathbf{u}_{\text{😘}}^\top \mathbf{v}_{\text{😎}})$$

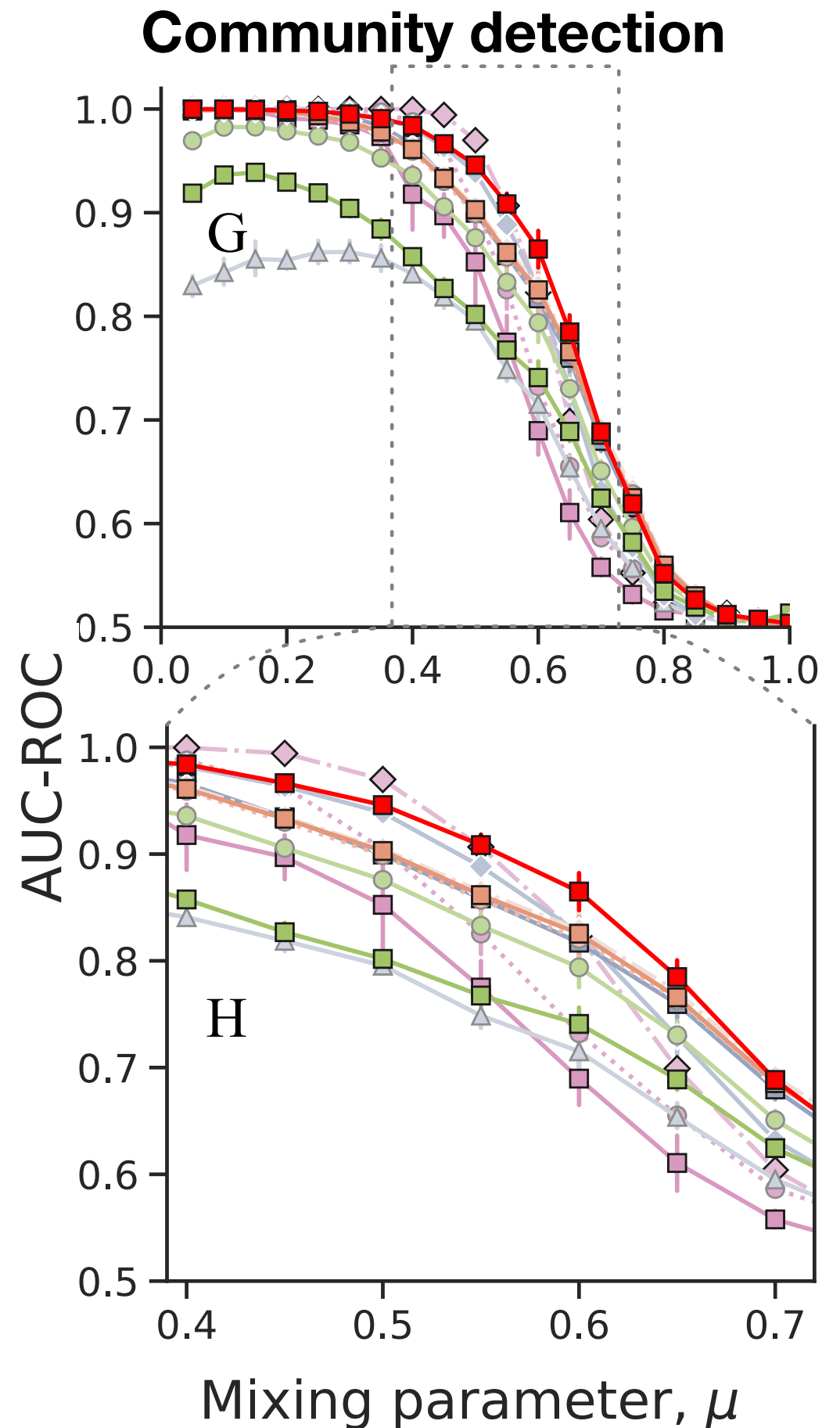
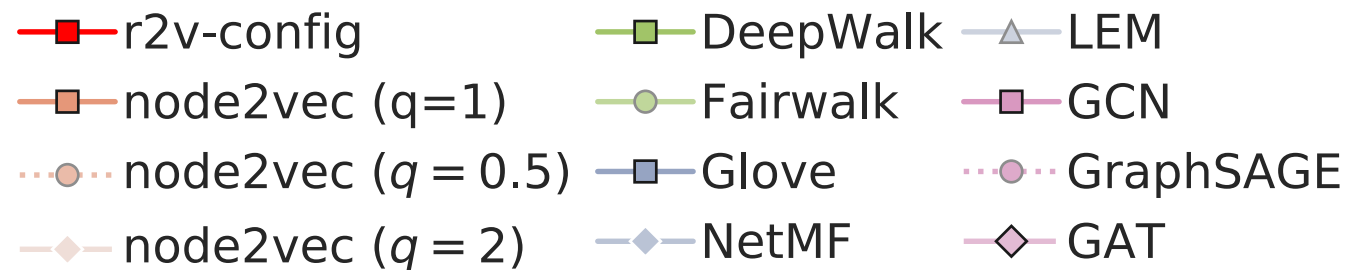


Can be done analytically without simulations!

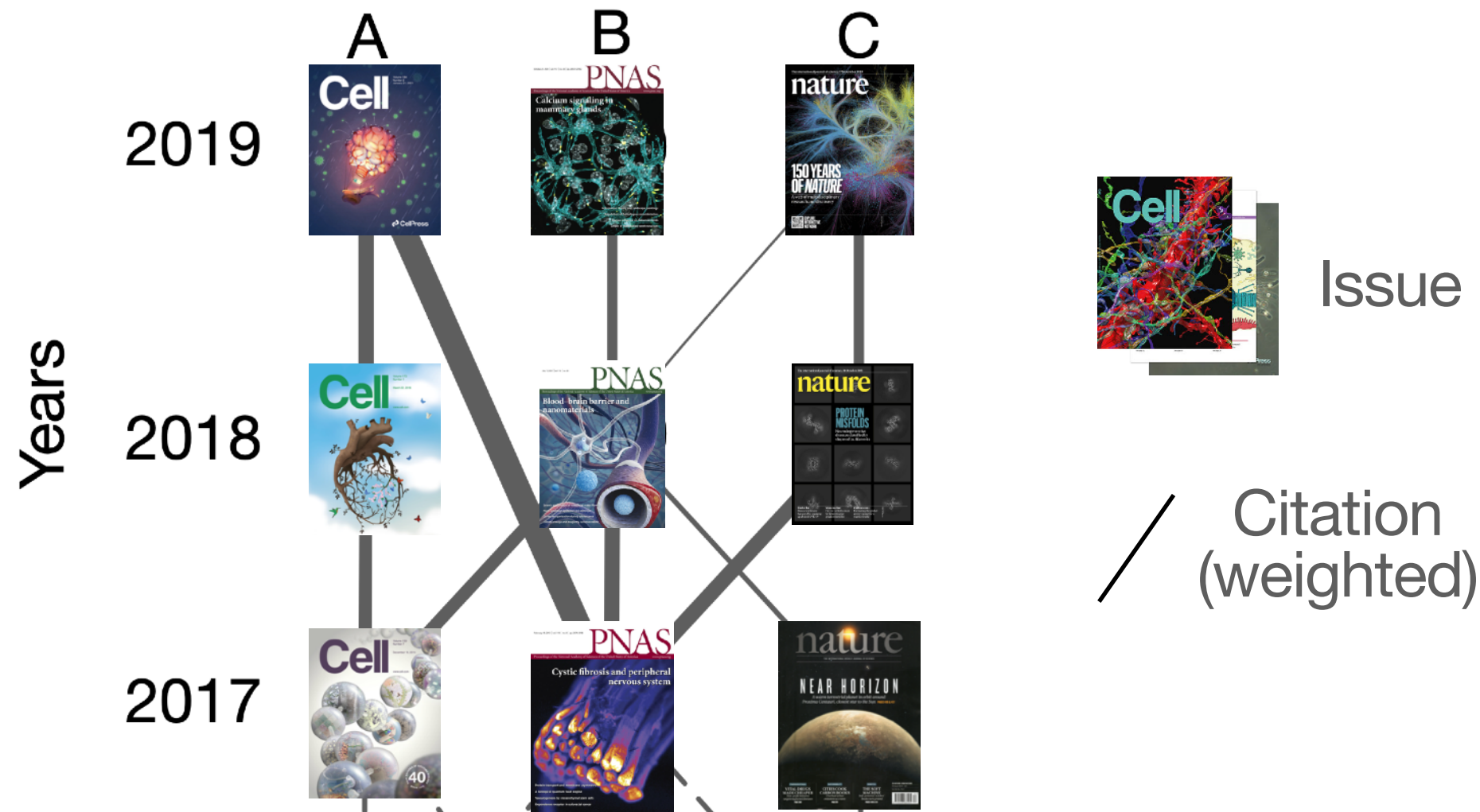
Performed the best or nearly the best for all the six graphs of different domains



The best or the second best performer for a community detection benchmark

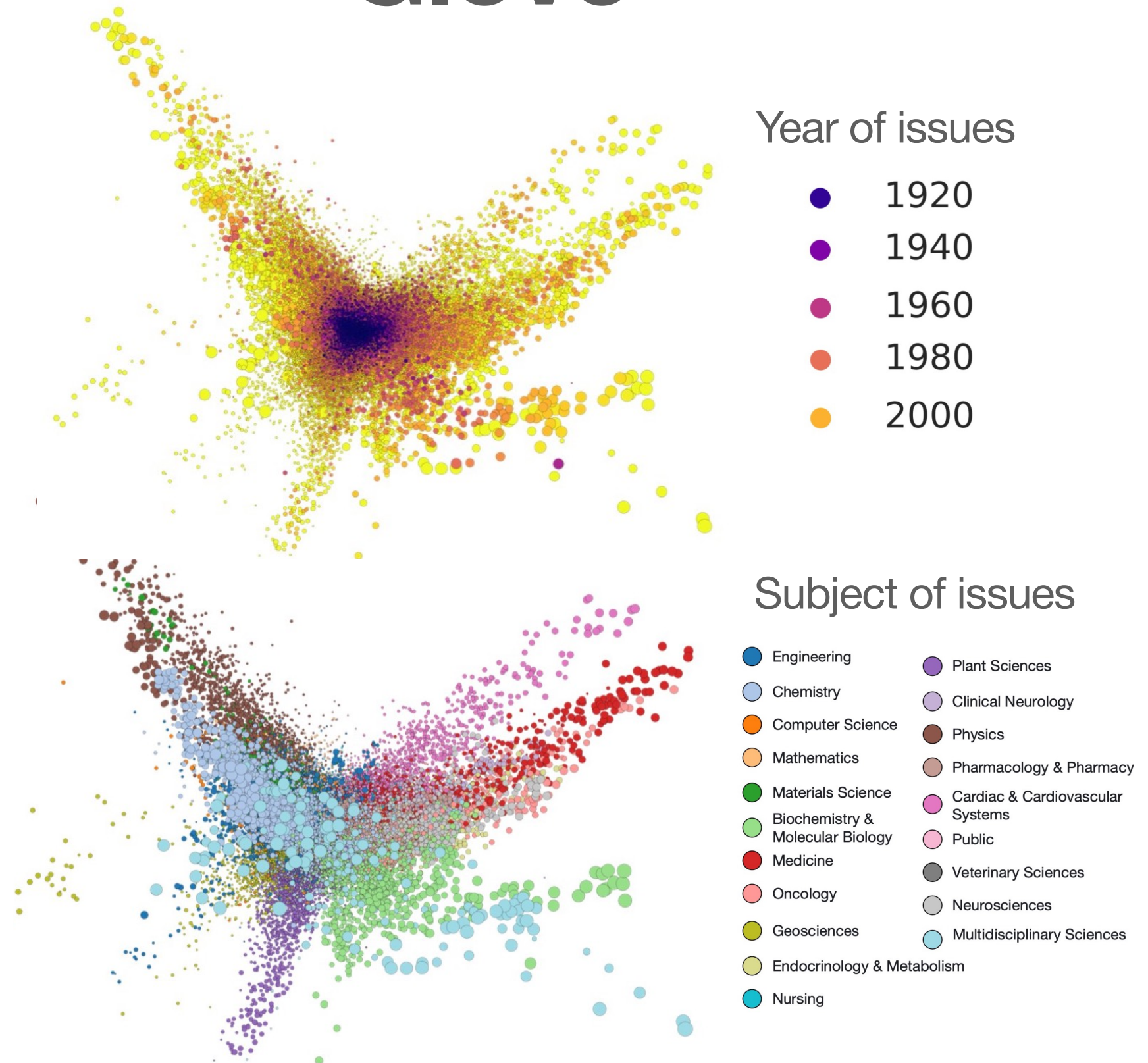
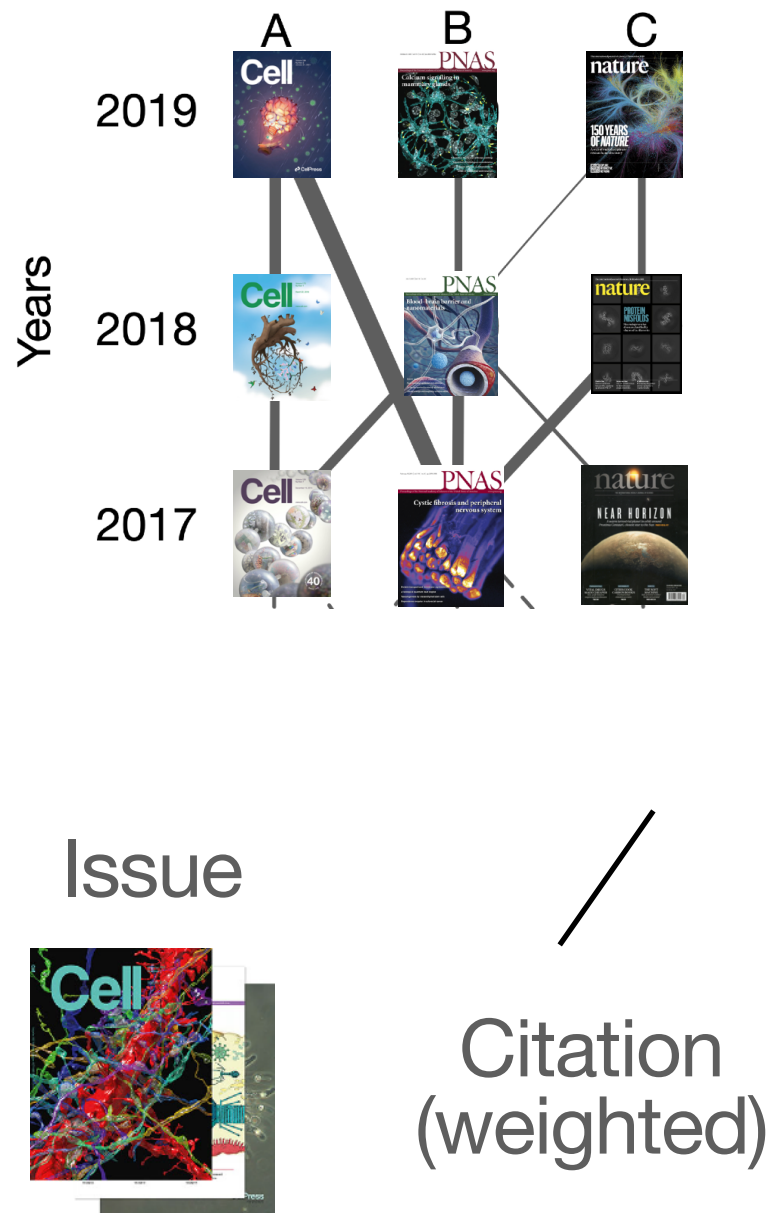


Case studies



Citation graph of academic issues
Constructed using the Web of Science
240K issues and 250M citations

Glove*



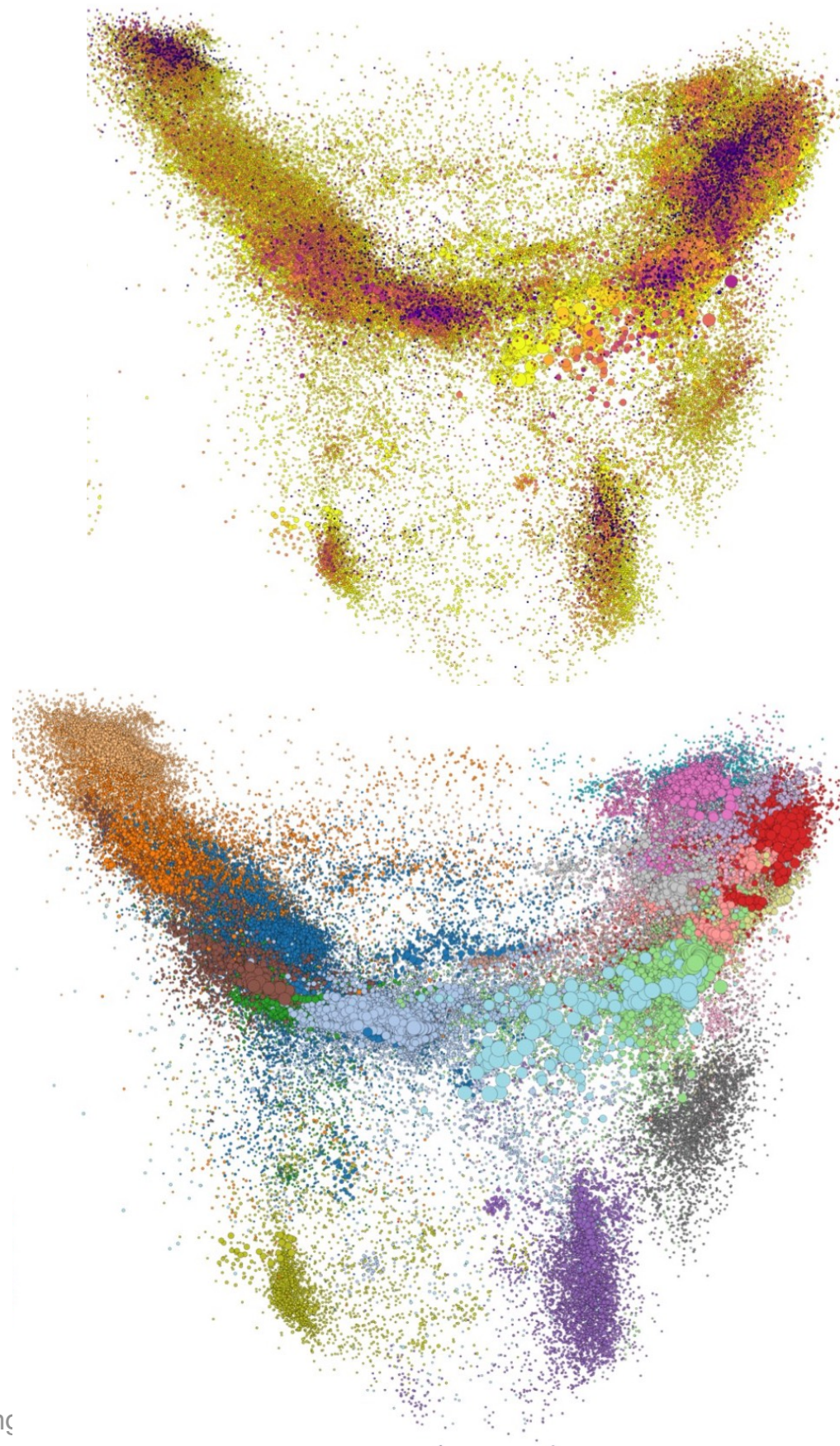
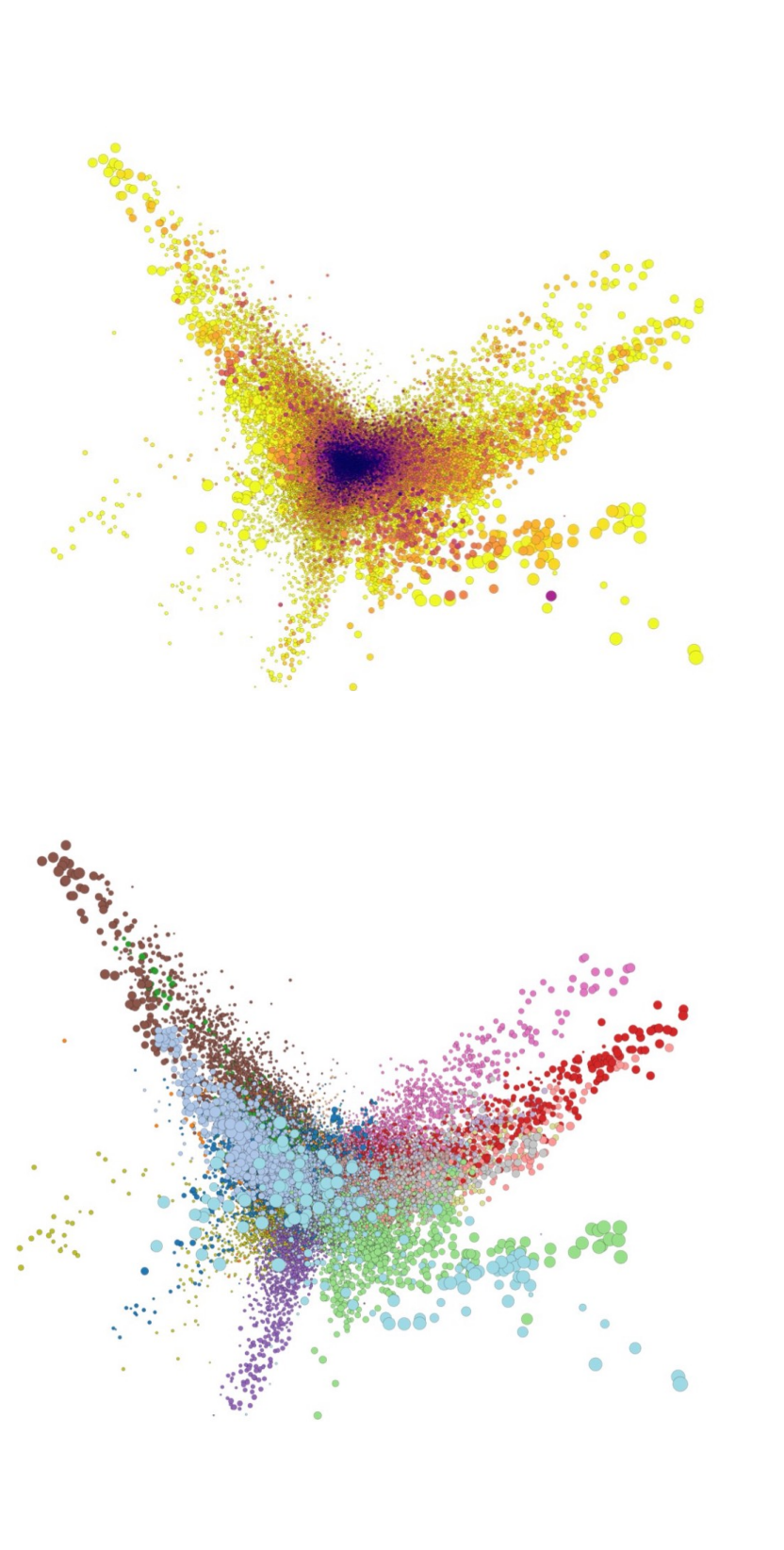
*Jeffrey Pennington, Richard Socher, and Christopher D. Manning. "Glove: Global vectors for word representation." *Proceedings of the 2014 conference on empirical methods in natural language processing (EMNLP)*. 2014.

Sadamori Kojaku, Jisung Yoon, Isabel Constantino, and Yong-Yeol Ahn. Residual2Vec: Debiasing graph embedding with random graphs. *NeurIPS* (2021)

Residual2Vec

(temporal biases are debiased)

Glove



Year of issues

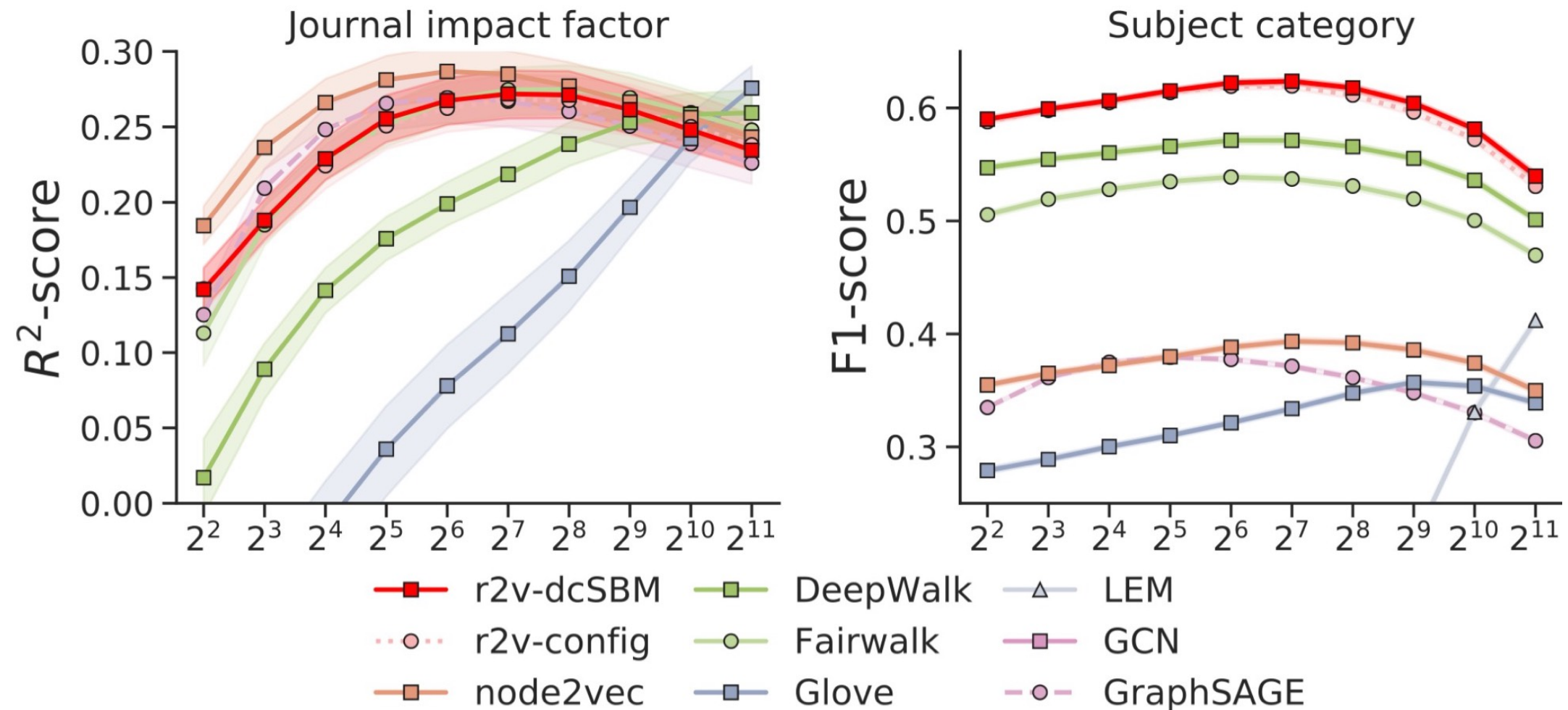
- 1920
- 1940
- 1960
- 1980
- 2000

Subject of issues

- Engineering
- Chemistry
- Computer Science
- Mathematics
- Materials Science
- Biochemistry & Molecular Biology
- Medicine
- Oncology
- Geosciences
- Endocrinology & Metabolism
- Nursing
- Plant Sciences
- Clinical Neurology
- Physics
- Pharmacology & Pharmacy
- Cardiac & Cardiovascular Systems
- Public
- Veterinary Sciences
- Neurosciences
- Multidisciplinary Sciences

Predicted the impact factor and the subject of journals well

Prediction based on embedding

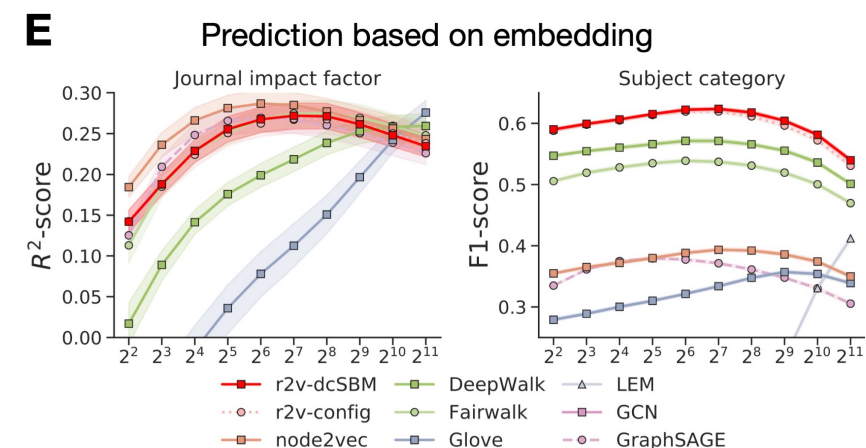
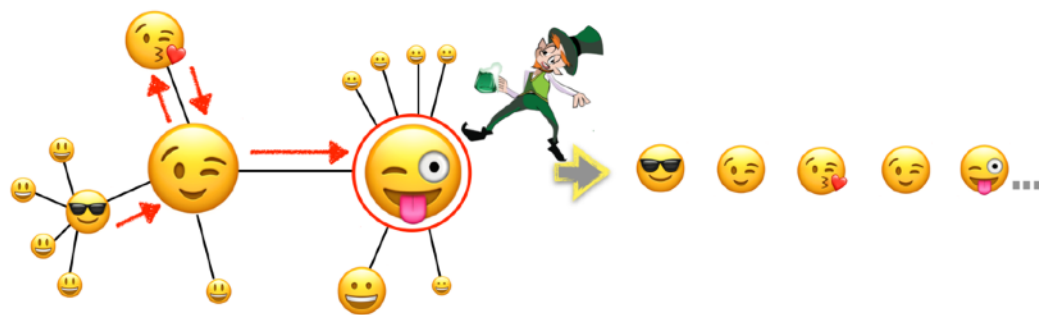


Summary

word2vec has a built-in debiasing feature attributed to negative sampling

Inspired by this finding, propose *residual2vec* that can negate other types of structural biases

Good performance and enable more control on the biases in embedding.



Discussion

We demonstrated a new potential of **negative sampling** as a way to mitigate **bias in representations**



This figure is taken from <https://www.bbc.co.uk/programmes/w3ct1ls5>

Sadamori Kojaku, Jisung Yoon, Isabel Constantino, and Yong-Yeol Ahn. Residual2Vec: Debiasing graph embedding with random graphs. NeurIPS (2021)

Residual2Vec: Debiasing graph embedding using random graphs



arXiv.org

<https://arxiv.org/abs/2110.07654>



github.com/skojaku/residual2vec



skojaku@iu.edu



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Jisung Yoon



Isabel Constantino



Yong-Yeol Ahn