Solving Large Sequential Games with the Excessive Gap Technique

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Extensive-Form Games
Applications - poker

Nash Equilibrium approximation used in recent breakthroughs

– Heads-Up Limit Texas Hold’Em [Bowling et al. 2015]


CFR, or variants, used to compute equilibria
How compute a zero-sum Nash equilibrium

**Linear programming** [Stengel 96]

Simplex and IPM too slow in practice

**CFR and variants** [Zinkevich et al. 07, Tammelin et al 15]

\[ \frac{1}{\sqrt{T}} \]

in theory

Better than \( \frac{1}{T} \) in practice

**First-order methods**, [Hoda et al 10, Kroer et al 18]

\[ \frac{1}{T} \]

in theory

\[ \frac{1}{T} \]

in practice
We introduce a practical variant of EGT

- EGT constructs smoothed approximations to the optimization problems faced by each player [Nesterov05, Hoda10, Kroer18]
- We use dilated entropy DGF from [Kroer18]
- Aggressive stepsizing
- Balancing of smoothing on each player
- Numerically-friendly smoothed best response computation
- GPU parallelization across different hands dealt
Experiments

Real-time subgames from Brains vs AI competition
  Last betting round of game
  43k/86k actions per player, 54M leaves

Excessive Gap Technique [Nesterov 05] with our smoothing function

Our Aggressive EGT

Three CFR variants
Comparison to existing algorithms

Endgame 7

\[ \epsilon \text{ (regret sum) [mbb]} \]

- CFR\(^+\)
- EGT
- EGT/AS
- CFR(RM)
- CFR(RM\(^+\))

Gradient computations

\[ 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \]
Conclusion

• We introduce aggressive EGT variant
• Give first comparison of FOMs and CFR on real, large-scale games
• First-order methods can be made faster than all but the best practical variant of CFR

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