Overlapping Clustering Models, and One (class) SVM to Bind Them All

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(Poster: Today 10:45 AM – 12:45 PM @ Room 517 AB #114)
Stochastic Blockmodel

\[
P = \Theta B \Theta^T
\]

Limitations:
- Each node belongs to exactly one community
- All nodes in the same community have the same expected degree
Extensions of Stochastic Blockmodel

- Mixed membership blockmodels (Airoldi et al. 2008) extend this to **allow overlap**
  - $\theta_i$ is a distribution over $K$ communities

- Degree-corrected blockmodels (Karrer and Newman 2011) extend this to **allow heterogeneous degree distributions**
  - Each node has a degree parameter $\gamma_i$

- There are many other extensions to model the above two properties
  - DCMMSB (Jin et al., 2017)
  - OCCAM (Zhang et al. 2014)
  - SBMO (Kaufmann et al. 2016)
Overlapping clustering model

\[ P = \Gamma \Theta B \Theta^T \Gamma \]

This covers many well-known overlapping clustering models:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( |\theta_i|_1 = 1 )</td>
<td>DCMMSB</td>
</tr>
<tr>
<td>( |\theta_i|_2 = 1 )</td>
<td>OCCAM</td>
</tr>
<tr>
<td>( \theta_i \in {0, 1}^K )</td>
<td>SBMO</td>
</tr>
</tbody>
</table>

The LDA topic model (Blei et al. 2003) is also a special case.
### Main idea

<table>
<thead>
<tr>
<th>Model</th>
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<tbody>
<tr>
<td>OCCAM</td>
<td>$k$-median on regularized eigenvectors</td>
</tr>
<tr>
<td>SBMO</td>
<td>Alternating minimization</td>
</tr>
<tr>
<td>MMSB</td>
<td>Finding $K$ corners of a simplex in $\mathbb{R}^K$</td>
</tr>
<tr>
<td>DCMMSB</td>
<td>Finding $K$ corners of a simplex in $\mathbb{R}^{K-1}$</td>
</tr>
<tr>
<td>Topic Models</td>
<td>Finding $K$ corners of a simplex in $\mathbb{R}^V$</td>
</tr>
<tr>
<td>All</td>
<td>Finding extreme rays of a <strong>convex cone</strong></td>
</tr>
</tbody>
</table>

Let $\mathbf{V} \in \mathbb{R}^{n \times K}$ be the top-$K$ eigenvectors of $\mathbf{P}$

Rows of $\mathbf{V}$ form a **cone**

**Figure:** Each point is a row of $\mathbf{V}$
Main idea

- SVM-cone:
  - Normalize rows \( v_i \) of \( V \) to unit \( \ell_2 \) norm
  - Each node lies on the intersection of the cone and the unit sphere
  - Run a one-class SVM \( \implies \text{support vectors are the corners} \)
  - Estimate community memberships by regression \( v_i \) on these corners
- This is for the ideal “population” version
  - Similar ideas provably work for the “empirical” version
This one algorithm yields consistency guarantees for community memberships of each node. Most algorithms show guarantees for the whole matrix for all overlapping clustering models mentioned earlier.

Example

Per-node consistency guarantee for DCMMSB (informal)

If $\theta_i \sim \text{Dirichlet}(\alpha)$, under a broad parameter regime, with high probability,

$$\max_i \| \hat{\theta}_i - \theta_i \| = \tilde{O} \left( \frac{g}{\sqrt{\rho n}} \right),$$

where $g$ depends on model parameters.
Conclusions

- A simple and scalable algorithm

\[
\text{Eigendecomposition} \Rightarrow \text{Row-normalize} \Rightarrow \text{One-class SVM} \Rightarrow \text{Regression}
\]

- infers community memberships for a **broad class** of overlapping clustering models
- with **per-node** consistency guarantees

- Good performance on several large scale real-world datasets.

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