Efficient Online Portfolio with Logarithmic Regret

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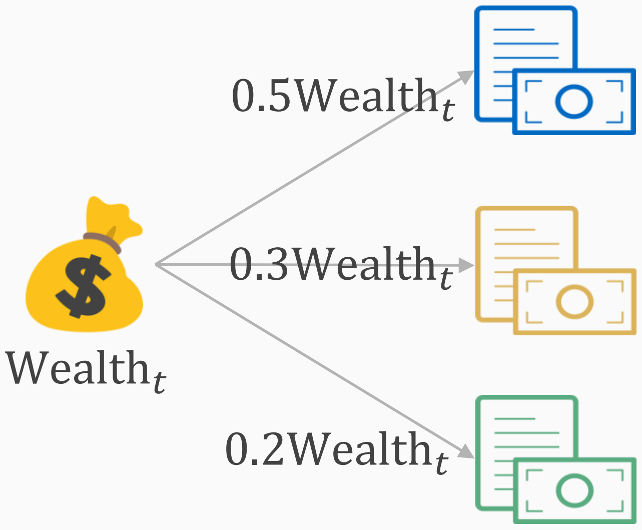
Online Portfolio

Wealth_t

Online Portfolio



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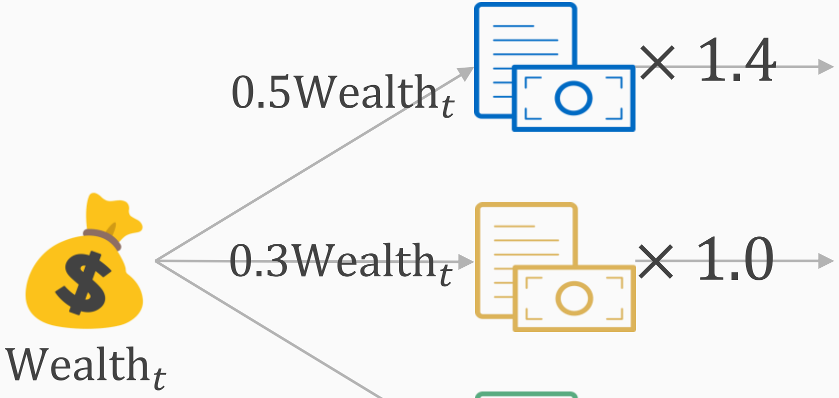

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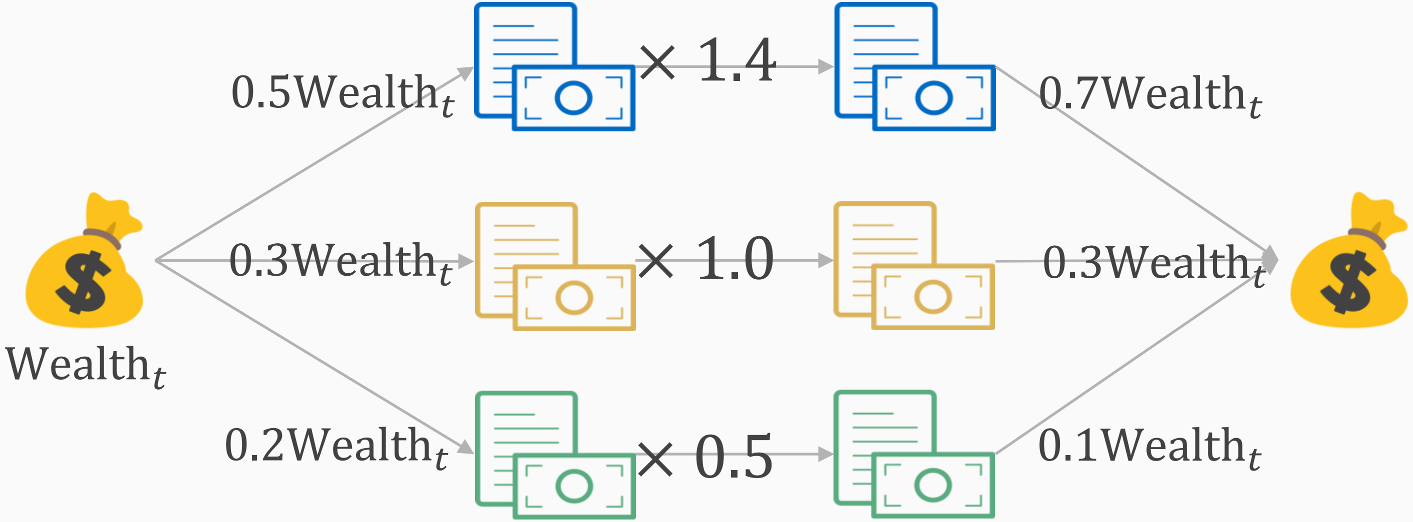

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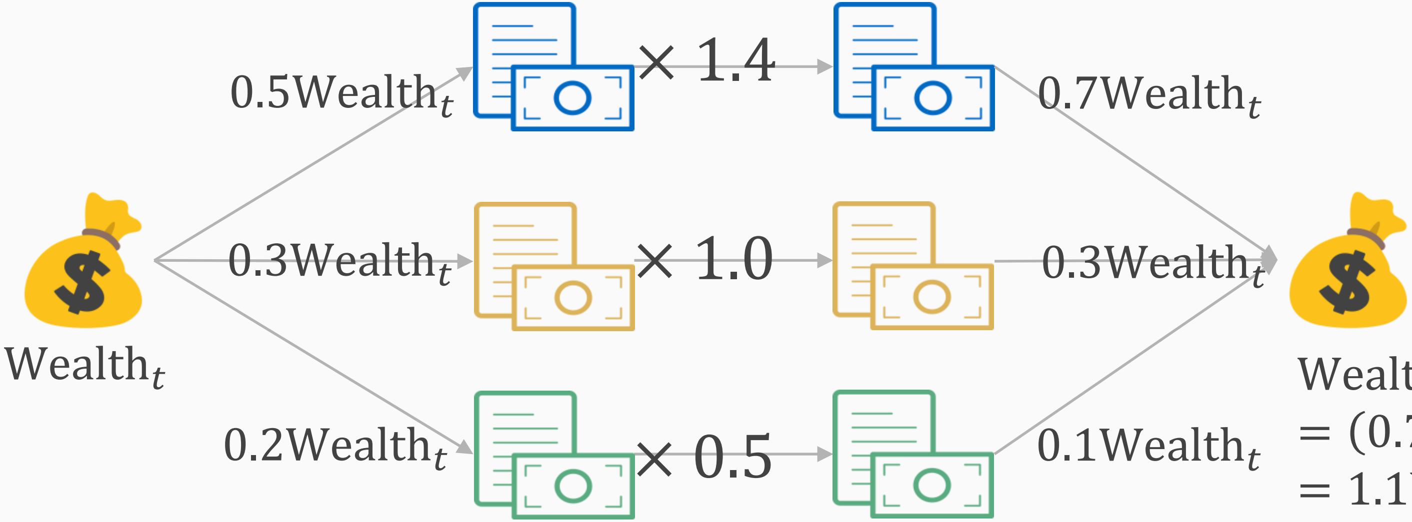

$$\text{Wealth}_{t+1} = (0.7 + 0.3 + 0.1)\text{Wealth}_t = 1.1\text{Wealth}_t$$

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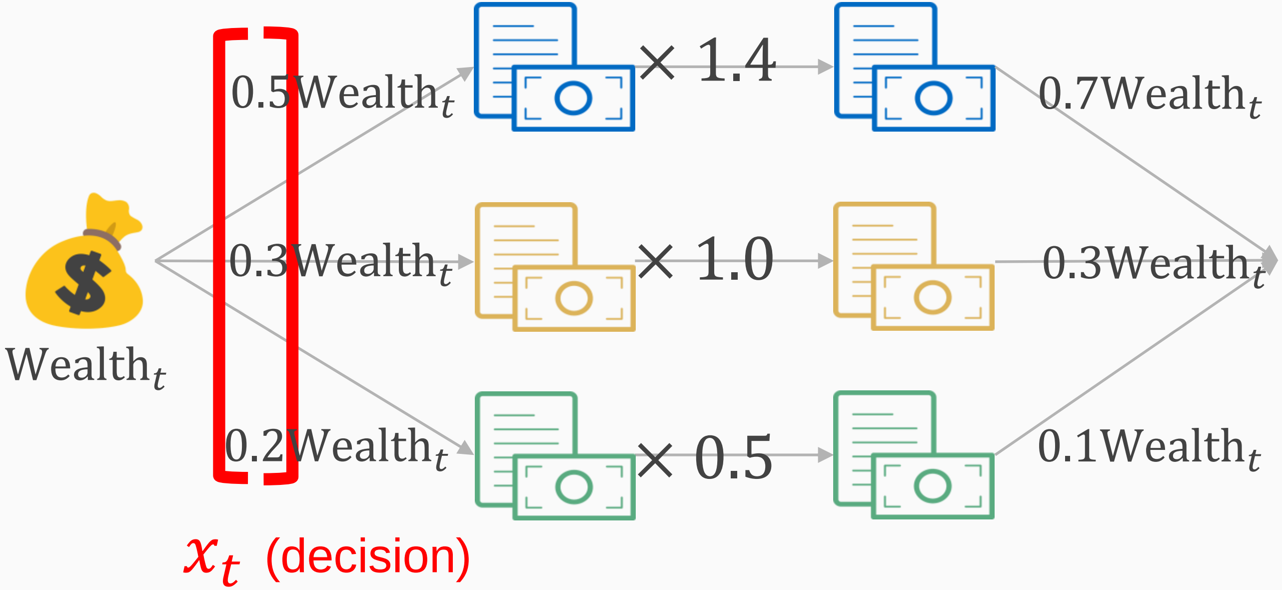

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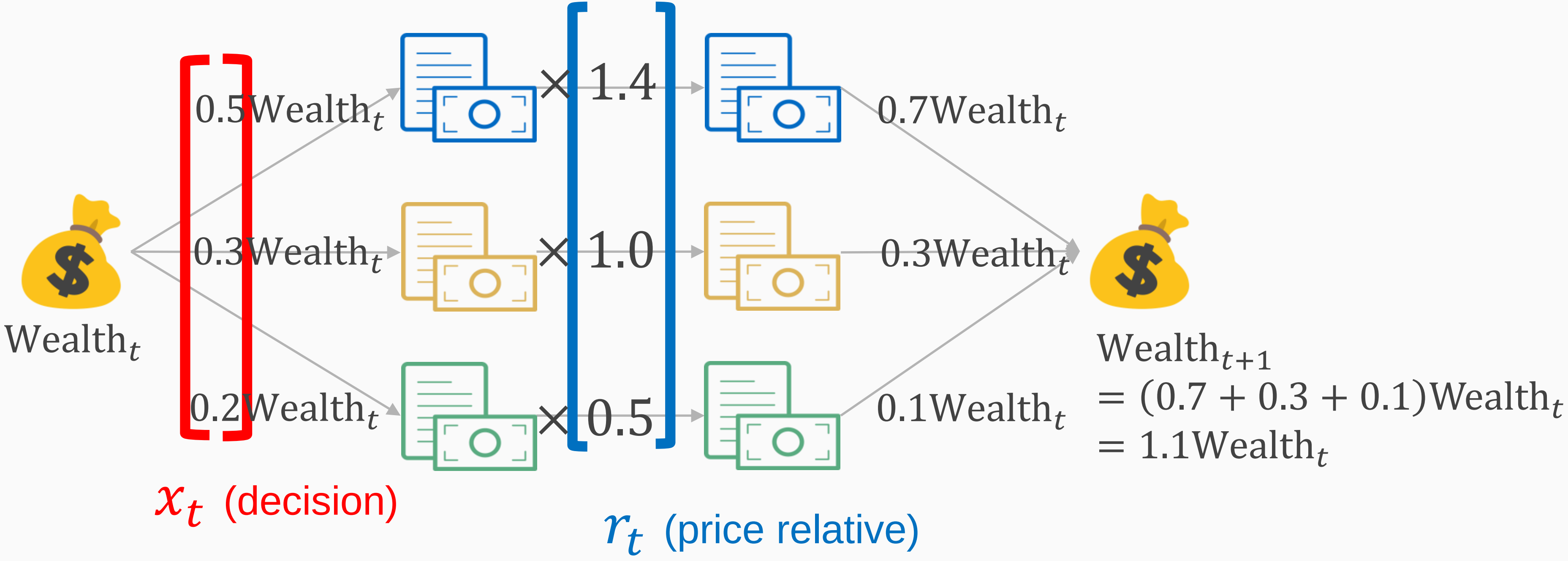

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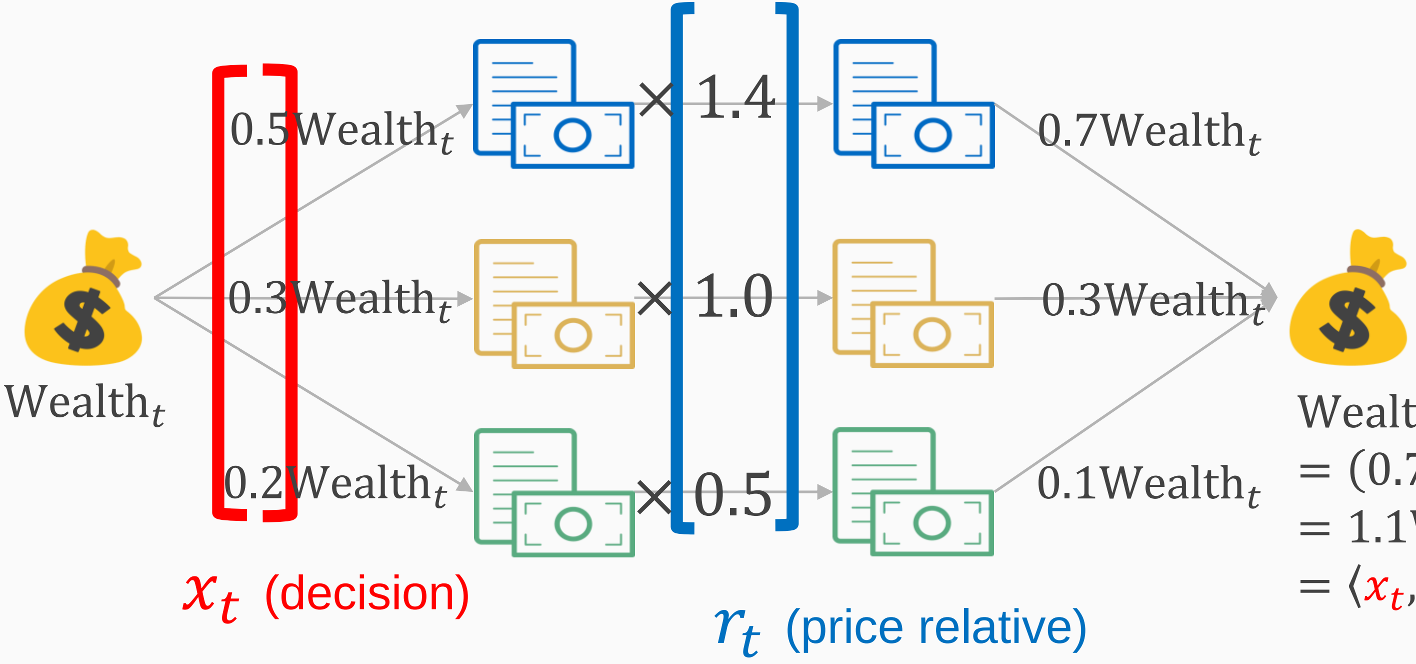

$$\begin{aligned}\mathrm{Wealth}_{t+1} &= (0.7 + 0.3 + 0.1)\mathrm{Wealth}_t \\ &= 1.1\mathrm{Wealth}_t \\ &= \langle x_t, r_t \rangle \mathrm{Wealth}_t\end{aligned}$$

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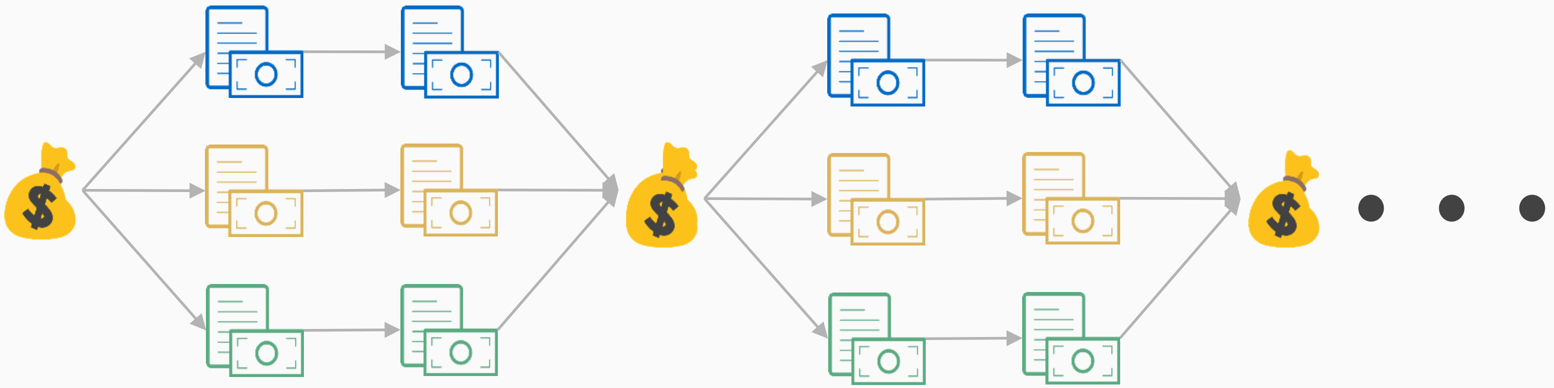
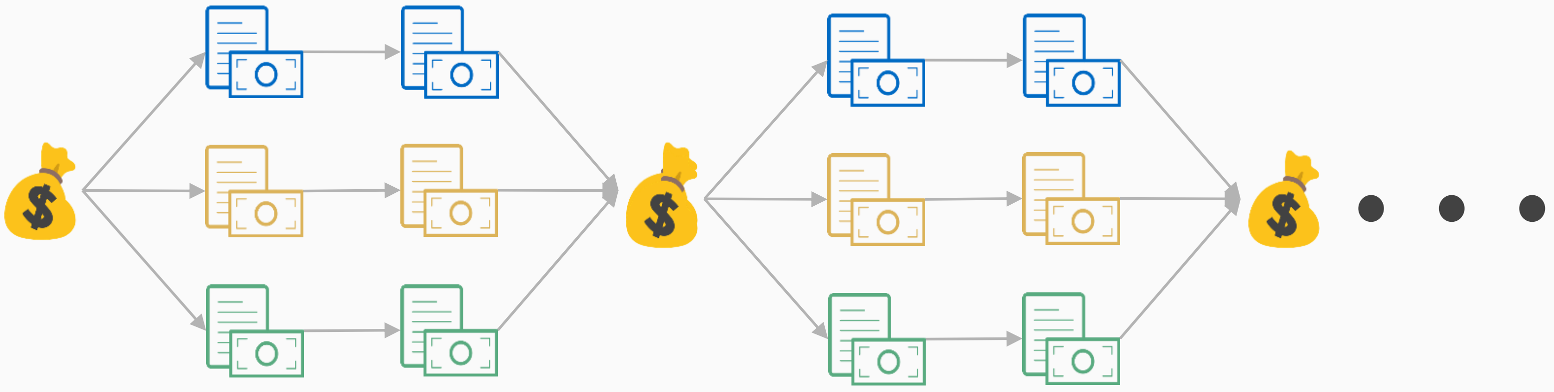


W_1

T periods

Online Portfolio

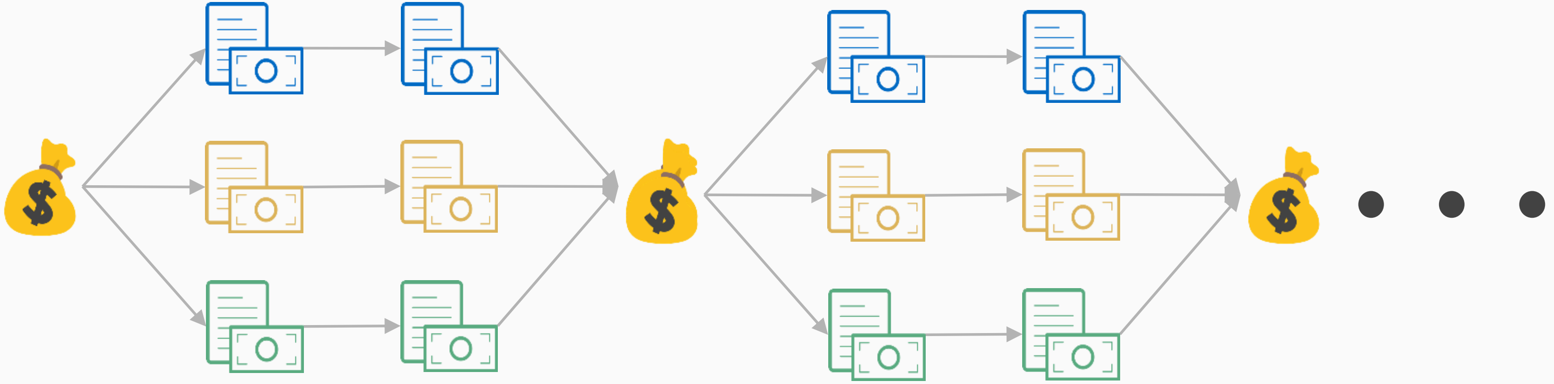


W_1

W_2
$= \langle x_1, r_1 \rangle W_1$

T periods

Online Portfolio



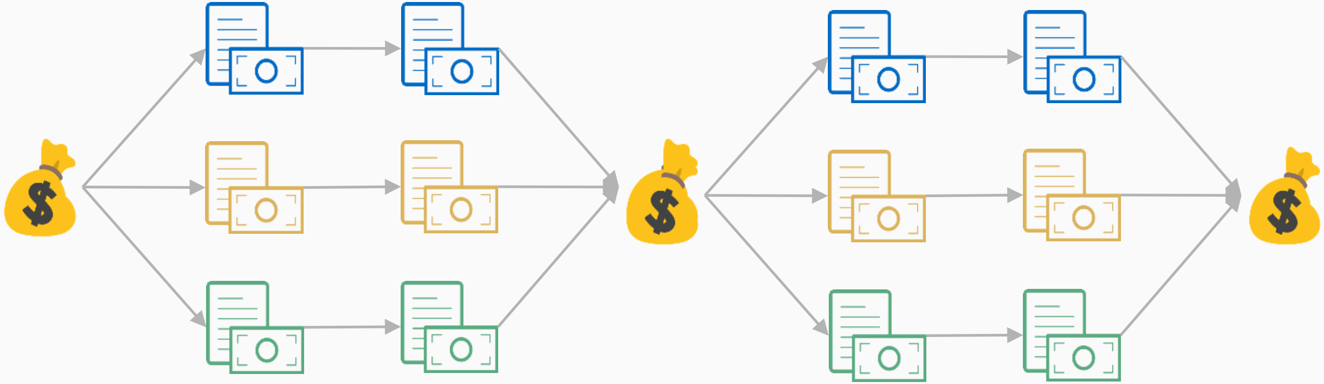
W_1

$W_2 = \langle x_1, r_1 \rangle W_1$

$W_3 = \langle x_2, r_2 \rangle W_2$

T periods

Online Portfolio



• • • T periods

W_1

$W_2 = \langle x_1, r_1 \rangle W_1$

$W_3 = \langle x_2, r_2 \rangle W_2$

Final wealth →

Initial wealth →

$$\frac{W_{T+1}}{W_1} = \prod_{t=1}^{T} \langle x_t, r_t \rangle$$

Online Portfolio

Gain: $$\frac{W_{T+1}}{W_1} = \prod_{t=1}^{T} \langle x_t, r_t \rangle$$

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Benchmark: $$\frac{W^*_{T+1}}{W_1} = \max_{u \in \Delta_N} \prod_{t=1}^{T} \langle u, r_t \rangle$$

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Minimize $$\ln\left(\frac{W^*_{T+1}}{W_{T+1}}\right) = \sum_{t=1}^{T} \ell_t(x_t) - \sum_{t=1}^{T} \ell_t(u)$$ **(Regret)**

$$\ell_t(x) = \ln \frac{1}{\langle x, r_t \rangle}$$

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Online Convex Optimization [Zinkevich'03]

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Online Convex Optimization [Zinkevich'03]

But with possibly unbounded gradient

$$\|\nabla \ell_t(x)\|_\infty \lesssim G \triangleq \max_{i,j} \frac{r_{t,i}}{r_{t,j}}$$

Maximum Relative Ratio

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Previous Results and Our Results

- Lower bound: $\Omega(N \log T)$

N: number of stocks

T: number of rounds

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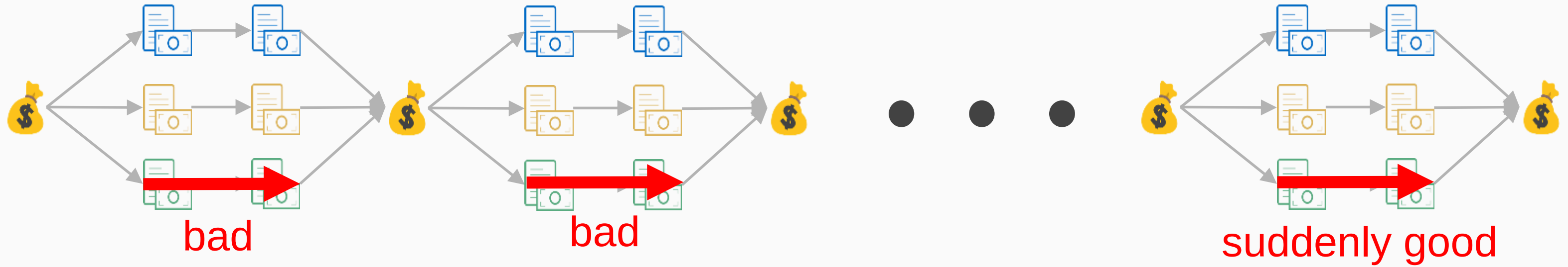
BarrONS (this work)	$N^2 (\log T)^4$	$TN^{2.5}$

Key Components of Our Algorithm

Main Challenge:



But player puts little weight on it

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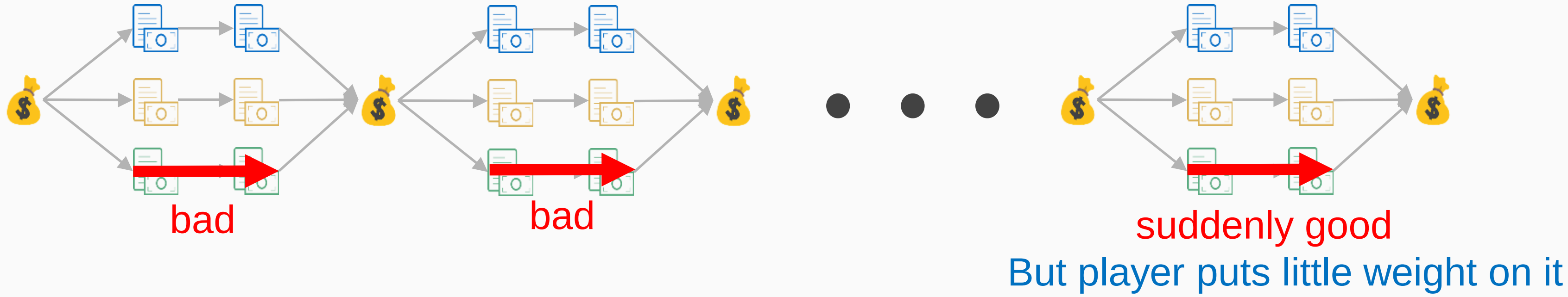

Barrons (**Bar**rier-**R**egularized-**ONS**) compared to ONS:

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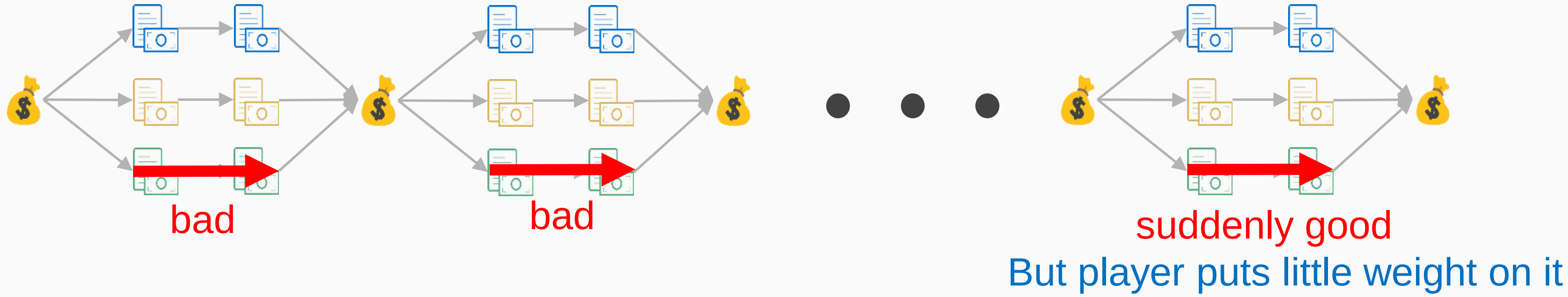

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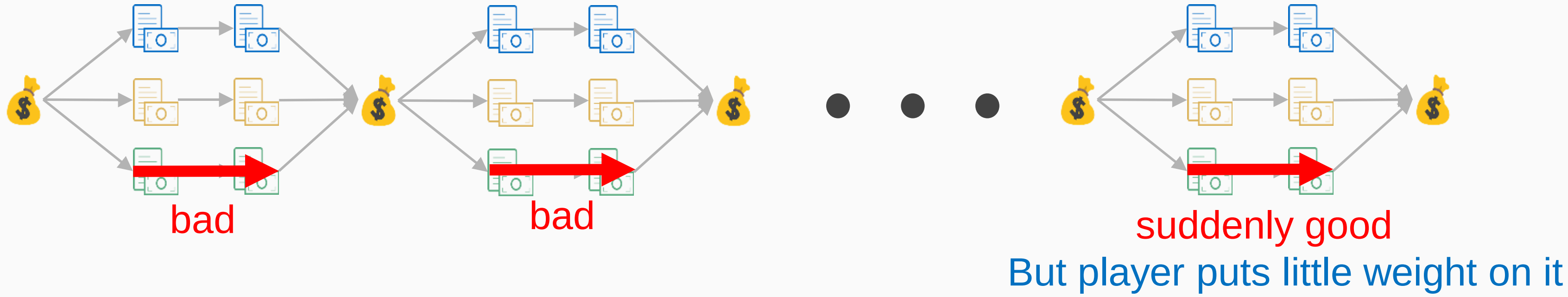

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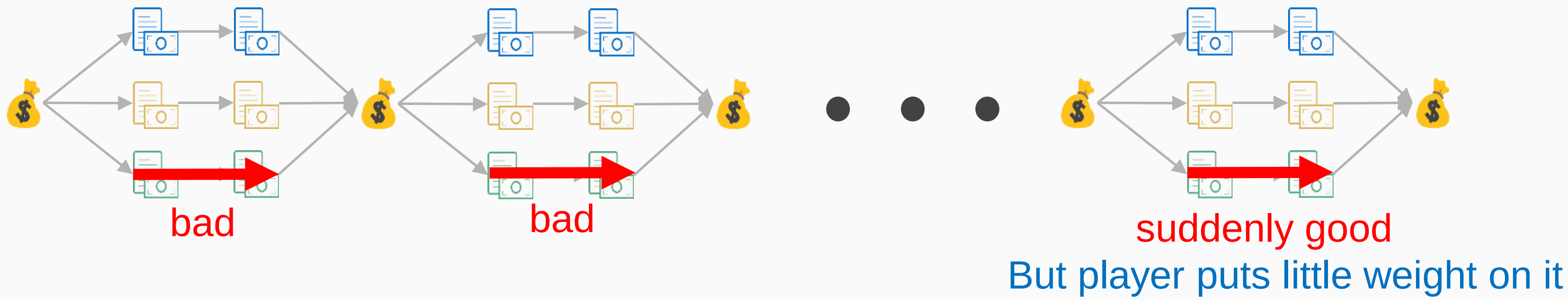

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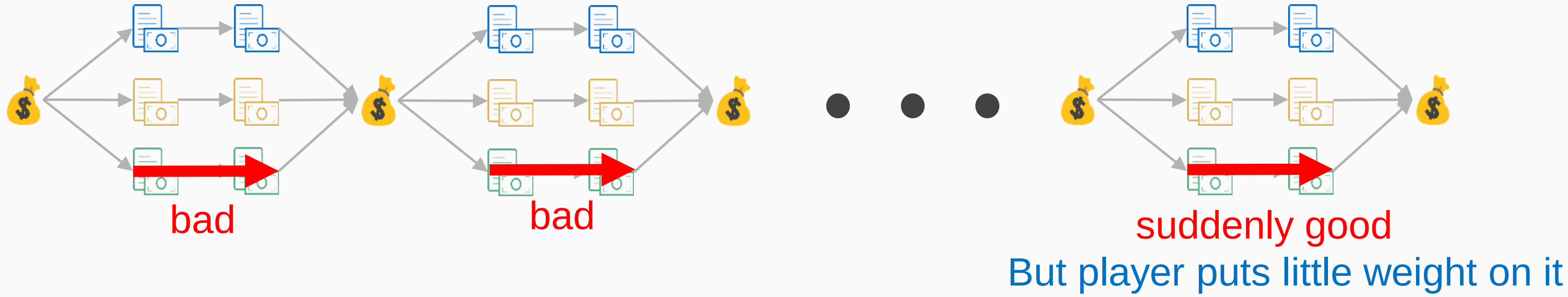

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