Almost Optimal Algorithms for Linear Stochastic Bandits with Heavy-Tailed Payoffs

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Linear Stochastic Bandits (LSB)

Previous setting

\[ x_1 = \{ \text{This is an old arm} \} \]

or \[ x_1 = [0, 1, 0, \cdots, 1] \]

or \[ x_1, t \in \mathbb{R}^d \]

Exploration \[ \xrightarrow{} \] True Optimal

Empirically Optimal at \( t \)

\[ x_{4, t} \]

Learning setting

1. At \( t \), an algorithm is given \( D_t \subseteq \mathbb{R}^d \)
2. Select an arm \( x_t \in D_t \), and observe \( y_t(x_t) = \langle x_t, \theta_* \rangle + \eta_t \)
3. \( \eta_t \) follows sub-Gaussian distributions
4. The goal is to maximize \( \sum_{t=1}^{T} \mathbb{E}[y_t(x_t)] \)
What Is A Heavy-Tailed Distribution?

Practical scenarios

- High-probability extreme returns in financial markets

- Many other real cases
  1. Delays in communication networks (Liebeherr 2012)
  2. Analysis of biological data (Burnecki 2015)
  3. ...
LSB with Heavy-Tailed Payoffs

Problem definition

- Multi-armed bandits with heavy-tailed payoffs (Bubeck, 2013)
  \[ \mathbb{E}[\eta_t^{1+\epsilon}] < +\infty, \]  
  (1)

where \( \epsilon \in (0, 1] \)

- Our setting: LSB with \( \eta_t \) satisfying Eq. (1)
  - Weaker assumption
  - Medina and Yang (2016) cannot recover the sub-Gaussian case when \( \epsilon = 1 \)
Algorithm: **Median of means under OFU (MENU)**
Framework comparison with MoM by Medina and Yang (2016)

(a) Framework of MENU

(b) Framework of MoM
Experimental Results

Figure 1: Comparison of cumulative payoffs for a synthetic dataset.
Table 1: Comparison of four algorithms.

<table>
<thead>
<tr>
<th>algorithm</th>
<th>MoM</th>
<th>MENU</th>
<th>CRT</th>
<th>TOFU</th>
</tr>
</thead>
<tbody>
<tr>
<td>regret</td>
<td>$\tilde{O}(T^{\frac{1+2\epsilon}{1+3\epsilon}})$</td>
<td>$\tilde{O}(T^{\frac{1}{1+\epsilon}})$</td>
<td>$\tilde{O}(T^{\frac{1}{2}+\frac{1}{2(1+\epsilon)}})$</td>
<td>$\tilde{O}(T^{\frac{1}{1+\epsilon}})$</td>
</tr>
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Lower bound: $\Omega(T^{\frac{1}{1+\epsilon}})$

Poster: Dec. 5th, 10:45 AM – 12:45 PM
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