Probabilistic Watershed

Sampling all spanning forests for seeded segmentation and semi-supervised learning

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What do we do?

We count forests!

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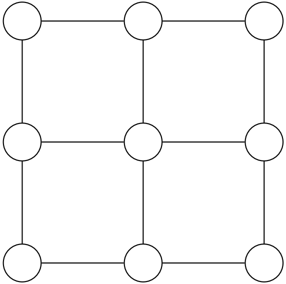
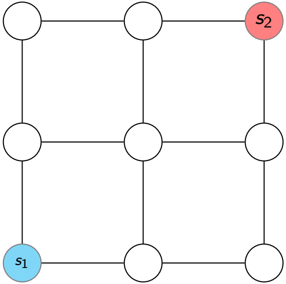


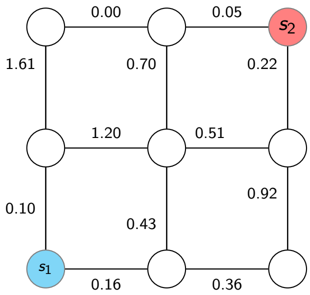
- **Graph**

Framework

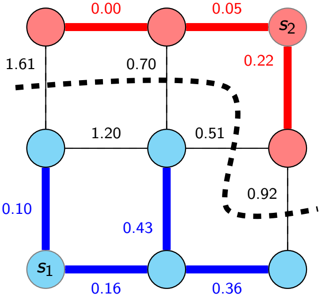


- Graph
- **Seeds (labeled nodes)**

Framework

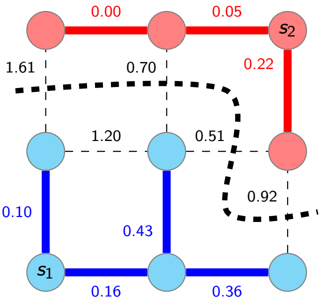


- Graph
- Seeds (labeled nodes)
- **Edge-Costs∼affinity between nodes**

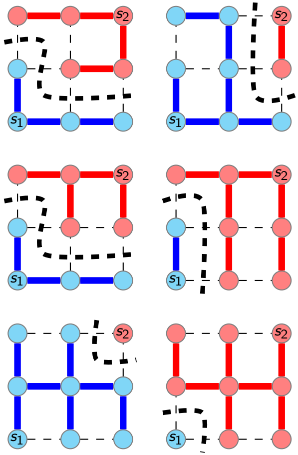


- Graph
- Seeds (labeled nodes)
- Edge-Costs∼affinity between nodes
- **Forest**

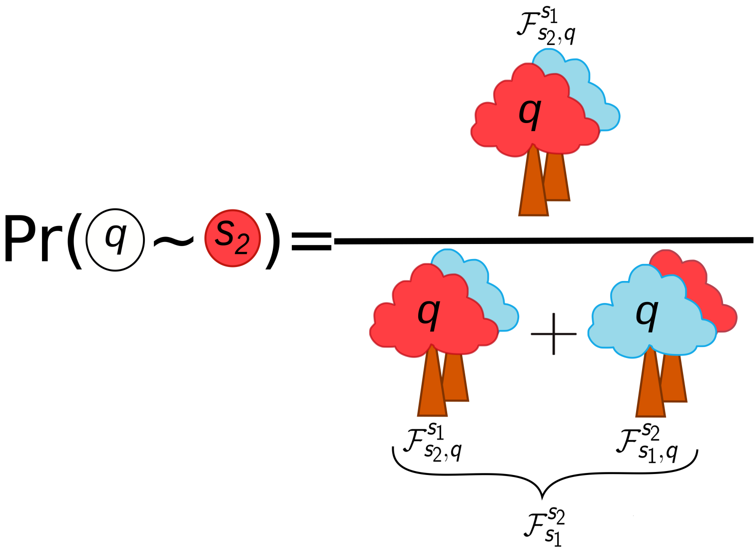
Forests

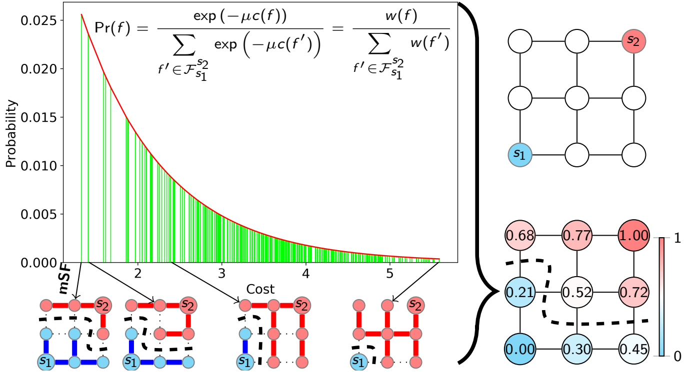


Watershed forest/
minimum cost Spanning Forest (**mSF**)



Counting Forests

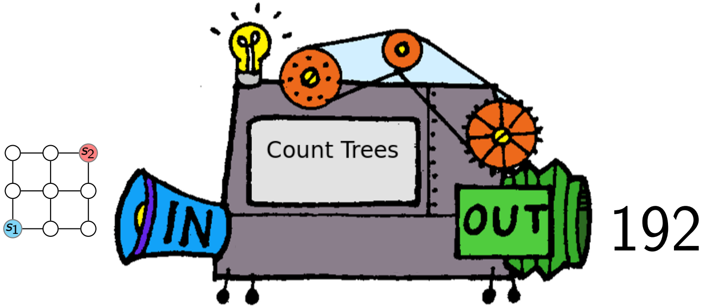




Probabilistic Watershed

$$\rightarrow \Pr(q \sim s_2) := \sum_{f \in \mathcal{F}_{s_2,q}^{s_1}} \Pr(f) = \frac{w\left(\mathcal{F}_{s_2,q}^{s_1}\right)}{w\left(\mathcal{F}_{s_2}^{s_1}\right)} = \frac{\sum_{f \in \mathcal{F}_{s_2,q}^{s_1}} w(f)}{\sum_{f \in \mathcal{F}_{s_2}^{s_1}} w(f)}.$$

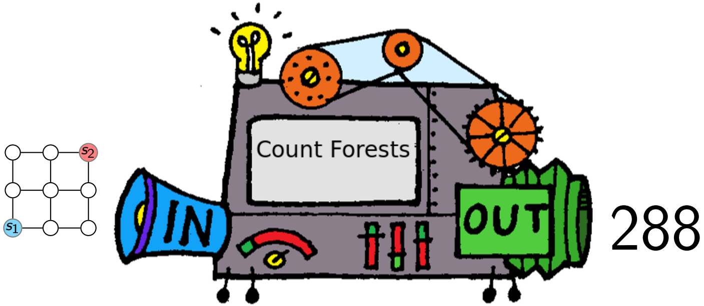
How do we count the forests?



Matrix Tree Theorem [Kirchhoff, 1847]

Let $G = (V, E, w)$ an edge-weighted multigraph, $w(\mathcal{T})$,the sum of the weights of the spanning trees of G, $\mathbb{1}$ is a column vector of 1's, L is the Laplacian matrix and $L^{[v]}$ is the Laplacian matrix after removing an abitrary row and column v, then

$$w(\mathcal{T}) := \sum_{t \in \mathcal{T}} w(t) = \sum_{t \in \mathcal{T}} \prod_{e \in E_t} w(e) = \frac{1}{|V|} \det\left(L + \frac{1}{|V|} \mathbb{1}\mathbb{1}^\top\right) = \det(L^{[v]}).$$

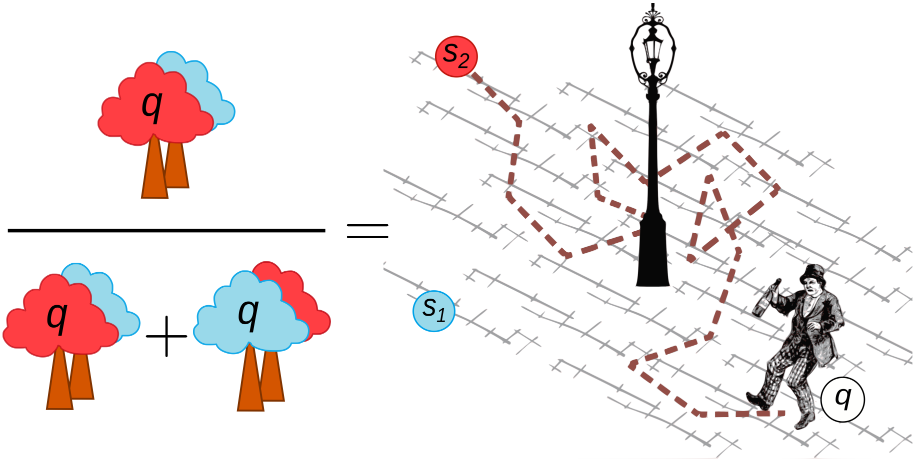


Modification Matrix Tree Theorem

Let $G = (V, E, w)$ be an undirected edge-weighted connected graph, r^{eff}_{uv} the effective resistance distance between $u, v \in V$ arbitrary vertices and $w(\mathcal{F}^v_u)$ the sum of the weights of the 2-trees spanning forests separating u and v, then

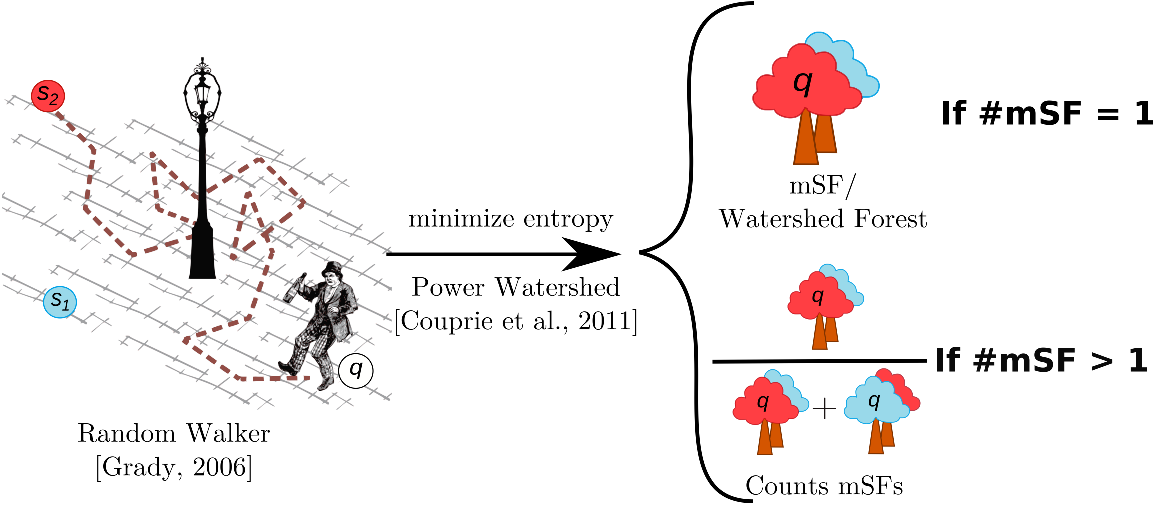
$$w(\mathcal{F}^v_u) = w(\mathcal{T}) r^{\text{eff}}_{uv}.$$

Probabilistic Watershed=Random Walker[Grady, 2006]



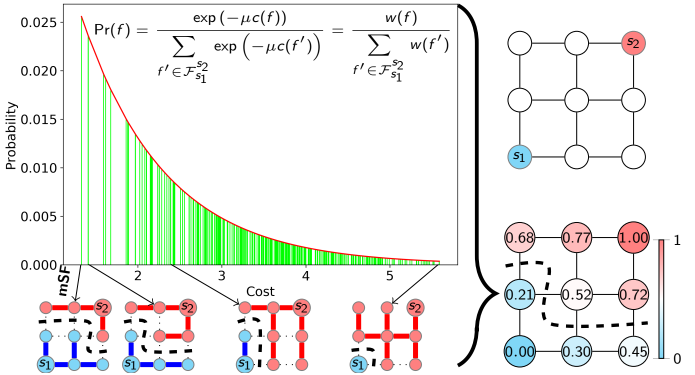
Probabilistic Watershed Random Walker [Grady, 2006]

Power Watershed new interpretation



Summary

- Probabilistic Watershed=Random Walker [Grady, 2006].
- New interpretations of the Power Watershed [Couprie et al., 2011].
- Technique to count forests.



Poster

Room East Exhibition Hall B + C #81

10:45 AM – 12:45 PM

References

Couprie, C., Grady, L., Najman, L., and Talbot, H. (2011).
Power watershed: A unifying graph-based optimization framework.
IEEE Transactions on Pattern Analysis and Machine Intelligence.

Grady, L. (2006).
Random walks for image segmentation.
IEEE Transactions on Pattern Analysis and Machine Intelligence.

Kirchhoff, G. (1847).
Über die Auflösung der Gleichungen, auf welche man bei der Untersuchung der linearen Vertheilung galvanischer Ströme geführt wird.
Annalen der Physik, 148:497–508.