Scalable Bayesian inference of dendritic voltage via spatiotemporal recurrent state space models

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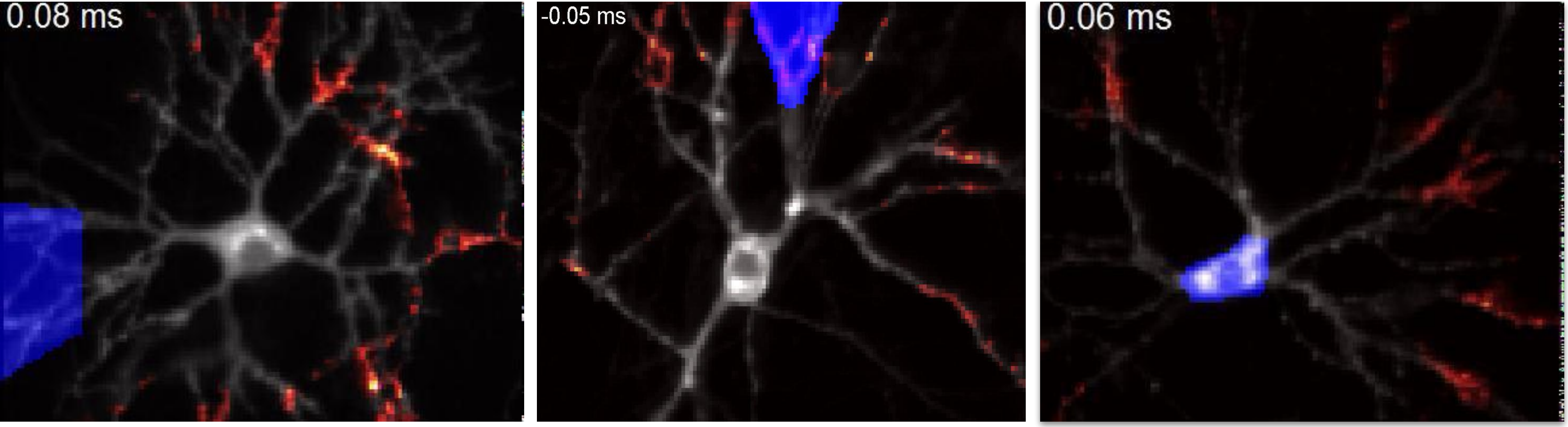
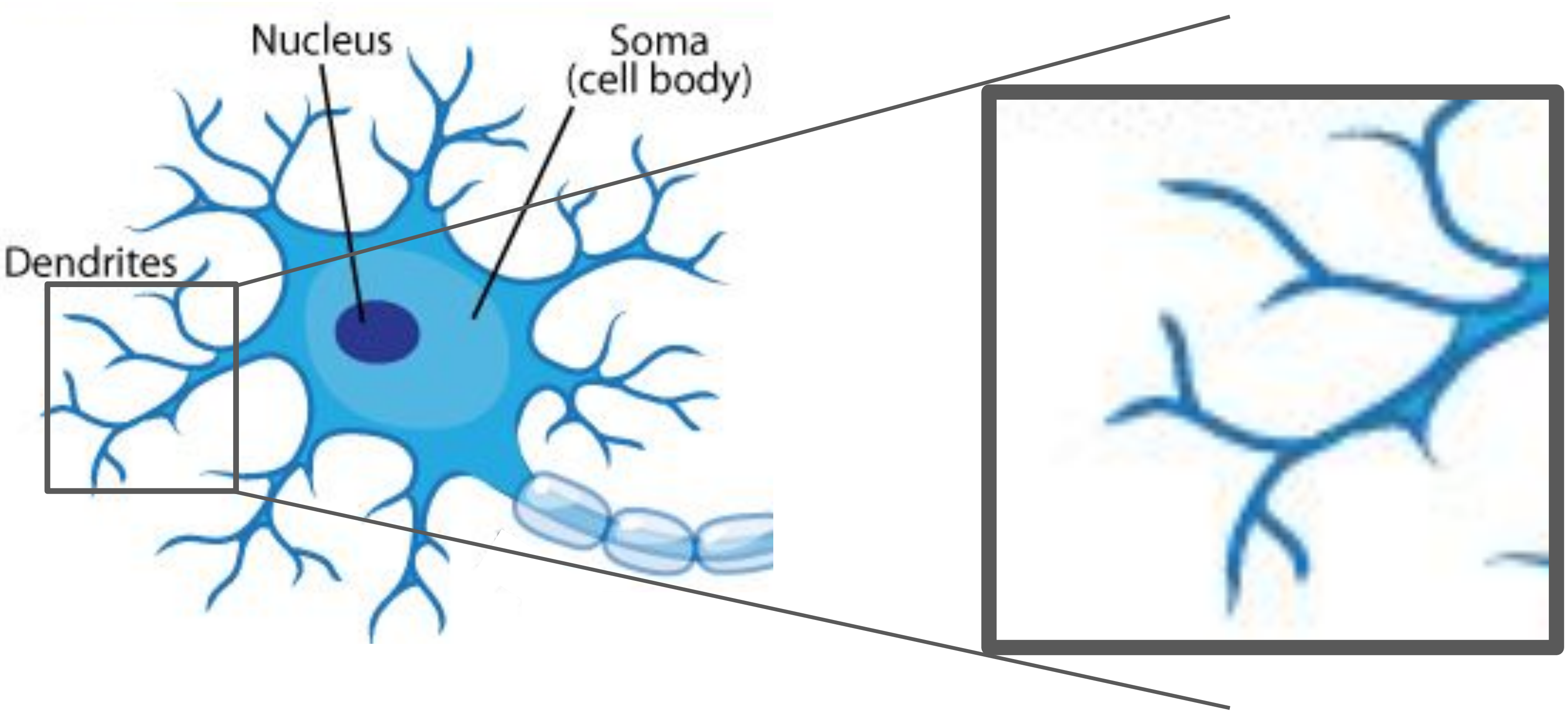
NeurIPS 2019

Dendritic voltage imaging



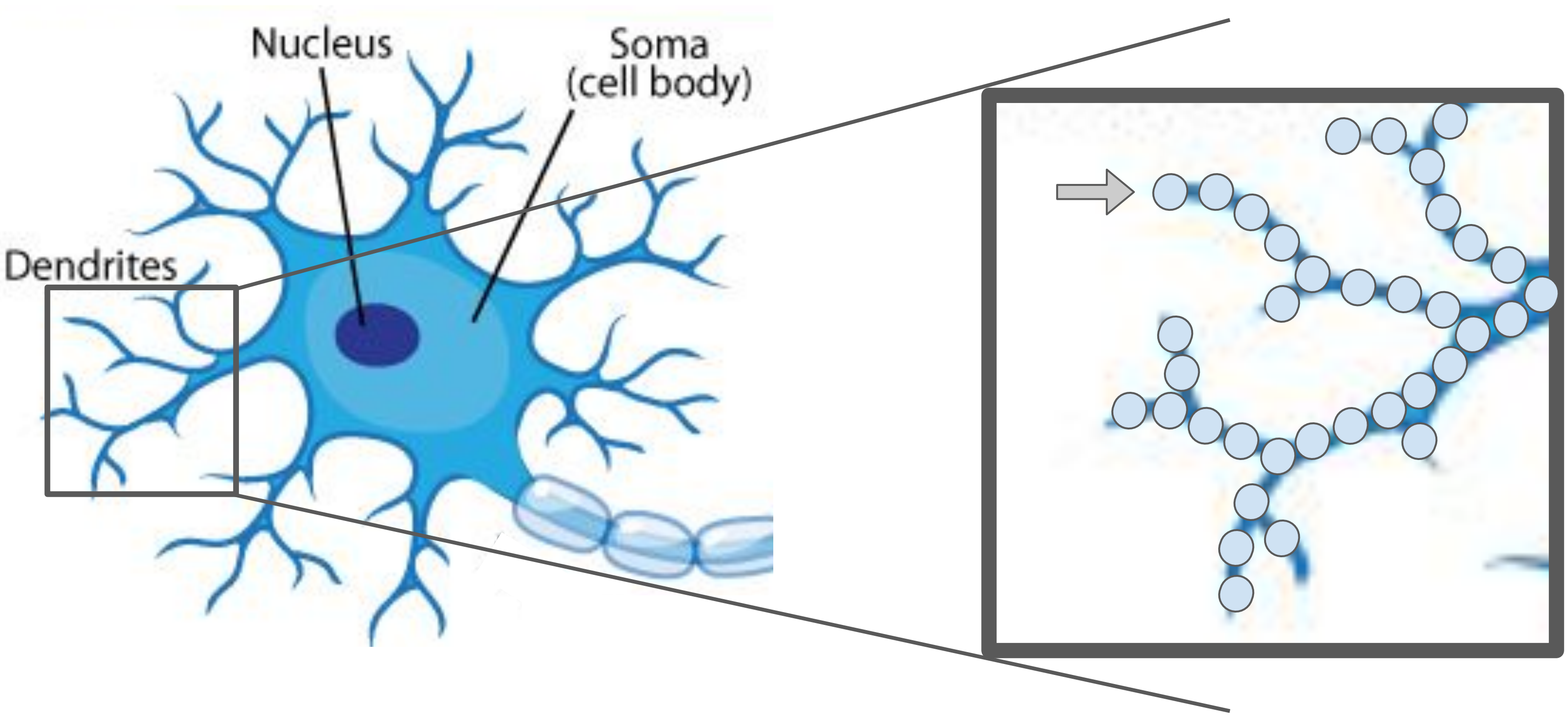
Hochbaum et al *Nature Methods, 2014*

Multiple Compartment models



https://biology.stackexchange.com/questions/44082/can-the-dendrites-of-sensory-neurons-be-a-meter-long

Multiple Compartment models



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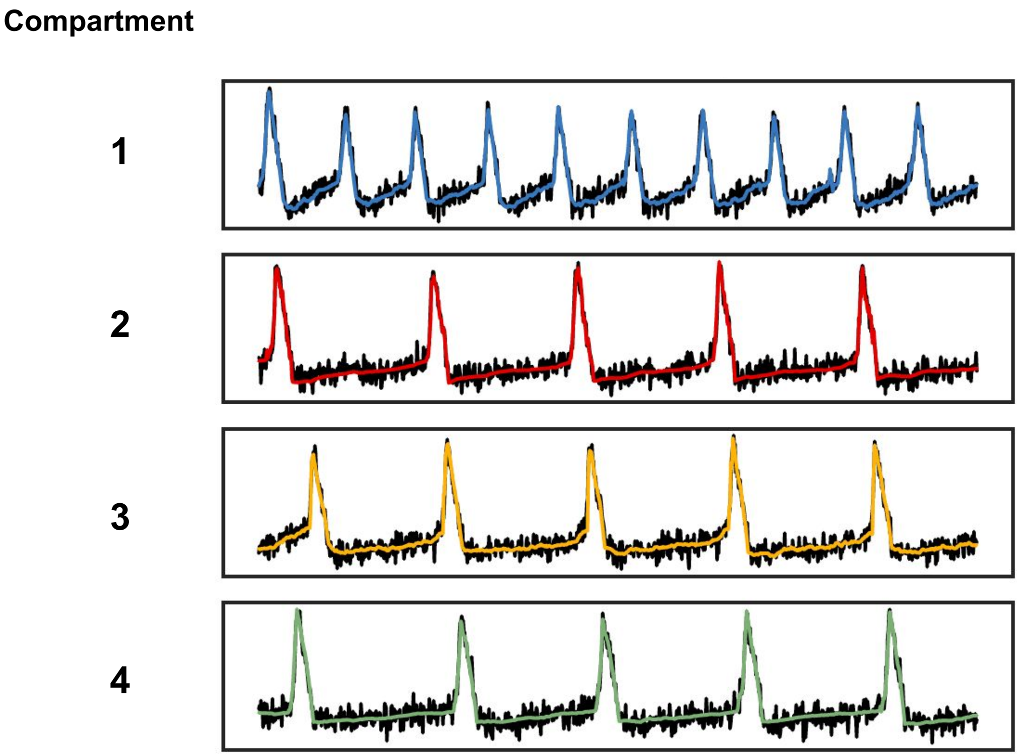

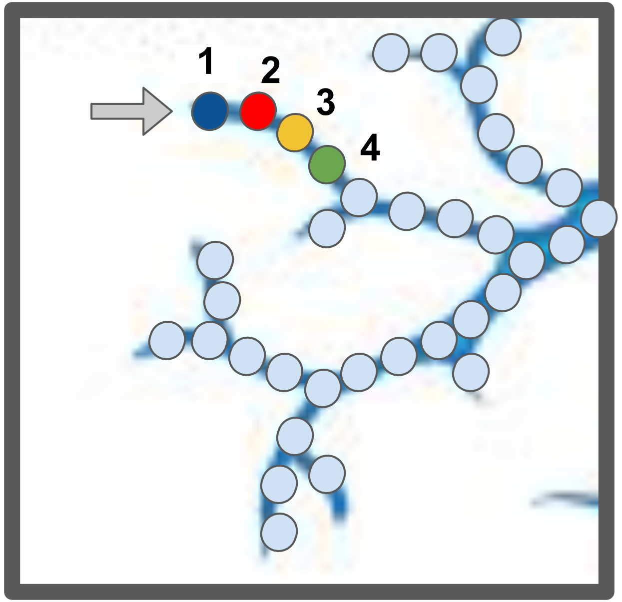

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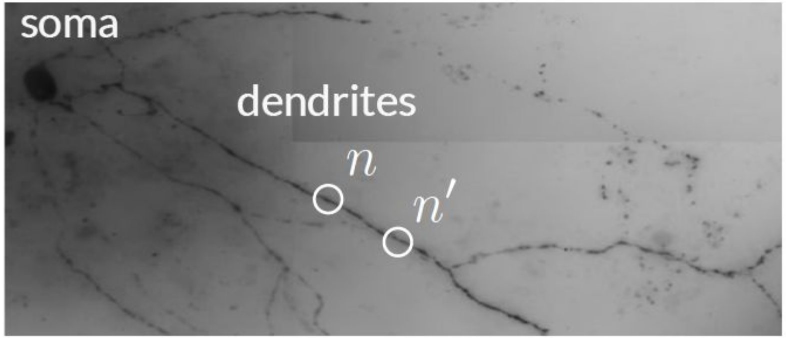
Biophysics



Cable equation theory

$$I = \frac{\Delta V}{R} = g\Delta V$$

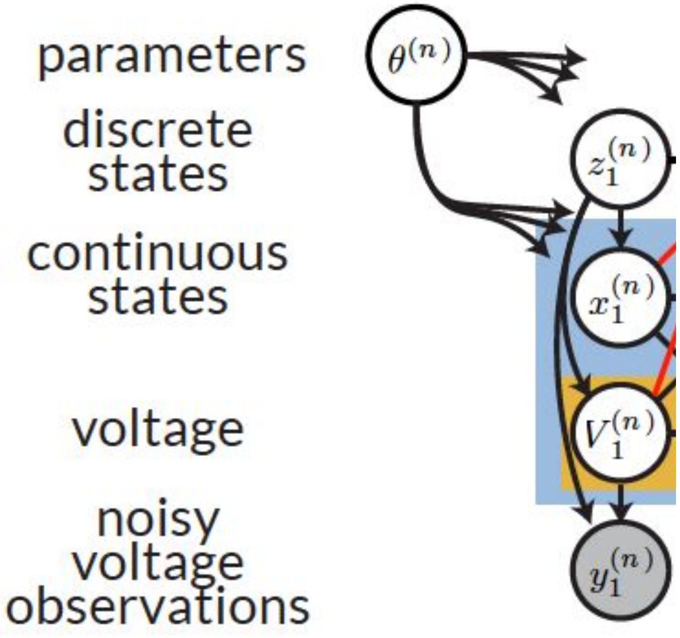
g: conductance; **I**: current; **R**: resistance; **V**: voltage; **C**: capacitance

Compartment n:

$$V_{t+\Delta t}^{(n)} \approx V_t^{(n)} + \frac{\Delta t}{C_n}\left[\sum_j I_t^{(n,j)} + \sum_{n'=1}^{N} g_{nn'} \cdot (V_t^{(n')} - V_t^{(n)})\right]$$

Biophysics to Statistics Model

Model Single Compartment Dynamics one time step



theta: parameters

Z: discrete latent variable

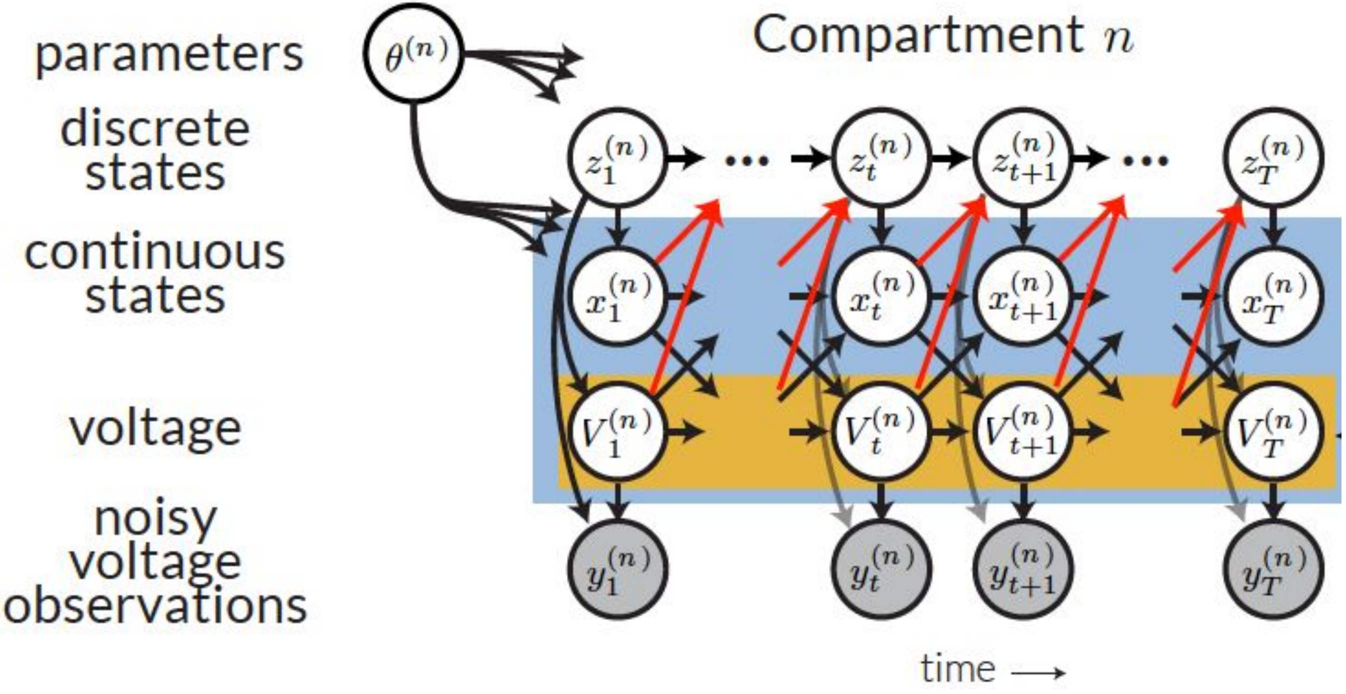
X: continuous latent variable (cycle parameters)

V: continuous latent variable (denoised voltage)

Y: observed variables

Model Single Compartment Dynamics

- Recurrent **Switching** Linear Dynamical System (rSLDs)



Statistical Model

- **Recurrent Switching Linear Dynamical System (rSLDs)**

Physical model

$$V_{t+\Delta t}^{(n)} \approx V_t^{(n)} + \frac{\Delta t}{C_n}\left[\sum_j I_t^{(n,j)} + \sum_{n'=1}^{N} g_{nn'} \cdot (V_t^{(n')} - V_t^{(n)})\right]$$

$$\mathbb{E}\left[\begin{pmatrix} V_{t+\Delta t}^{(n)} \\ x_{t+\Delta t}^{(n)} \end{pmatrix} \middle| z_t^{(n)} = k, \left\{V_t^{(n')}\right\}_{n' \neq n}\right] = \begin{pmatrix} V_t^{(n)} \\ x_t^{(n)} \end{pmatrix} + \frac{\Delta t}{C_n}\left[\underbrace{A_k^{(n)}\begin{pmatrix} V_t^{(n)} \\ x_t^{(n)} \end{pmatrix} + b_k^{(n)}}_{\approx \sum_j I_t^{(n,j)}} + \begin{pmatrix} \sum_{n'=1}^{N} g_{nn'} \cdot \left(V_t^{(n')} - V_t^{(n)}\right) \\ 0 \end{pmatrix}\right]$$

theta: parameters; **Z**: discrete latent variable; **X**: continuous latent variable (cycle parameters);
V: continuous latent variable (denoised voltage); **Y**: observed variables

Statistical Model

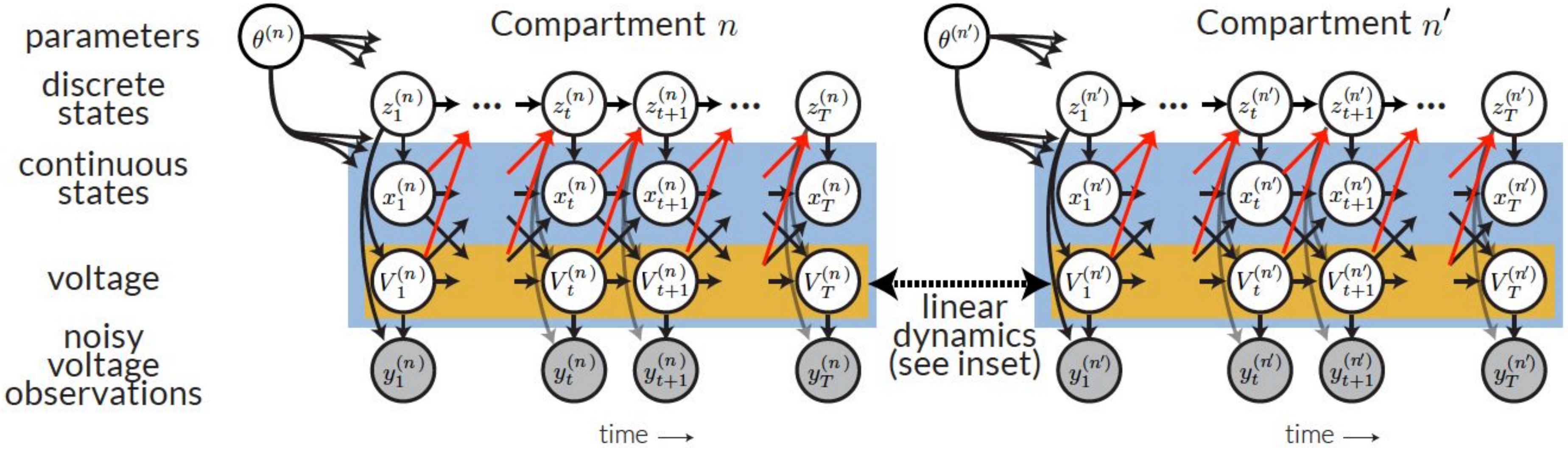
- **Recurrent Switching Linear Dynamical System (rSLDs)**

$$p(z_{t+\Delta t}^{(n)} = k \mid V_t^{(n)}, x_t^{(n)}) \propto \exp\left\{ w_k^{(n)\top} \begin{pmatrix} V_t^{(n)} \\ x_t^{(n)} \end{pmatrix} + d_k^{(n)} \right\}$$

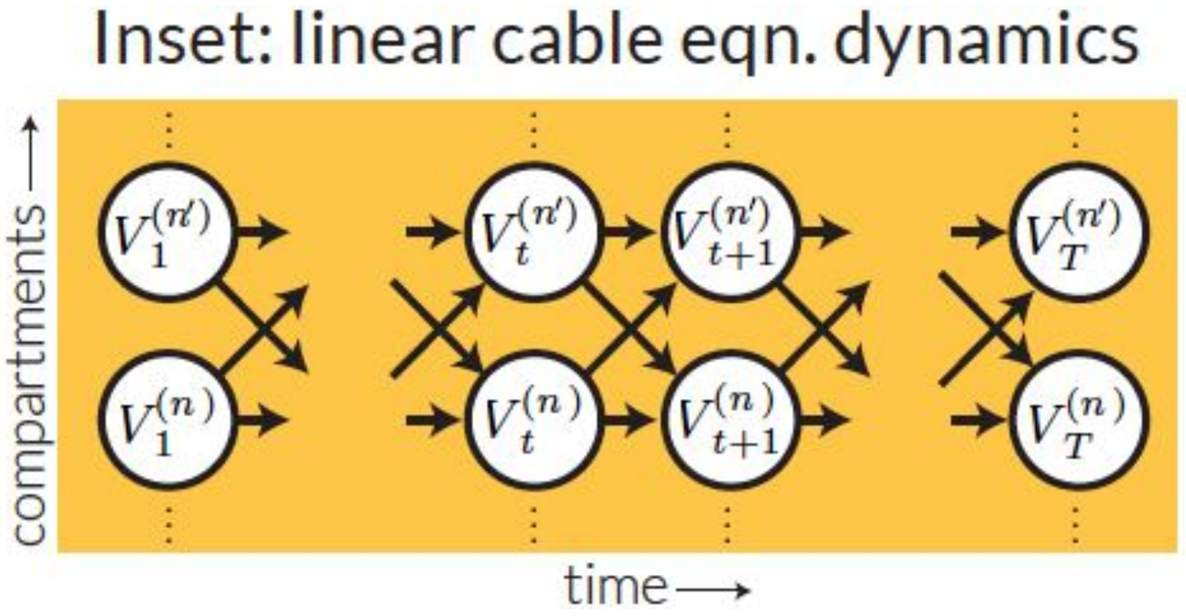
theta: parameters; **Z**: discrete latent variable; **X**: continuous latent variable (cycle parameters);
V: continuous latent variable (denoised voltage); **Y**: observed variables

Linderman et al (AISTATS 2017)

Model Inter-Compartment Dynamics



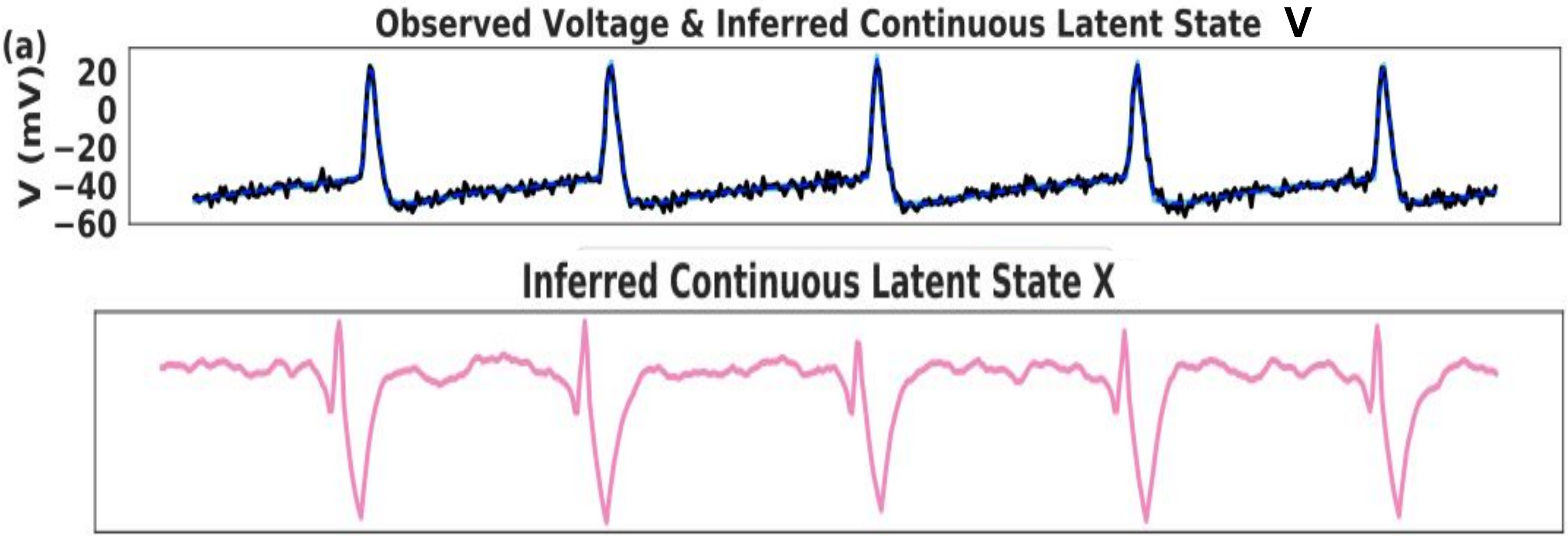
Linear Dependency between Adjacent Compartments



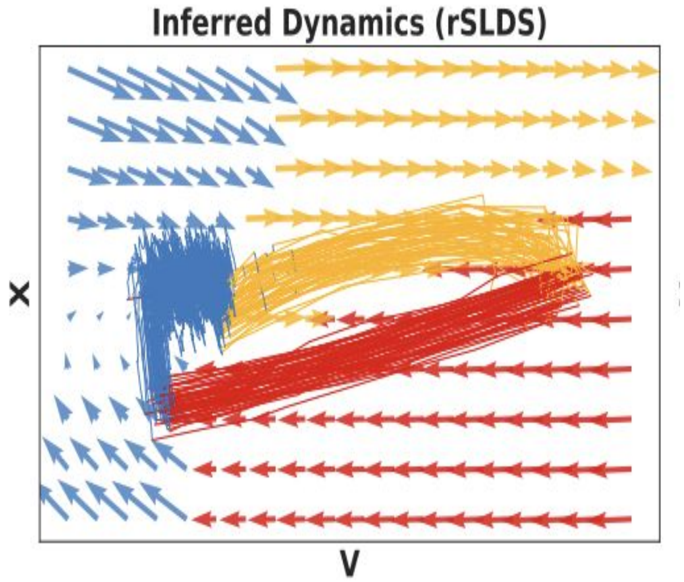
Results: Single Compartment

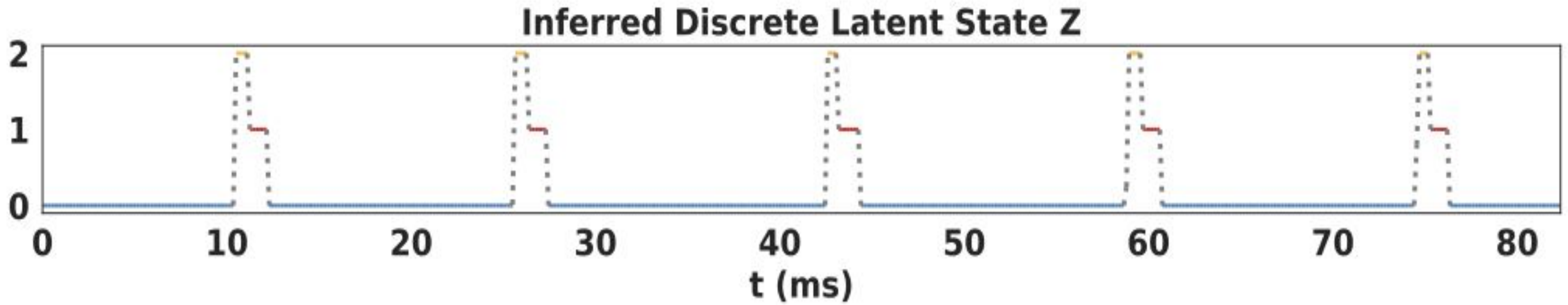
Output of the model for Single Compartment model

- Observed Voltage (y)
- Inferred Continuous Latent State: V (voltage) and X (cycle)

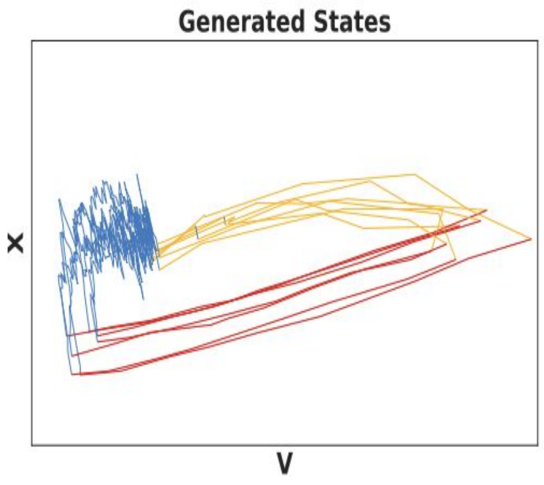


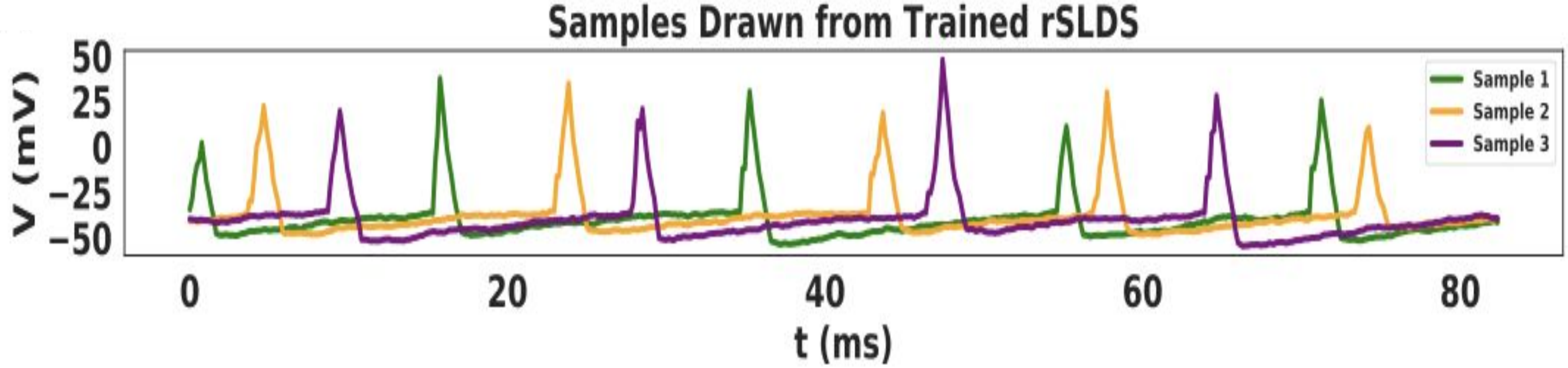
- Inferred Discrete Latent State (Z)





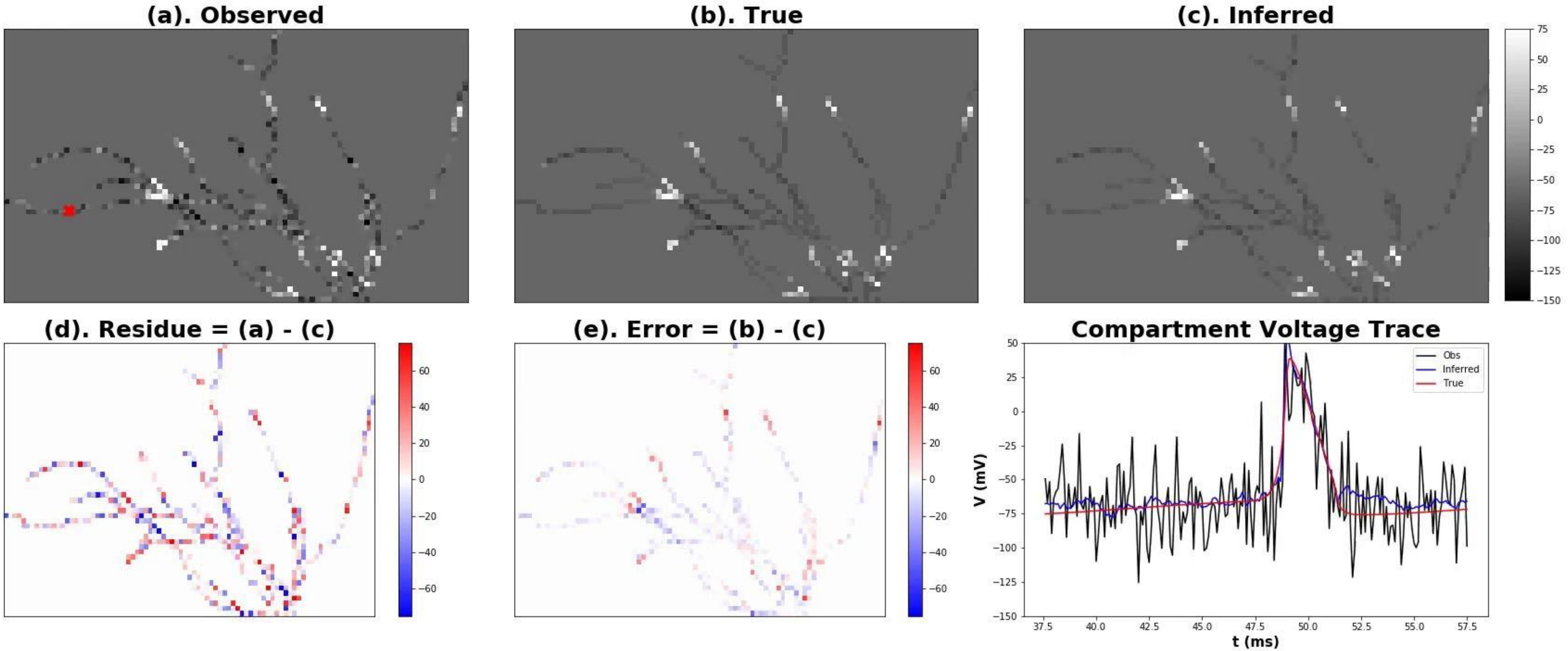
- Generated new spike (voltage)



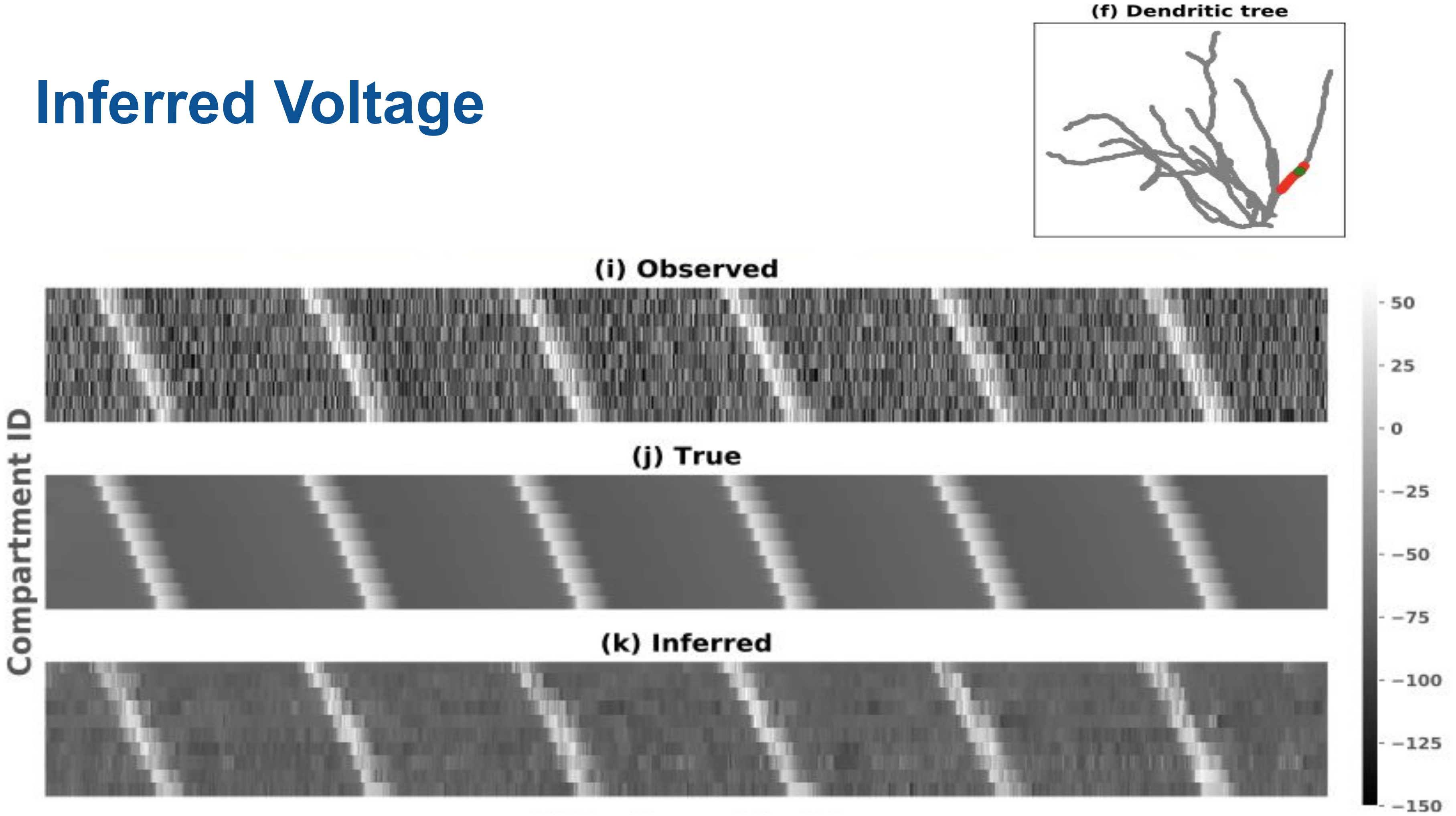


Results: Multiple Compartments

Multiple Compartment denoising



Inferred Voltage



Thank you!

Poster: #147

Code: https://github.com/SunRuoxi/Voltage_Smoothing_with_rSLDS

Previous Biophysical work

- Hodgkin Huxley
- Fitzhugh-Nagumo