Batched Multi-armed Bandits Problem

Yanjun Han (Stanford EE)

Joint work with:

Zijun Gao	Stanford Stats
Zhimei Ren	Stanford Stats
Zhengqing Zhou	Stanford Math

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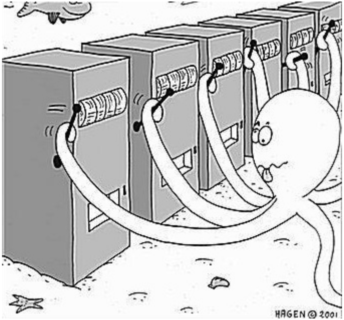
Background: Multi-armed Bandits (MAB)

- sequential decision making
- time horizon T
- action space: K arms
- random reward for each action
- target: maximize the cumulative rewards



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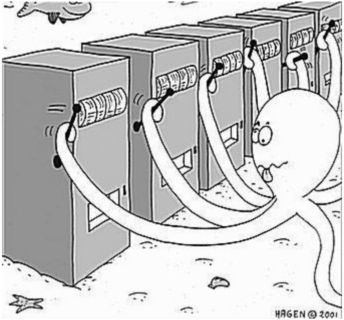

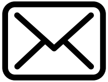

Spam filtering

Dynamic pricing

Recommender system

Partial Information in the "Space" Domain

Space Domain: Bandit Feedback

Only the reward of the pulled arm is revealed.

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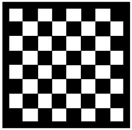

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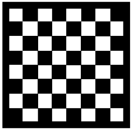

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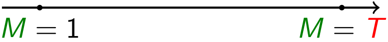

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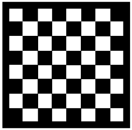

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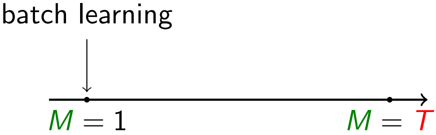

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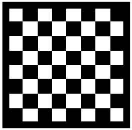

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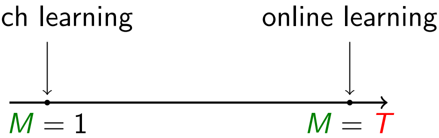

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- adaptive grid: the next grid point determined by historic data
- task: design policy + grid

Two Types of Regrets

Tight analysis of stochastic MAB [Vog'60, LR'85, AB'09]:

$$\mathbb{E}[R(\pi^1)] \leq C \cdot \sqrt{KT}$$

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$$R_{\text{pro-dep}}(K, M, T) = \inf_{\pi, \mathcal{T}} \sup_{\Delta > 0} \Delta \cdot \sup_{\Delta_i \in \{0\} \cup [\Delta, \sqrt{K}]} \mathbb{E}[R(\pi)]$$

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Full online case:

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Two-armed case with static grid [PRCS'16]:

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Lower bounds typically very challenging [JJNZ'16, AAAK'17, DRY'18, ...].

Main Result I: Upper Bound

Theorem 1 (Upper Bound)

There exist policies π^1, π^2 such that

$$\mathbb{E}[R(\pi^1)] \le \text{polylog}(K, T) \cdot \sqrt{K} T^{\frac{1}{2-2^{1-M}}}$$

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- $M = \log\log T$ batches sufficient for centralized minimax regret
- $M = \log T$ batches sufficient for centralized problem-dependent regret

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  estimate the mean reward for each active arm;
  eliminate all probably suboptimal arms from $\mathcal{A}$.
end for

Optimal Grid Design

Minimax Grid

$\mathcal{T}_{\text{minimax}} = \{t_1, \cdots, t_M\}$ with

$$t_1 = a, \qquad t_m = \lfloor a\sqrt{t_{m-1}} \rfloor,$$

where a is chosen such that $t_M = T$.

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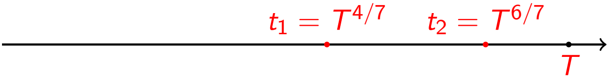

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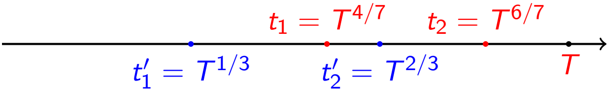

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Theorem 2 (Static Lower Bound)

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- proof uses a max-min approach: find multiple fixed reward distributions under which no policy performs uniformly well

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Indistinguishability Lemma

Let $Q_1, \cdots, Q_n$ be probability measures on some common probability space. Then for any tree $T = ([n], E)$ and test Ψ,

$$\frac{1}{n}\sum_{i=1}^{n} Q_i(\Psi \neq i) \geq \sum_{(i,j)\in E} \frac{1}{2n} \exp(-D_{\mathrm{KL}}(Q_i \| Q_j)).$$

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- max-min approach breaks down even for static but randomized grid
- use a min-max approach instead: construct corresponding reward distributions after a policy is given

Min-max: More Details

Construct reward distributions $P_1, P_2, \cdots, P_M$ and events $A_1, \cdots, A_M$.

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For any policy, if $P_m(A_m)$ is not too small for some m, then the policy incurs a large regret in the worst case.

Lemma 2 (Covering of Events)

For any policy it holds that

$$\sum_{m=1}^{M} P_m(A_m) \geq \frac{1}{2}.$$

Concluding Remarks

Take-home message:

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Thank you!