Solving Large Sequential Games with the Excessive Gap Technique

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Extensive-Form Games
Applications - poker

Nash Equilibrium approximation used in recent breakthroughs

– Heads-Up Limit Texas Hold’Em [Bowling et al. 2015]

CFR, or variants, used to compute equilibria
How compute a zero-sum Nash equilibrium

Linear programming [von Stengel 96]
  Simplex and IPM too slow in practice

CFR and variants [Zinkevich et al. 07, Tammelin et al 15]
  \[ \frac{1}{\sqrt{T}} \] in theory
  Better than \[ \frac{1}{T} \] in practice

First-order methods, [Hoda et al 10, Kroer et al 18]
  \[ \frac{1}{T} \] in theory
  \[ \frac{1}{T} \] in practice
Practical Excessive Gap Technique

We introduce a practical variant of EGT

- EGT constructs smoothed approximations to the optimization problems faced by each player [Nesterov 05, Hoda et al 10, Kroer et al 18]
- We use dilated entropy DGF from [Kroer et al 18]
- Aggressive stepsizing
- Balancing of smoothing on each player
- Numerically-friendly smoothed best response computation
- GPU parallelization across different hands dealt
Experiments

Real-time subgames from Brains vs AI competition
  Last betting round of game
    43k/86k actions per player, 54M leaves
EGT with Kroer et al 18 smoothing function
Our Aggressive EGT
Three CFR variants
Comparison to existing algorithms

Endgame 7

\[ \begin{align*}
\epsilon \text{ (regret sum) [mbb]} \\
\text{Gradient computations}
\end{align*} \]
Conclusion

• We introduce aggressive EGT variant
• Give first comparison of FOMs and CFR on real, large-scale games
• First-order methods can be made faster than all but the best practical variant of CFR

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