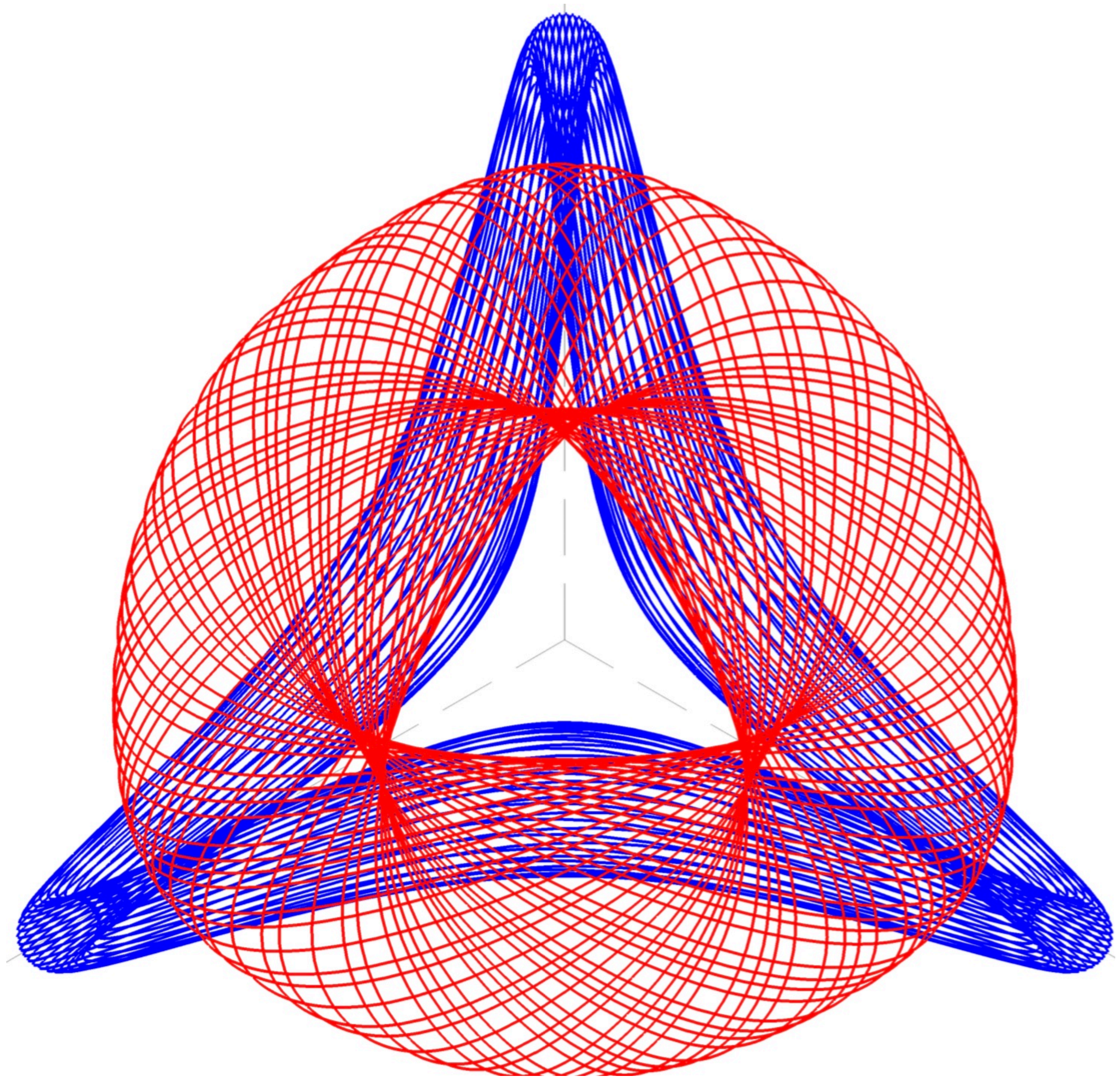
Poincaré Recurrence, Cycles and Spurious Equilibria in Gradient Descent Ascent for Non-Convex Non-Concave Zero-Sum Games.



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Our work

This is the first **theoretical** paper that analyzes **vanilla GDA** in non-convex non-concave zero-sum games:

Takeaways:

i) GDA **does not solve** always zero-sum games

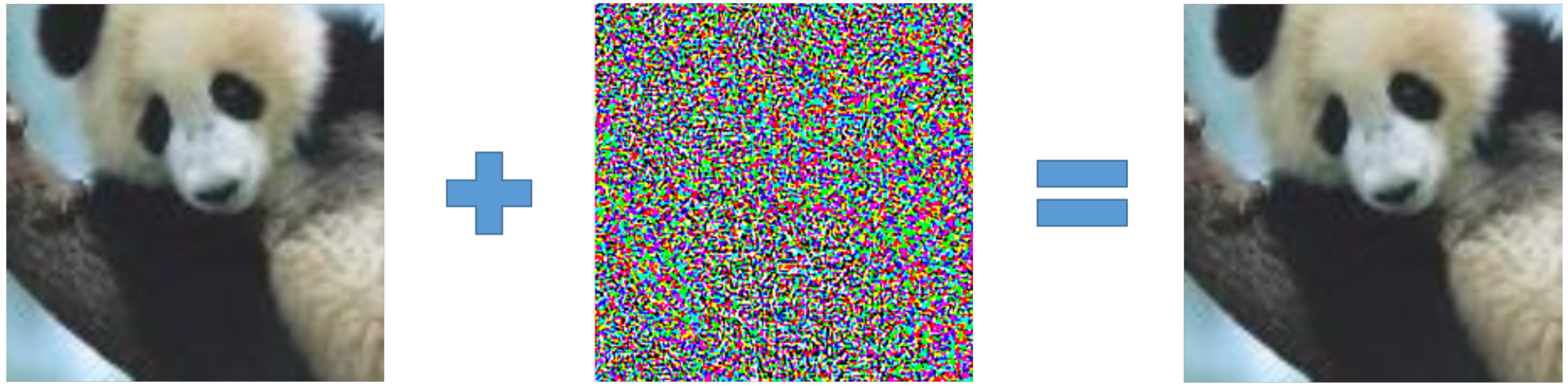
ii) Many distinct failure modes **provably exist** including **cycles** and **spurious equilibria**.

iii) To understand these settings we need **physics** + **non-convex optimization** combined.

Motivation

i) Generative Adversarial Networks

ii) Adversarial Learning



"panda"
57.7% confidence

perturbation

"gibbon"
99.3% confidence

iii) Multi-agent Reinforcement learning

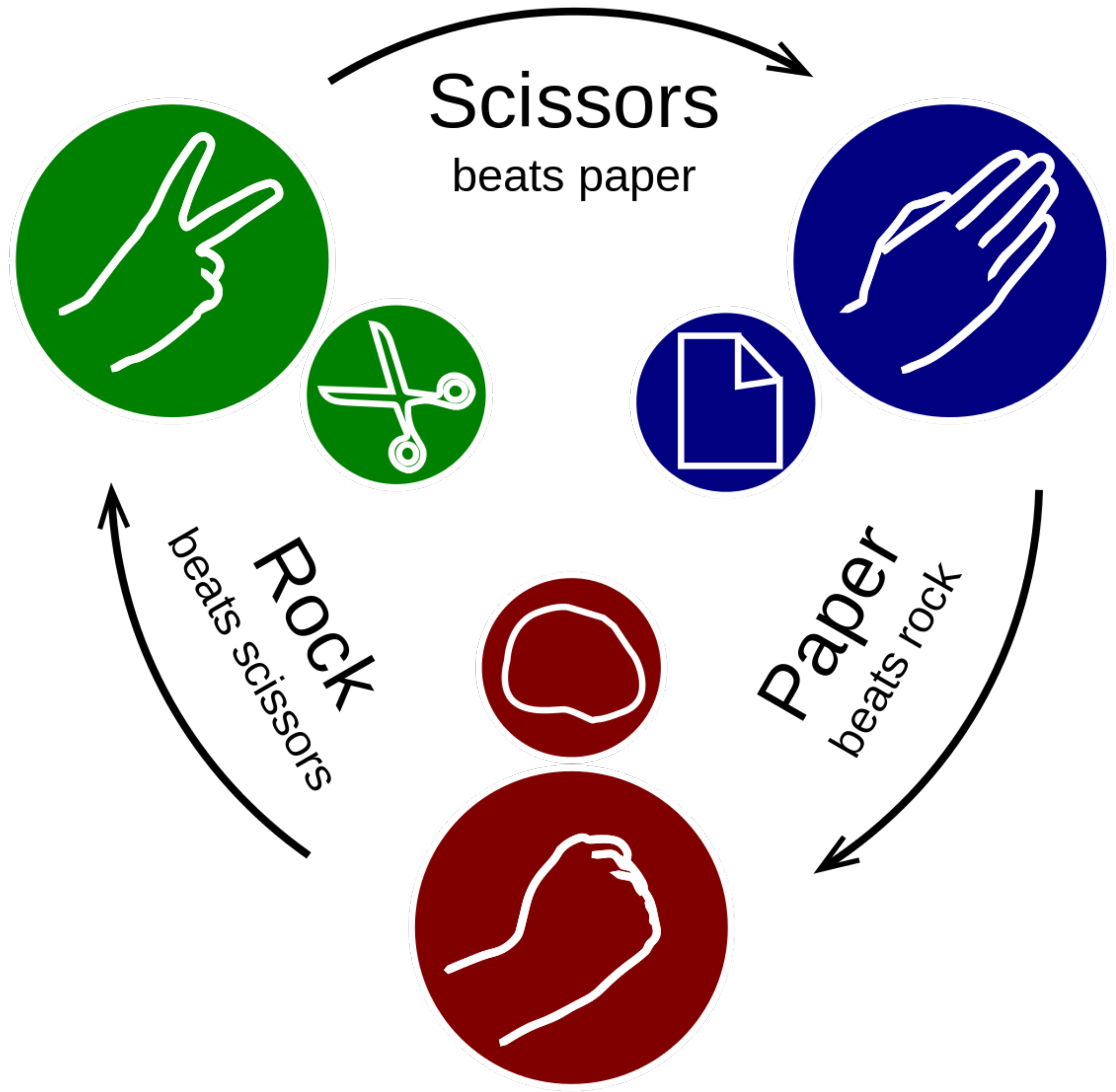
Prior work: Bilinear Games

Zero Sum Game

$$\min_{\mathbf{x}\in\Delta_n} \max_{\mathbf{y}\in\Delta_m} \mathbf{x}^T U \mathbf{y}$$

Example:

$$U = \begin{array}{c} \\ \text{Rock} \\ \text{Paper} \\ \text{Scissors} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Rock} & \text{Paper} & \text{Scissors} \end{array} \\ \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \end{array}$$



This work: Hidden Bilinear Games

Hidden Zero Sum Game

$$F(\mathbf{x}) = \left[f_1(\mathbf{x}_1) \cdots f_n(\mathbf{x}_n)\right] \qquad G(\mathbf{y}) = \left[g_1(\mathbf{y}_1) \cdots g_m(\mathbf{y}_m)\right]$$

$$\min_{F(\mathbf{x}) \in \Delta_n} \max_{G(\mathbf{y}) \in \Delta_m} F(\mathbf{x})^T U G(\mathbf{y})$$

- ❖ This is a well-defined problem.
- ❖ The hidden structure identifies the **correct equilibrium** that is also **meaningful**.
- ❖ It is clear that the min/max solution **does not depend on the operator**.
- ❖ GDA corresponds to the **indirect competition of players in the parameter level**.

Our Results

GDA results in a variety of behaviors antithetical to convergence

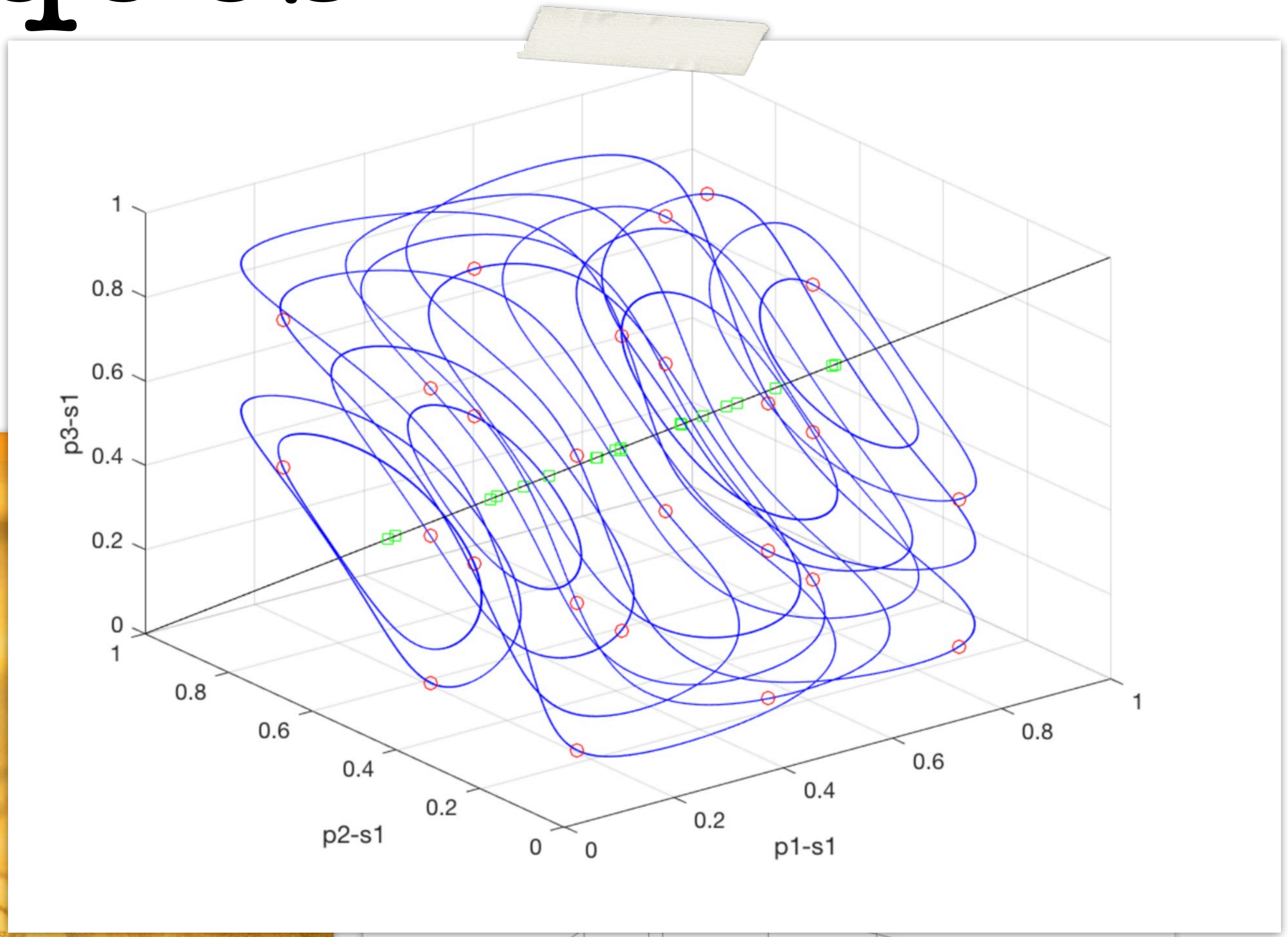
i) Convergence to **spurious equilibria** corresponding to stationary points of the operators F and G.

ii) **Cycling behavior** around the equilibrium for continuous time GDA.

iii) **Divergence** from equilibrium fo discrete time GDA.

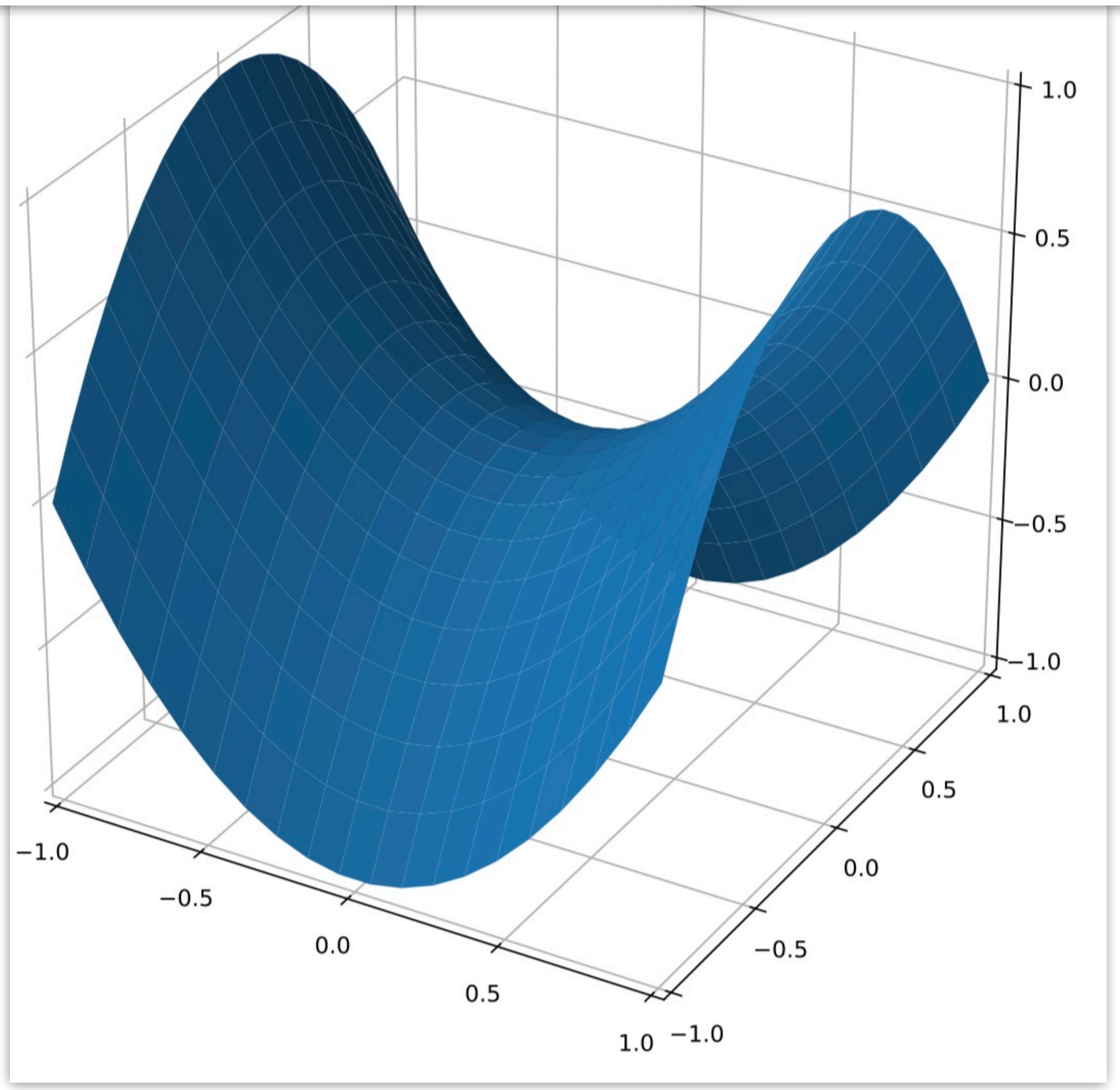
Our Techniques

- ❖ Poincaré Recurrence Theorem



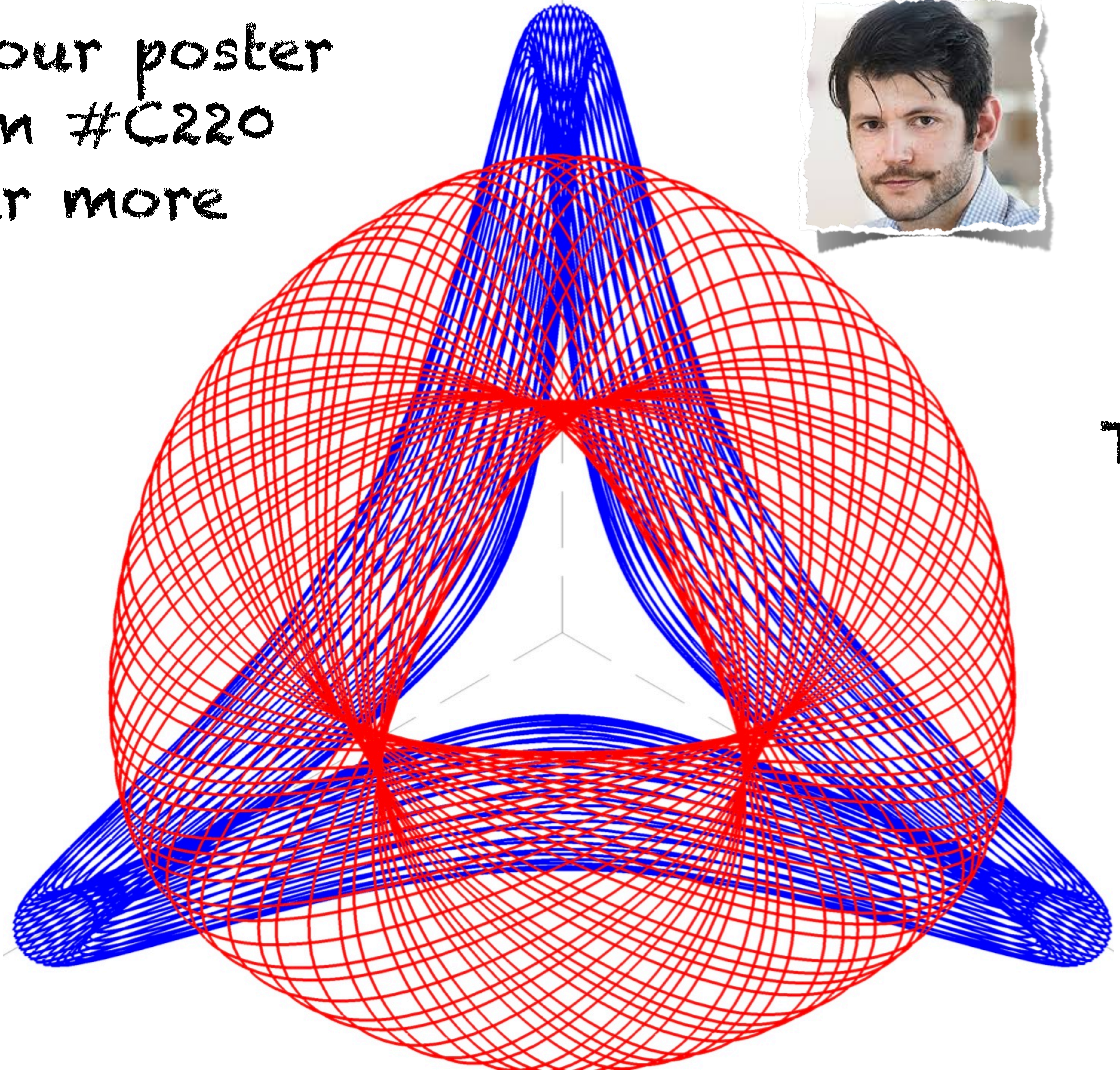
- ❖ Energy conservation

- ❖ Stable-Center Manifold Theorem



... and many more

Come to our poster
Wed 5pm #C220
To hear more



Thank you