Wasserstein Distributionally Robust Kalman Filtering

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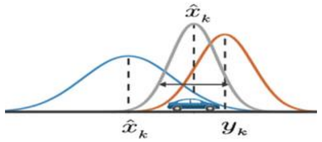
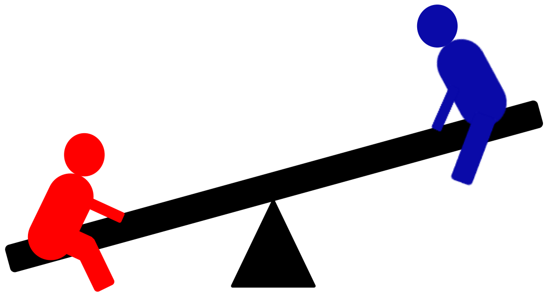
Minimum mean square error estimation

$$\inf_{\psi(\cdot)} \mathcal{R}(\psi, \mathbb{P}) \text{ for } \mathcal{R}(\psi, \mathbb{P}) := \mathbb{E}^{\mathbb{P}}\left[\|x - \psi(y)\|^2\right]$$



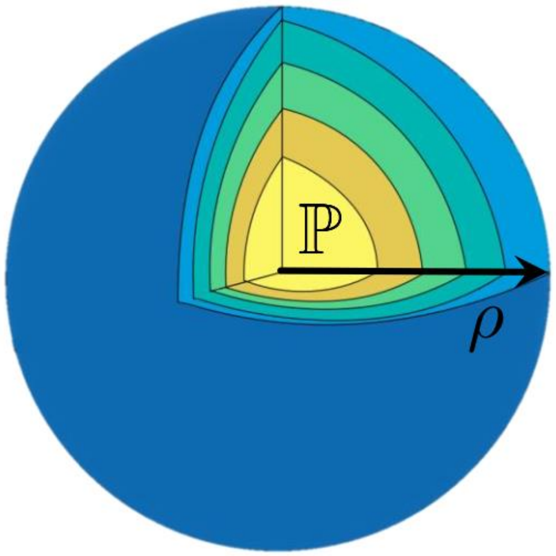
Zero-sum game against nature



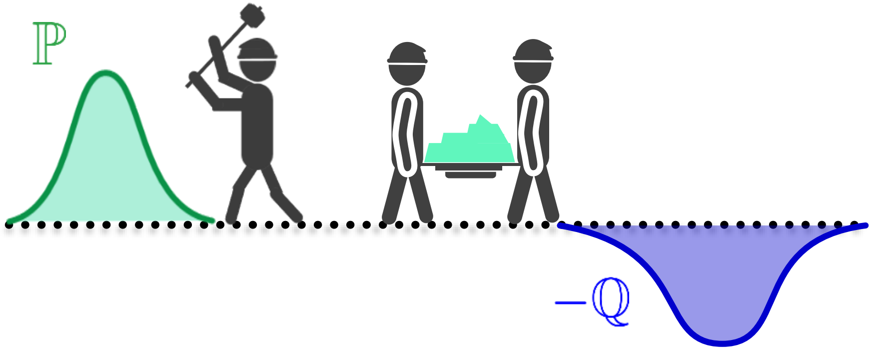
$$\inf_{\psi(\cdot)} \sup_{\mathbb{Q} \in \mathcal{P}} \mathcal{R}(\psi, \mathbb{Q})$$

$$\mathcal{P} := \{\mathbb{Q} \text{ is normal} : W(\mathbb{Q}, \mathbb{P}) \leq \rho\}$$

Wasserstein distance



$$W(\mathbb{Q}, \mathbb{P}) = \sqrt{\inf_{\substack{\xi_1 \sim \mathbb{Q} \\ \xi_2 \sim \mathbb{P}}} \mathbb{E}\|\xi_1 - \xi_2\|_2^2}$$



Optimal estimator

$$\psi^\star \in \operatorname*{argmin}_{\psi(\cdot)} \sup_{\mathbb{Q}\in\mathcal{P}} \mathcal{R}(\psi,\mathbb{Q})$$

Least favorable prior

$$\mathbb{Q}^\star \in \operatorname*{argmax}_{\mathbb{Q}\in\mathcal{P}} \inf_{\psi(\cdot)} \mathcal{R}(\psi,\mathbb{Q})$$

Structural results

Result 1. $\inf_{\psi(\cdot)} \sup_{\mathbb{Q}\in\mathcal{P}} \mathcal{R}(\psi,\mathbb{Q}) = \sup_{\mathbb{Q}\in\mathcal{P}} \inf_{\psi(\cdot)} \mathcal{R}(\psi,\mathbb{Q}) \implies (\psi^\star,\mathbb{Q}^\star) = \text{Nash equilibrium}$

Result 2. $\sup_{\mathbb{Q}\in\mathcal{P}} \inf_{\psi(\cdot)} \mathcal{R}(\psi,\mathbb{Q}) \iff$ SDP, $\psi^\star$ is affine, $\mathbb{Q}^\star = \mathcal{N}(\mu, S^\star)$

Frank-Wolfe algorithm

Result 3. Analytically solvable oracle subproblem

Result 4. Guaranteed convergence speed $\mathcal{O}\left(\frac{1}{k}\right)$

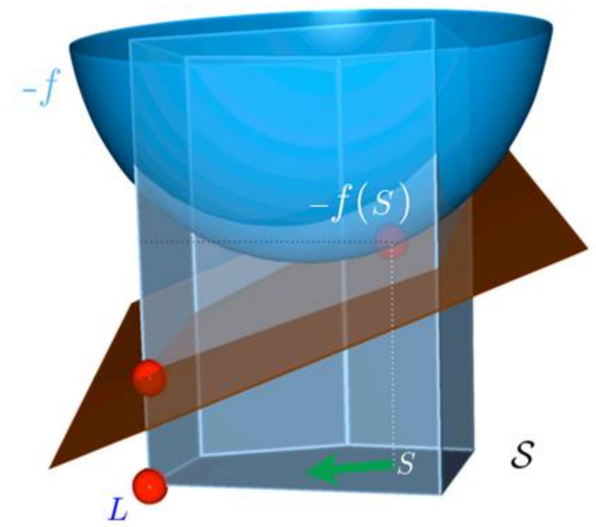
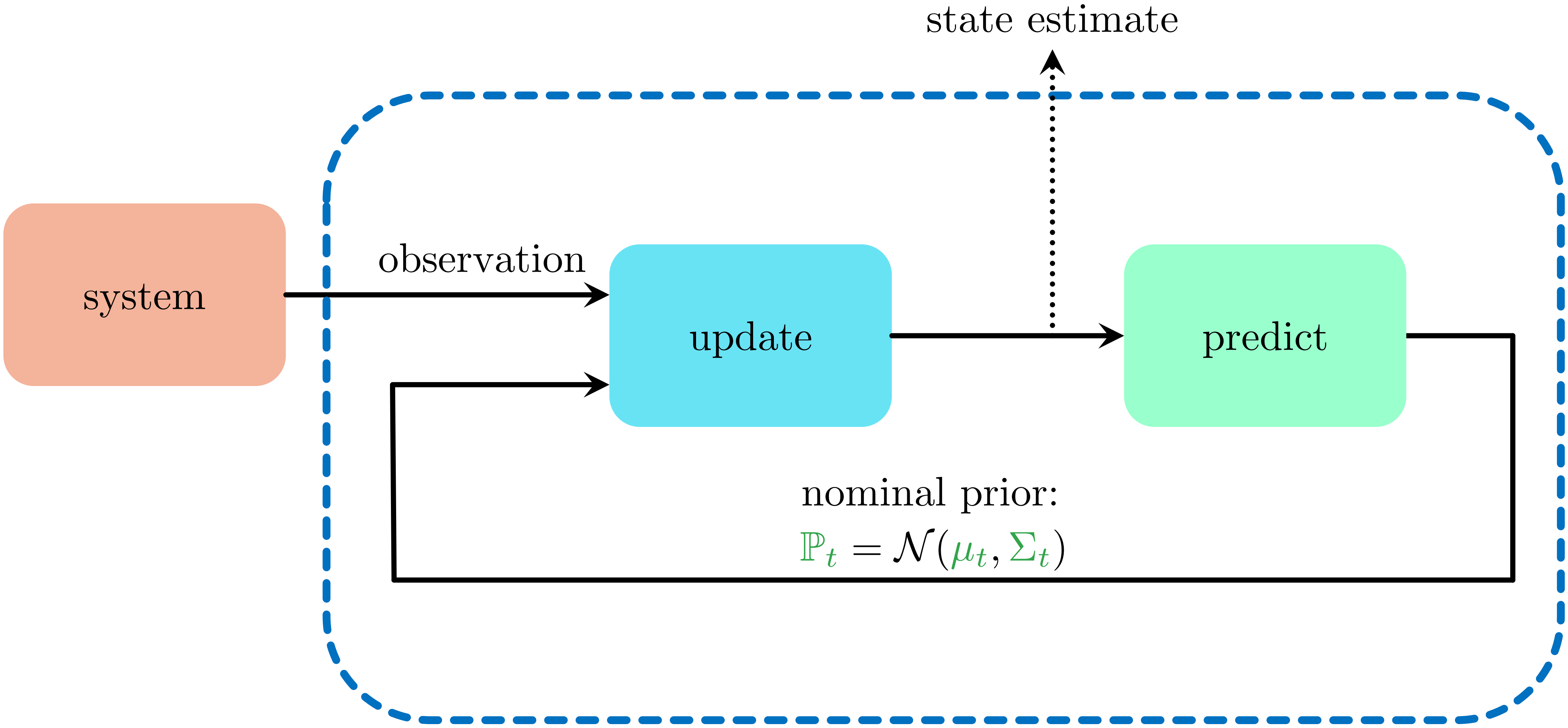
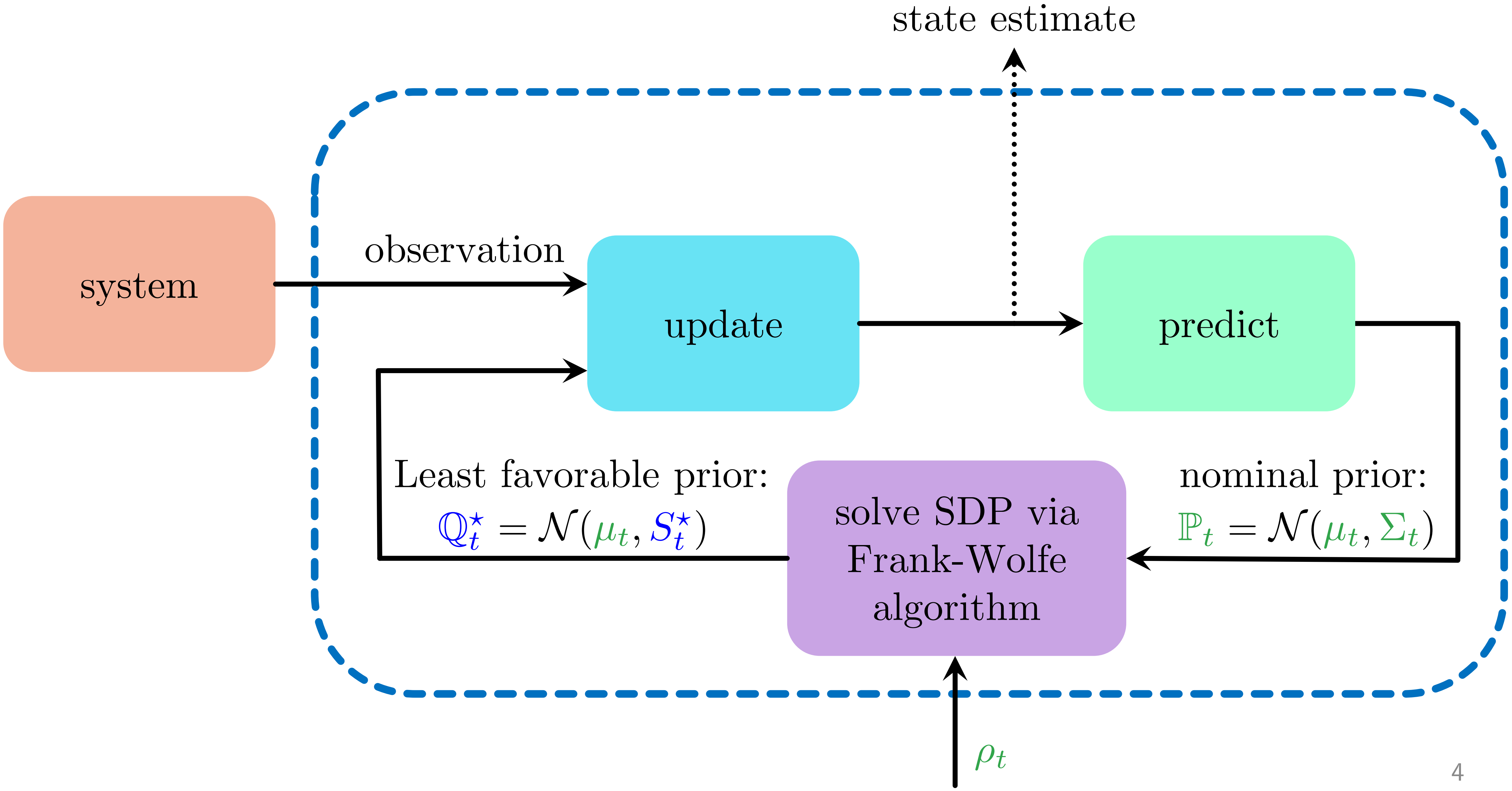


Image source: Jaggi, ICML (2013)

Classical Kalman filter

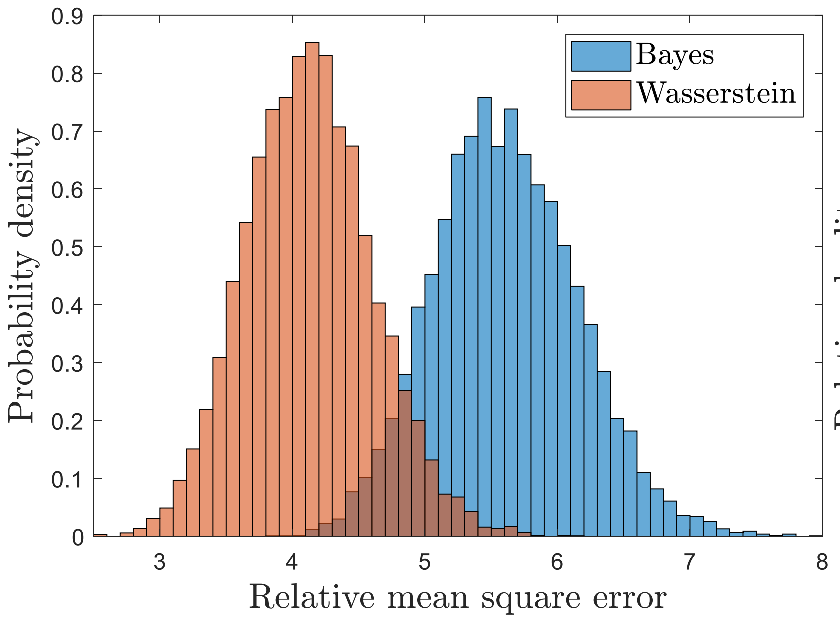


Distributionally robust Kalman filter



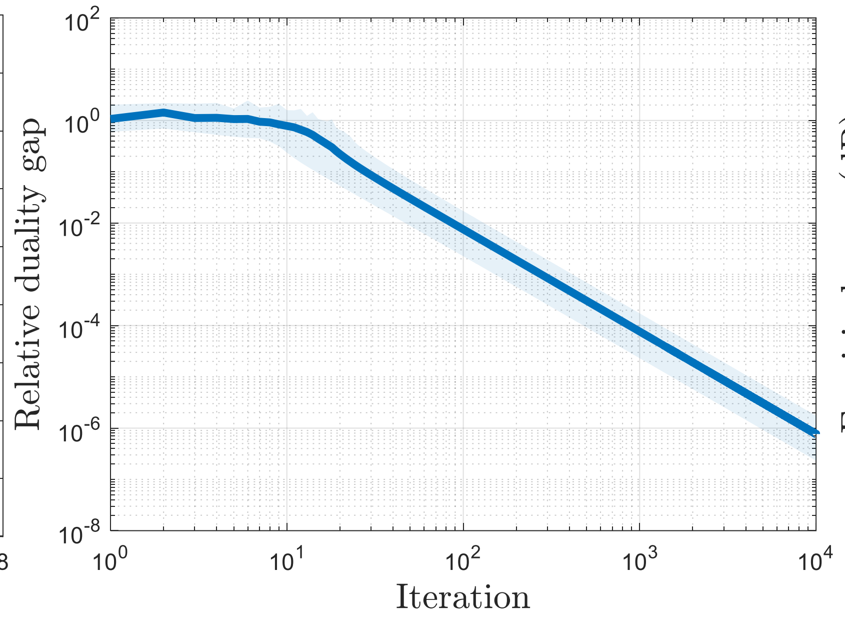
Numerical results

MMSE estimation



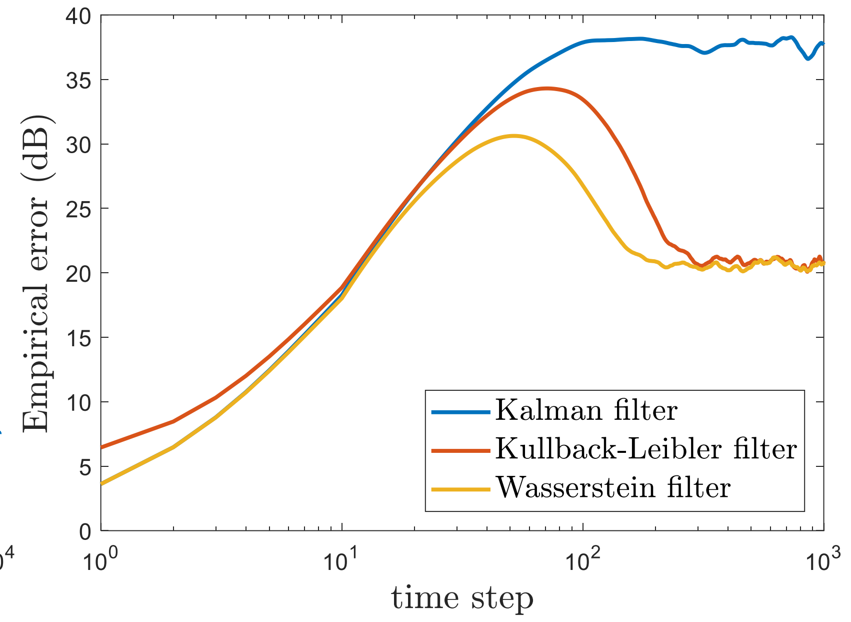
Robustness reduces regret

Frank-Wolfe algorithm



Empirical convergence speed $\mathcal{O}(\frac{1}{k})$

Kalman filtering



Wasserstein filter displays:
Lowest steady-state error
Fastest convergence

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